

**REPORT OF “MATHEMATICAL ANALYSIS” DEPARTMENT  
OF IMM ANAS FOR THE YEAR OF 2016  
SCIENTIFIC PART.**

The following research works were done:

**Subject:** Modern problems of harmonic analysis.

The following works were considered:

**Work 1:** Investigating multilinear rough kernel fractional integral in generalized Morrey type spaces.

**Executor:** Head of department, corr. member of ANAS, prof. V.S. Guliyev

**Work 2:** Global regularity of non divergent elliptic equations with VMO coefficients in generalized weighted Morrey spaces.

**Executor:** Corr. member of ANAS, prof. V.S. Guliyev, PhD in math. s. r. a. M.N. Omarova

**Work 3:** Two weight inequalities for  $B_{k,n}$  sublinear potential type operators.

**Executor:** r.a. F.A. Isayev, cand. ph. m. s. E.A. Hajiyeva

**Work 4:** Boundedness of  $(\Phi, \Psi)$  potential type sublinear operators and their commutators in Orlic-Morrey spaces.

**Executor:** cand. ph. m. s. lead. r. a. J.J. Hasanov, r.a. A.N. Mammadova.

**Work 5:** Riesz-Kolmogorov type compactness theorems in variable degree Lebesgue spaces determined in metric space.

**Executor:** doct. of math. sc. ass. prof., lead. r. a. R.A. Bandaliyev, r.a. A.E. Abdullayeva

**Work 6:** Boundedness of rough kernel multilinear fractional integral operators in generalized Morrey spaces.

**Executor:** cand. ph. m. s. lead. r. a., Z.V. Safarov

**Work 7:** Jacobian transformation and its application to approximation theory.

**Executor:** cand. ph. m. s., lead. r. a. E.J. Ibrahimov

**Work 8:** Continuity of logarithmic potentials.

**Executor:** Phd in math., j. r. a. L. Aliyeva.

2 of 16 department employees are doctor of sciences including one corr. member of ANAS, 7 cand. of ph. math. sci. and PhD on mathematics.

According to the plan, 8 scientific research works are executed on 1 subject.

**Work 1:** Investigating fractional integral with multilinear rough kernel in generalized Morrey type spaces.

**Executor:** Corr. member of ANAS, prof. V.S. Guliyev

Assume that the function  $\Omega \in L_q(S^{n-1})$ ,  $q > 1$  is a zero order homogeneous function and its mean value on  $S^{n-1}$  unit sphere equals zero. In the paper, a rough kernel  $T_\Omega$  singular integral operator and the boundedness of its  $[b, T_\Omega]$  commutator in generalized weight Morrey space  $M_{p,\varphi}(w)$  is considered. So, under the conditions  $q' \leq p < \infty$ ,  $p \neq 1$  and  $w \in A_{p'/q'}$ , or  $1 < p < q$  and  $w^{1-p'} \in A_{p'/q'}$  sufficient conditions on the pair  $(\varphi_1, \varphi_2)$  for bounded action of the operator  $T_\Omega$  from the generalized Morrey space  $M_{p,\varphi_1}(w)$  to the space  $M_{p,\varphi_2}(w)$  was found, for  $1 < p < \infty$ . For  $b \in BMO(\mathbb{R}^n)$  and  $q' \leq p < \infty$ ,  $p \neq 1$ ,  $w \in A_{p'/q'}$  or under conditions  $1 < p < q$ ,  $w^{1-p'} \in A_{p'/q'}$  necessary condition on the pair  $(\varphi_1, \varphi_2)$  for bounded action of the operator  $[b, T_\Omega]$  from the space  $M_{p,\varphi_1}(w)$  to the space  $M_{p,\varphi_2}(w)$  was found for  $1 < p < \infty$ . The obtained results were published in “Mathematical Inequalities and Applications” journal in the form of papers.

The obtained results were published in the following authoritative foreign journals.

