Report on scientific activity of
"Differential Equations" department of 2014.

"Differential equations" department includes 14 collaborators, including 4 professors, 3 associate professor, 12 research associate. In 2014 year 7 scientific works were performed on one theme.

Theme: Investigation of boundary value problems for differential equations.

a) "Existence and non-existence of the global solution with non-linear Klein-Gordon system".

Executants: Aliyev A.B., Mammadzade K.S.

According to the plan the Cauchy problem for the following Klein-Gordon system was investigated:

$$u_{itt} - \Delta u_i + u_i + \gamma u_{it} = \sum_{j=1}^{m} \left|u_j\right|^{p_j+1} \left|u_i\right|^{p_i-1} u_i, \quad i = 1,2,...,m \quad , \tag{1}$$

$$u_i(0,x) = u_{i0}(x), \quad u_{it}(0,x) = u_{i1}(x), \quad x \in R^n, \quad \text{i=1,2,...,m,} \tag{2}$$

here (u_1, u_2) are real functions dependent on variables $(t,x) \in R^+ \times R^n$:

$$n \ge 2, \ p_j \ge 0, \ j = 1,..., m, \tag{3}$$

In addition when $n \ge 3$

$$0 < p_j \le \frac{2}{n-2} 0, \ j = 1,..., m \tag{4}$$

In the report year, the non-existence of the global solution for the initial data prossesing positive energy.

Denote the following energy function by $E(t)$

$$E(t) = \sum_{j=1}^{m} \frac{p_j+1}{2} \left[\left|\dot{u}_{jt}(t,\cdot)\right|^2 + \left\|u_j(t,\cdot)\right\|^2 + 2\gamma \int_0^t \left|\dot{u}_{jt}(s,\cdot)\right|^2 ds \right] - \\ - \sum_{i,j=1}^{m} \int_{R^n} \left|u_i(t,x)\right|^{p_i+1} \cdot \left|u_j(t,x)\right|^{p_j+1} dx \ .$$

Here $|.|$ is the norm of the space $L_2(R^n)$, $\|.\|$ is the norm of the Sobolev space $H^1 = W_2^1(R^n)$, i.e. $\|u\| = \left[\|\nabla u|^2 + |u|^2\right]^{1/2}$, $\nabla u = (\frac{\partial u}{\partial x_1},..., \frac{\partial u}{\partial x_n})$ is a gradient.

Let's introduce the following functional:

$$I(\phi_1,....,\phi_m)=\sum_{j=1}^{m}\frac{p_j+1}{\sum_{j=1}^{m}p_j+m}\|\phi_j\|^2-\sum_{i,j=1}^{m}\int_{R^n}|\phi_i|^{p_i+1}\cdot|\phi_j|^{p_j+1}dx$$

The main result consists of the followings.

Theorem 1. Assume that conditions (3), (4) are satisfied, and the $u_{i0}(.)\in H^1,\quad u_{i1}(.)\in L_2(R^n)\,,\,i=1,2,....,m$. Assume that the following conditions also satisfied

$$E(0)>0\,, \qquad (5)$$

$$I(u_{10},...,u_{m0})<0\,, \qquad (6)$$

$$\sum_{j=1}^{m}\langle u_{j0},u_{j1}\rangle>0\,, \qquad (7)$$

$$\sum_{j=1}^{m}\frac{p_j+1}{2}|u_{j0}|^2>\frac{\sum_{j=1}^{m}p_j+m}{\sum_{j=1}^{m}p_j}E(0)\quad . \qquad (8)$$

Then the solution of the Cauchy problem fails at finite time.

Furthermore, the following problems were considered.

1)
$$\left.\begin{aligned} u_{1tt}+u_{1t}+(-1)^{l_1}\Delta^{l_1}u_1&=f_1(u_1,u_2)\\ u_{2tt}+u_{2t}+(-1)^{l_2}\Delta^{l_2}u_2&=f_2(u_1,u_2)\end{aligned}\right\} \qquad (1)$$

$$\left.\begin{aligned} u_1(0,x)=\varphi_1(x),\quad u_{1_t}(0,x)=\psi_1(x),\\ u_2(0,x)=\varepsilon\varphi_2(x),\quad u_{2_t}(0,x)=\varepsilon\psi_2(x)\end{aligned}\right\}\,,\ x\in R^N\,, \qquad (2)$$

here $\varepsilon\in R^N$ such growth conditions on non-linear functions $f_i:R^2\to R,\,,i=1,2$ were found that under these conditions for any initial data

$(\varphi_i,\psi_i)\in(H^{l_i}(R^N)\cap L_{m_i}(R^N))\times(L_2(R^N)\cap L_{m_i}(R^N)),i=1,2 \quad m_i\in[1,2]$

the problem given in sufficient values of $\varepsilon\in R^N$ has a global solution.