1. C. Aykol, V.S. Guliyev, A. Kucukaslan, A. Serbetci, The boundedness of Hilbert transform in the local Morrey-Lorentz spaces, **Integral Transforms and Special Functions**, 27 (2016), no. 4, 318-330. (Impact Factor-0.723) (TR)
2. V.S. Guliyev, M.N. Omarova, Parabolic oblique derivative problem with discontinuous coefficients in generalized weighted Morrey spaces, **Open Mathematics**, 14 (2016), 49-61. (Impact Factor 0.578) (TR)
3. V.S. Guliyev, M.N. Omarova, Corrigendum to : Parabolic oblique derivative problem with discontinuous coefficients in generalized weighted Morrey spaces, **Open Mathematics**, 14 (2016), 283-285. (Impact factor 0.578) (TR)

4. V.S. Guliyev, F. Deringoz, J.J. Hasanov,  $(\Phi, \Psi)$ -admissible potential operators and their commutators on vanishing Orlicz-Morrey spaces, **Collectanea Mathematica**, 67 (2016), no. 1, 133-153. (Impact Factor 0.843) (TR)
5. V.S. Guliyev, S. Samko, Maximal operator in variable exponent generalized Morrey spaces on quasi-metric measure space, **Mediterranean Journal of Mathematics**, 13 (2016), 1151-1165. (impact factor 0.656) (TR)
6. V.S. Guliyev, Sh. Muradova, M.N. Omarova, L. Softova, Generalized weighted Morrey estimates for the gradient of divergence form parabolic operators with discontinuous coefficients, **Acta Mathematica Sinica**, 32 (8), 2016, 911–924. (impact factor 0.386) (TR)
7. R.A. Bandaliyev, V.S. Guliyev, I.G. Mamedov, A.B. Sadigov, The optimal control problem in the processes described by the Goursat problem for a hyperbolic equation in variable exponent Sobolev spaces with dominating mixed derivatives, **Journal of Computational and Applied Mathematics**, 305 (2016), 11-17. (Impact Factor 1.266) (TR)
8. V.S. Guliyev, V.H. Hamzayev, Rough singular integral operators and its commutators on generalized weighted Morrey spaces, **Mathematical Inequalities and Applications**, 19 (3) (2016), 863–881 (Impact Factor 0.645) (TR)
9. V.S. Guliyev, S.G. Hasanov, Y. Sawano, T. Noi, Nonsmooth atomic decompositions for generalized Orlicz-Morrey spaces of the third kind, **Acta Applicandae Mathematicae**, 145 (1) (2016), 133-174 (Impact Factor 0.853) (TR)
10. V.S. Guliyev, C. Aykol, A. Kucukaslan, A. Serbetci, *Maximal operator and Calderon-Zygmund operators in local Morrey-Lorentz spaces*, **Integral Transforms and Special Functions**, 27 (11) (2016), 866-877. (Impact Factor-0.723) (TR)
11. V.S. Guliyev, F. Deringoz, S.G. Hasanov, *Characterizations for the Riesz potential and its commutators on generalized Orlicz-Morrey spaces*, **Journal of Inequalities and Applications**, (2016) 2016:248 (Impact factor 0.630) (TR)

12. V.S. Guliyev, Ali Akbulut, V. H. Hamzayev, Okan Kuzu, *Commutators of marcinkiewicz integrals associated with Schr'odinger operator on generalized weighted Morrey spaces*, **Journal of Mathematical Inequalities**, 10(4) (2016), 947–970 (Impact factor 0.632) (TR)
13. V.S. Guliyev, Generalized Morrey regularity for parabolic equations with discontinuous data, 2nd International Conference on Analysis and its Applications, July 12-15, 2016 Kırşehir/Turkey, pp. 33
14. V.S. Guliyev, M.N. Omarova, Parabolic oblique derivative problem with discontinuous coefficients in generalized weighted Morrey spaces, 2nd International Conference on Analysis and its Applications, July 12-15, 2016 Kırşehir/Turkey, pp. 306
15. V.S. Guliyev, Y. Zeren, H. Armudcu, Boundedness of the Calderon-Zygmund singular integral operator and its commutators on modified Morrey spaces, 2nd International Conference on Analysis and its Applications, July 12-15, 2016 Kırşehir/Turkey, pp. 145
16. V.S. Guliyev, Characterizations for the Riesz potential and its commutators on generalized Orlicz-Morrey spaces, Intern. Workshop on "Non-harmonic Analysis and Differential Operators", May 25-27, 2016, Baku, Azerbaijan, p. 38.

- On July 09-July 18, 2016 was invited to International Conference of Analysis and its Applications (ICAA) (Kırşehir) and gave the talk “Generalized Morrey regularity for parabolic equations with discontinuous data”.

- On 18-22 December, 2016 was invited to International Conference “Sobolev Reading” as an invited lecturer. He will give a talk on “Characterizations for the integral operators of harmonic analysis in generalized Orlicz-Morrey spaces on Carnot groups”.

- Research works on the project "Modern problems and applications of harmonic analysis in local Morrey type spaces" on scientific-research programs of ANAS (two year, 2015-2017) are continued. Within the framework of the project, prof. V.S. Guliyev was on a professional trip at Ankara University and during this visit he carried out research works on "Modern problems and applications of

harmonic analysis in local Morrey type spaces" and completed the papers " Maximal operator and Calderon-Zygmund operators in local Morrey-Lorentz spaces", "Boundedness of the Riesz potential on the local Morrey-Lorentz spaces and some applications". During this visit prof. Vagif Guliyev in collaboration with research associate of the Institute of Mathematics of the Academy of sciences of Czech republic Dr. Amiran Gogatashvili, prof. Ayhan Sherbetchi and others conducted a workshop.

- The one year grand project of the Science Development Foundation "Boundedness of integral operators of real analysis in some function spaces and applications" was completed with successes.

- He is a member of the Presidium of the Higher Certificate Commission at the President of the Republic of Azerbaijan.

- He is a member of the international journal "Eurasian mathematical Journal", "Fractional Differential Calculus" and the Republican journal of "Proceedings of Institute of Mathematics and Mechanics of NAS of Azerbaijan". He is one of the editor-in-chief of the journal "Caspian journal of applied mathematics, ecology and economics". He is the editor-in chiefs of "Transactions of Azerbaijan National Academy of Sciences", Issue Mathematics, Series of physical-technical and mathematics science".

- The number of citations to his papers in MathSciNet Mathematical reviews is 487 (349); in Google Scholar 1701 (1412).

- At BSU substitutes the post of professor at "Mathematical analysis" faculty and gives lectures on harmonic analysis, selected problems of mathematical analysis.

- "Vol. 36, V1, 2016 of "Transactions of Azerbaijan National Academy of Science, Issue Mathematics, Series of physical-technical and mathematics science" was published and vol.36, No4, 2016 is on print.

V.S. Guliyev has written reviews to the papers from foreign journals:

1 paper **Journal of Mathematical Analysis and Applications** (Impact Factor –1,014), 1 paper **Journal of Fourier Analysis and Applications**, 1 paper **Mathematische Nachrichten** (Impact Factor –0,910), 1 paper **Journal of**

**Mathematical Inequalities** (Impact Factor –0,632), 1 paper **Integral Transforms And Special Functions** (Impact Factor – 0.814), 1 paper **Palestine Journal of Mathematics**, 2 paper **Journal of Function Spaces** (Impact Factor – 0.656); 1 paper **Communications in Mathematical Analysis** (Impact Factor –0,370), 1 paper **Complex Variables and Elliptic Equations** (Impact Factor – 0.650); 1 paper **Journal of Inequalities and Applications** (Impact Factor –0,630), 2 paper **Collectanea Mathematica** (Impact Factor –0,593), 2 məqalə **Fractional Differential Calculus**; 1 paper **Journal of Fourier Analysis and Applications** (Impact Factor –0,912), 1 paper **Ukrain Mathematical Journals** (Impact Factor – 0,189), 1 paper **FILOMAT** (Impact Factor –0,638), 1 paper **Turkish Journal of Mathematics** (Impact Factor –0,311), 1 paper **Facta Universitatis (NIS) Ser. Math. Inform**;

**Work 2:** Global regularity of VMO coefficients non divergent elliptic equations in generalized weighted Morrey spaces.

Executor: Corr. member of ANAS, prof. V.S. Guliyev, PhD in math. s. r. a. M.N. Omarova.

In the paper, global regularity of solutions of VMO coefficient non-divergent elliptic equations was studied. It was shown that when the function given in the right hand side of the equation is from the Morrey space  $M_{2,1}^{p,\varphi}(Q, \omega)$  the solution is from the generalized weighted Sobolev-Morrey space  $M^{p,\varphi}(Q, \omega)$ .