2) The following cauchy problem was studied

$$\left.\begin{aligned} u_{1tt}+u_{1t}+\Delta_{I_1}^2 u_1-\Delta_{J_1}u_1=\sum_{k=1}^{l_1} f_{1k}(u_1,u_2) \\ u_{2tt}+u_{2t}+\Delta_{I_2}^2 u_2-\Delta_{J_2}u_2=\sum_{k=1}^{l_2} f_{2k}(u_1,u_2) \end{aligned}\right\}, \tag{1}$$

$$u_i(0,x)=\varphi_i(x),\ u_{it}(0,x)=\psi_i(x),\ x\in R_N,\ i=1,2\ , \tag{2}$$

here $\Delta_{I_i}=\sum_{s\in I_i}\frac{\partial^2}{\partial x_s^2},\ \Delta_{J_i}=\sum_{s\in J_i}\frac{\partial^2}{\partial x_s^2},\quad I_i\subset N_n=\{1,\ldots,n\},\ \ J_i=N_n\setminus I_i,\ i=1,2$. Such growth conditions on the functions $f_{ik}:R^2\to R, k=1,2,\ldots, l_i\ , i=1,2$ were found that under these conditions the given problem has global solutions.

b) "On the spectrum and trace of a boundary value problem with a spectral parameter contained in boundary conditions".

Executants: Bayramoglu M.B., Aslanova N.M.

In the work the spectrum and trace of differential operators with unbounded operator coefficient is studied.

1) In the space $L_2(H,(0,\pi))$ the problem

$$y^{IV}+Ay+p(x)y=\lambda y$$

$$y(0)=0, y'(\pi)+hy(\pi)=0,\ y''(0)=0,\ y'''(\pi)+hy''(\pi)=0,\ h>0$$

is considered. Here A is a definite-continuous acting on separable Hilbert space H, p(x) are bounded self-adjoint operators acting in H. By $\varphi_1,\varphi_2,\ldots$, we denote eighen functions, and $\gamma_1\le\gamma_2\le$ ... eighen numbers of the operator A. Assume that

$$\gamma_k\sim rk^{\alpha}, r>0, \alpha>\frac{4}{3}. \tag{1}$$

p(x) is a weak measurable acting in H for each x and satisfies the following conditions:

1) p(x) has the second weak derivative and $[p^{(l)}(x)]^*=p^{(l)}(x),\qquad \|p^{(l)}(x)\|<const$
2) $\sum_{j=1}^{\infty}|(p^{(l)}(x)\varphi_j,\varphi_j)|<const, l=\overline{0,2}$
3) $p'(0)=p'(\pi)=0$
4) $\int_0^{\pi}(p(x)f,f)dx=0,\ \forall f\in H.$

For $p(x) = 0$, the operator L_0 is determined in the space $L_2(H,(0,\pi))$:

$$D(L_0) = \left\{\frac{y(x)}{□} y^{IV} + Ay \in L_2(H,(0,\pi)), y(0) = 0, y'(\pi) + hy(\pi) = 0, y''(0) = 0, y'''(\pi) + hy''(\pi) = 0\right\}$$

$L_0 y = y^{IV} + Ay$.

we denote the operator corresponding to the case $p(x) \neq 0$ by : $L = L_0 + p$. The member of the work is the calculation of the first regularized trace of the operator L. We prove the following theorem.

Theorem. In the eigen numbers of the operator A satisfy condition (1), subject to the conditions 1)-4) for operator-valued function p(x), the following formula is true:

$$\lim_{n_m \to \infty} \sum_{n=1}^{n_m} (\lambda_n - \mu_n) = \sum_{k=1}^{\infty} \frac{p_k(\pi) - p_k(0)}{4}$$

This work was published in “Ukrainian Mathematical Journal”.