The results obtained by M.N. Omarova are in the following papers:

1. V.S. Guliyev, M.N. Omarova, *Parabolic oblique derivative problem with discontinuous coefficients in generalized weighted Morrey spaces*, **Open Mathematics**, 14 (2016), 49-61. (Impact Factor 0.578)
2. V.S. Guliyev, M.N. Omarova, *Corrigendum to: Parabolic oblique derivative problem with discontinuous coefficients in generalized weighted Morrey spaces*, **Open Mathematics**, 14 (2016), 283–285. (Impact Factor 0.578)
3. V.S. Guliyev, Sh. Muradova, M.N. Omarova, L. Softova, *Generalized weighted Morrey estimates for the gradient of divergence form parabolic*

operators with discontinuous coefficients, **Acta Mathematica Sinica**, 32 (8), 2016, 911–924. (Impact Factor 0.386)

4. A. Eroglu, A.A. Hasanov, M.N. Omarova, *(p,q)-admissible multilinear fractional integral operators and their commutators in product generalized local Morrey spaces*, Trans. of NAS of Azerb., Issue Mathematics, 36 (4) (2016), 77-98.
5. C. Aykol, H. Armutcu, M.N. Omarova, *Maximal commutator and commutator of maximal function on modified Morrey spaces*, Trans. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci. 36 (2016), no. 1, Mathematics, 29-35.
6. V.S. Guliyev, M.N. Omarova, *Parabolic oblique derivative problem with discontinuous coefficients in generalized weighted Morrey spaces*, 2nd International Conference on Analysis and its Applications, July 12-15, 2016 Kırşehir/Turkey, pp. 306

- M.N. Omarova regularly participates at the institute seminars and the department seminar “Actual problems of harmonic analysis” she regularly participates at training seminar held every Tuesday at the department.

- She is one of the main technicians of "Transactions of Azerbaijan National Academy of Sciences, Issue of Mathematics, Series of physical-technical and mathematics sciences".

**Work 3:** Two weight inequalities for  $B_{k,n}$  sublinear potential type operator.

Executor: r.a. F.A. Isayev, cand. ph. m. s. r. a., E.A. Hajiyeva

In the paper, strong and weak boundedness of potential type operators generated by  $(p, q)$  admissible  $B_{k,n} = \sum_{i=1}^n \frac{\partial^2}{\partial x_j^2} + \sum_{j=1}^k \frac{\gamma_j}{x_j} \frac{\partial}{\partial x_j}$  Laplace-Bessel differential operator in  $((p, q)$  admissible  $B_{k,n}$ -potential integral operator) weighted  $L_{p,\omega,\gamma}(R_{k,+}^n)$  spaces was studied. The considered class of sublinear operators contain some operators of harmonic analysis as  $B_{k,n}$ -fractional maximal operators,  $B_{k,n}$ -potential integral operator. Thus, necessary conditions for bounded  $(p, q)$  type action of  $(p, q)$

admissible  $B_{k,n}$  potential integral operators from  $L_{p,\omega,\gamma}(R_{k,+}^n)$  for  $1 < p < q < \infty$  of  $L_{p,\omega_1,\gamma}(R_{k,+}^n)$  and weak  $(p,q)$  admissible  $B_{k,n}$  -potential integral operators from  $L_{p,\omega,\gamma}(R_{k,+}^n)$  to  $WL_{p,\omega_1,\gamma}(R_{k,+}^n)$  were found for  $(\omega, \omega_1)$  weighted functions.

The obtained results found their reflection in the following theorems.

**Theorem 1.** Let,  $1 < p < q < \infty$  and  $T_{\alpha,\gamma}, 0 < \alpha < n + |\gamma|$  be a  $(p,q)$  admissible  $B_{k,n}$  -potential operator.