2) In the space $L_2(H,(-\infty,\infty))$ the problem ly≡ $-y''(x) + |x|y(x) + Ay + q(x)y(x) = \lambda y(x)$ is considered. Here A is a positive-definite self-adjoint operator acting on A separable Hilbert space H. We denote by $\varphi_1, \varphi_2, \dots$ the eigen functions and by $\gamma_1 \leq \gamma_2 \leq$ ... the eigen numbers of the operator A. Assume that $\gamma_k \sim rk^{\alpha}, r > 0$, $\alpha > 0$. q(x) satisfies the following conditions:

1) $\sum_{k=1}^{\infty} \int_{-\infty}^{\infty} |(q(x)\varphi_k, \varphi_k)| dx < const.$

2) $\frac{q_k(x)}{x}$ $(q_k(x) = (q(x)\varphi_k, \varphi_k))$ is summable on the interval $(-\infty, \infty)$ and $\int_{-\infty}^{0} \frac{q_k(x)}{x} dx = 0$, $\int_{0}^{\infty} \frac{q_k(x)}{x} dx = 0,$ $\forall k = \overline{1, \infty}$

3) $\int_{-\delta}^{\delta} \left|\frac{q_k(x)}{x^5}\right| dx < \infty,$ is a rather small number.

In the space $L_2(H,(-\infty,0))$ and $L_2(H,(0,\infty))$ we define the operators L_1 and L_2:

$D(L_1 1) = \{y \in L_1 2\ (H,(0,\infty))/l_1 1\ y \equiv -y''(x) + xy(x) + Ay + q(x)y(x) \in L_1 2\ (H,(0,\infty)), y(0) = 0\}$

$L_1(y) = -l_1 y$,

$D(L_1 2) = \{y \in L_1 2\ (H,(-\infty,0))/l_1 2\ y \equiv -y''(x) - xy(x) + Ay + q(x)y(x) \in L_1 2\ (H,(-\infty,0)), y'(0) = 0\}$

$L_2(y) = -l_2 y$.

The operators corresponding to the case $q(x) \equiv 0$ are denoted by L_1^0, L_2^0. This operators are positive definite self-adjoint operators with discrete spectrum. We define the operator $L_0 \coloneqq L_1^0 \oplus L_2^0$. The spectrum L_0 is unification of the operators L_1^0, L_2^0. We also define the operator L=$L_1 \oplus L_2$ the asymptotics of the spectrum of the operator L is studied, regularized trace formula is obtained. Denote the eigen numbers of the operator L by $\lambda_1, \lambda_2, \dots, L_0$ and the eigen numbers of the operator L_0 by $\mu_1, \mu_2, \dots$.

We prove the following main theorems.

Theorem 1. The following formula is true for the eigen numbers of the operator L:

$$\lambda_n \sim dn^{\frac{2\alpha}{2+3\alpha}}$$

Theorem 2. Subject to conditions 1)-3)

$$\sum_{n=1}^{\infty} (\lambda_n - \mu_n) = 0.$$

This work was accepted to be published in the journal "Annalele Stintifice Ale Univeritatii Al Cuza Din İasi Serie Noua-Matematica (Science Citation İndex Expanded)"

3) The boundary value problem containing a spectral parameter in the boundary conditions was considered. The spectrum of this problem was studied. In the next stage it is intended to find the trace of the considered spectral problem.

In the space $L_2(H,(0,1))$ we consider the problem

$$-y'' + Ay + q(x)y = \lambda y$$

$$y(0)=0, (1+\lambda)y(1)=(h+\lambda)y'(1), \ h>-1$$

Here A is a positive-definite self-adjoint operator with computely continous inverse acting in the separable Hilbert space H. q(t) is a bounded, self-adjoint operator acting in H. In $L_2 -$ də $q(t) \equiv 0$ is determined by self-adjoint operator L_0, and perturbed L operator.

In the work the spectrum of the operator L studied and regularized trace is calculated. Taking into account iterative order of eigen numbers of the operator l and L_0 they can be arranged in the form $\lambda_1 \le \lambda_2 \le \ldots$, $\mu_1 \le \mu_2 \le \ldots$, respectively we prove the following main theorems.

Theorem 1. The eigen numbers of the operator L generate two different segures:

$\lambda_k = \gamma_k + O(1), \lambda_{k,n} = \gamma_k + \alpha_n^2$, $\quad v\text{ə } kifay\text{ə}t\ q\text{ə}d\text{ə}r\ b\bar{o}y\bar{u}k\ n - l\text{ə}r$ üçün $\alpha_n \sim \frac{\pi}{2} + \pi n$

Theorem 2. The formula

$$\lim_{m\to\infty} \sum_{n=1}^{n_m} (\lambda_n - \mu_n) = \frac{trq(1) - trq(0)}{4}$$

Is true for the eigen numbers of the operator L.

This work was published in the Proceedings of the International Conference devoted to 55 years of IMM. (Baku, may 15-16, 2014).

c) "A priori estimation for generate elliptic-parabolic equations".

Executants: Gadjiev T.S., Aliyev O.S.

In the report year, a priori estimates for degenerate elliptic-parabolic equations were obtained. These works are the generalizations of the works O.A.Oleynik, F.Fikera, M.Keldysh in some sense.

d) The fredholm property of operator boundary condition boundary value problems for fourth order elliptic type differential operator equations".

Executants: Aliyev B.A.

It is considered a boundary value problem with boundary conditions containing unbounded operator. The Fredholm property of the considered problem was shown and the obtained result was applied to partial differential equations of elliptic type.

e) "Solvability of a boundary value problem for a class of differential operator equation with variable operator coefficient".

Executants: M.Balayev M.Q.

In the report year was studied differential operators with variable and non-limited differential coefficients and was studied correct solutions to the problem of differential-operator equations of arbitrary order.

f) Uniqueness, local and global solvability of the classic solution of one-dimensional mixed problem.

Executant: A.G.Aliyeva.

Some a priori estimations for almost everywhere solutions of the following one-mimensional mixed problem were obtained

$$\begin{cases} \{u_{txx}(t,x) - \alpha u_{xxxx}(t,x) = F(t,x,u(t,x),u_x(t,x),u_{xx}(t,x),u_{xxx}(t,x)), \ (0 \le t \le T, \ 0 \le x \le \pi) \\ u(0,x) = \varphi(x), \ (0 \le x \le \pi) \\ u(t,0) = u(t,\pi) = u_{xx}(t,0) = u_{xx}(t,\pi) = 0 \ \ (0 \le t \le T), \end{cases}$$

where $\alpha > 0$ is fixed number, $(0 < T < +\infty)$, F, φ - are given functions, $u(t,x)$ - a sought function.

g) "Investigation of optimal control problem for motion of sources for a wave equation".

Executant: R.A. Teymurov.

The optimal control of moving sources in control systems whose state is characterized by a wave equation.

The work, added to the plan of 2014:

h) "Solution of two-point boundary value problem for fractional order impulse differential equations"

Executant: M.J.Mardanov.

In the work the two-point boundary value problem for fractional order impulse differential equations was researched. At first an expression for the representation of the solution is found. Using the contracting mapping principle and the Shauder principle, sufficient conditions for the existence of the problem solution is found.

Participation at scientific seminars:

Each Tuesday, at 12.00 the seminar “Modern problems of theory of differential equations” is held under supervision of A.B.Aliyev.

All the collabrators of the seminar take part at the institute seminar.

A.B.Aliyev, T.S.Gadjiev, Nadir Suleymanov are the members of the specialized scientific seminars. The collaborator of the department prof. N.Suleymanov takes part at the scientific seminar under supervision of acad. Akif Gadjiev held at the Institute at each Tuesday and has given a talk at his seminar.

Grant projects.

A.B.Aliyev, N.M.Aslanova and K.S.Mamedzade have successfully completed the grant project of SOCAR “Dynamics of propagation of hydrocarbon contaminants in water” on September 30, 2014.

Prof. T.Gadjiev gained the grant of SOCAR “Computer program system for processing of elaboration map and optimal arrangement of new operation walls”.

R.Teymurov’s work “Complex investigation of optimal control of interlayer combustion processes in oil production” was the winner of the competititon of SOCAR on scientific research works for 2014.

Scientific social activity.

A.B.Aliyev has done some works in the editorial staff of the journals “Proceedings of IMM of ANAS” and “Azerbaijan Methematical Journal” and reviewed some works sent by the International journals. A.B.Aliev and T.S.Gadjiev are the members of the Defnce Council of the Institute. M.Mayramoglu is a member of the expert commission of HCC, he is also a member of the editorial board of the journal “Balkan Journal of Mathematics”. In 2014, the collaborators of the department B.A.Aliyev, N.M.Aslanova gained Doctor’s degree on Mathematics, R.Teymurov and Sh.Muradova the PhD degree.

Acad. Of RAN V.I.Il'in has written a review to Nadir Suleymanov's monograph in the journal "Turkic World Mathem. Society" published in the MSU publishing house.

A.B.Aliyev has been opponent of 1 doctor of science and 2 philosophy doctors.

The collaborator of the department O.S.Aliyev has submitted his PhD dissertation to Defense Council (supervisor T.S.Gadjiev).

Participation at Conferences.

All the collaborators of the Institute have given talks at the International Conference "Actual problems of mathematics and Mechanic's" devoted to 55 year of IMM of NAS of Azerbaijan, held on May 15-16, 2014.

The collaborators of the department prof. A.B.Aliyev, prof. M.J.Mardanov, prof. T.S Gadjiev have participated and given talks at the International Conference "Caucasian mathematics Conference CMCI" held in Georgian Republic on September, 2014.

R.Teymurov has participated at the following conferences:

1) Optimal control of system with the distributed parameters of hyperbolic type / V Congress of the Turkic World Mathematicians (TWMS), Issyk-Kul, Kyrgyzstan, 5-7 june, 2014.

2) Принцип максимума в одной задаче оптимального управления подвижными источниками / Четвертая Международная конференция «Математическая физика и ее приложение» (г.Самара, Россия, 25 августа - 01 сентября 2014 г.). Самарский Государственный Технический Университет,2014.

3)Оптимальное подвижное управление распределенными системами, описываемыми линейным параболическим уравнением / VII Международная конференция имени академика И.И.Ляшко «Вычислительная и прикладная математика» (Киев, 9-10 октября 2014 г.). – Киевский Национальный Университет им. Тараса Шевченко, 2014.

4)Задача оптимального управления движением источников для систем с распределенными параметрами / Международная научная конференция «Теоретические и прикладные аспекты математики, информатики и образования» (г.Архангельск, Россия, 16-21 ноября 2014г.). – Северный Арктический Федеральный Университет им. М.Ю.Ломоносова, 2014.

In 2014, 42 scientific works of the collaborators were published (including 16 papers). 5 of these papers have been published in the journals included in Thompson list; 26 of them abstracts and conference materials.

The most important results of the department.

Bayramoglu M.B., Aslanova N.M. “On spectrum and trace of a boundary value problem with a spectral parameter in the boundary conditions”.

B.A.Aliyev. “Fredholm property of operator boundary condition boundary value problem for elliptic type fourth order equations”.

The papers published in Thompson Reuters List Journals.

1. Misir J. Mardanov, Yagub A. Sharifov, Habib H. Molaei "Existence and uniqueness of solutions for first-order nonlinear differential equations with two-point and integral boundary conditions", ELECTRONIC JOURNAL OF DIFFERENTIAL EQUATIONS (EJDE),Vol. 2014 (2014), No. 259, pp. 1-8.

2. M.J.Mardanov, N.I.Mahmudov, Y.A.Sharifov. "Existence and uniqueness theorems for impulsive fractional differential equations with the two-point and integral boundary conditions". The Scientific World Journal, v. 2014, Article ID 918730,8 pages, 2014. doi:10.1155/2014/918730.

3. Алиев А.Б., Казимов А.А. Существование и несуществование глобальных решений задачи Коши для систем Клейна- Гордона. Доклады Академии Наук (Росии), 2014, т.259, № 2, с.1-3.

4. T.S.Hajiyev. The removability of compact of solutions in classes bounded functions. – Ukr. Math. Journal, 2014, vol.8, pp.38-44.

5. Б.А. Алиев, Я.С. Якубов. Фредгольмовость краевых задач для эллиптического дифференциально-операторного уравнения четвертого порядка с операторными граничными условиями. Диф.урав., 2014 том 50, № 2, с.210-216.

6. Б.А. Алиев, Я.С. Якубов. Разрешимость краевых задач для эллиптических дифференциально-операторных уравнений второго порядка со спектральным параметром и с разрывным коэффициентом при старшей производной. Диф.урав., 2014 том 50, № 4, с.468-479.

7. Н.М. Асланова, М.Байрамоглы. Об обобщенном регулязированном следе дифференциального оператора четвертого порядка с операторным коэффициентом. Укр. Мат. Журнал, 2014, том 66, № 1, сс. 128-134

Head of department **prof. A.B.Aliyev**