Moreover, let  $\omega(x'), \omega_1(x')$  be weighted functions on and the following three conditions be satisfied:

(a) there exists  $b > 0$  such that

$$\sup_{|x'|/8 < |y'| < 8|x'|} (\omega_1(y'))^{1/q} \leq b \omega(x')^{1/p} \quad \text{for a.e. } x' \in R_{++}^k,$$

(b)<sub>1</sub>

$$\begin{aligned} A_1 \equiv & \sup_{r>0} \left( \int_{E'(0,2r)} \omega_1(x') |x'|^{(n-k)(1+q/p') - (n+|\gamma|-\alpha)q} (x')^\gamma dx' \right)^{1/q} \\ & \times \left( \int_{E'(0,r)} \omega^{1-p'}(x') (x')^\gamma dx' \right)^{1/p'} < \infty, \end{aligned}$$

(c)<sub>1</sub>

$$\begin{aligned} B_1 \equiv & \sup_{r>0} \left( \int_{E'(0,r)} \omega_1(x') (x')^\gamma dx' \right)^{1/q} \\ & \times \left( \int_{E'(0,2r)} \omega^{1-p'}(x') |x'|^{-((n-k)(1/q+1/p') - n - |\gamma| + \alpha)(1-p')} (x')^\gamma dx' \right)^{1/p'} < \infty. \end{aligned}$$

Then there exists a constant  $c$ , independent of  $f$ , such that for all  $f \in L_{p,\omega}(R_{k,+}^n)$

$$\left( \int_{R_{k,+}^n} |T_\alpha f(x)|^q \omega_1(x') (x')^\gamma dx \right)^{1/q} \leq c \left( \int_{R_{k,+}^n} |f(x)|^p \omega(x') (x')^\gamma dx \right)^{1/p}.$$

Moreover, condition (a) can be replaced by the condition

(a')<sub>1</sub> there exists  $b > 0$  such that



$$\omega_1(x')^{1/q} \left( \sup_{|x'|/8 < |y'| < 8|x'|} \frac{1}{(\omega(y'))^{1/p}} \right) \leq b \quad \text{for a.e. } x' \in R_{++}^k .$$

**Theorem 2.** Let,  $1 \leq p < q < \infty$  and let  $T_{\alpha,\gamma}, 0 < \alpha < n + |\gamma|$  be a weak  $(p, q)$  admissible  $B_{k,n}$ -potential operator. Moreover, let  $\omega(x), \omega_1(x')$  be weight functions on  $R_{k,+}^k$  and conditions  $(a)_1, (b)_1, (c)_1$  be satisfied.

Then there exists a constant  $c$ , independent of  $f$ , such that for all  $f \in L_{p,\omega,\gamma}(R_{k,+}^n)$

$$\left( \int_{\{x \in R_{k,+}^n : |T_{\alpha,\gamma} f(x)| > \lambda\}} \omega_1(x')(x')^\gamma dx \right)^{1/q} \leq \frac{c}{\lambda^q} \left( \int_{R_{k,+}^n} |f(x)|^p \omega(x')(x')^\gamma dx \right)^{1/p} .$$

The obtained results were published in the following journals.

1. E.A. Gadjieva, F.A. İsayev, A. Kucukaslan, *Two-weighted inequality for  $(p, q)$ -admissible  $B_{k,n}$ -potential operators in weighted Lebesgue spaces*, Trans. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci. 36 (2016), no. 1, Mathematics, 36-53.
2. S.K. Abdullayev, E.A. Gadjieva, F.A. İsayev, *Two-weighted inequality for  $(p, q)$ -admissible  $B_{k,n}$ -potential operators in weighted Lebesgue spaces*, **Fractional Differential Calculus**, Volume 6, Number 2 (2016), 233–248
3. E.A. Gadjieva, A.A. Hasanov, Z.V. Safarov, *Multilinear fractional integral operators with rough kernel on modified Morrey spaces*, Proc. IMM of NAS of Azerb, 42, issue 1, 2016, 93-105.
4. F.A. İsayev, *Two-weighted inequality for  $(p, q)$ -admissible  $B_{k,n}$ -potential operators in weighted Lebesgue spaces*, International Workshop on Non-Harmonic Analysis and Differential Operators, Baku 2016, pp. 1

- r.a. F. İsayev has given a talk on “Two-weighted inequality for  $(p, q)$ -admissible  $B_{k,n}$ -potential operators in weighted Lebesgue spaces” at the International workshop held by IMM ANAS "Nonharmonic analysis and differential operators".

- Within the framework of scientific research program "Modern problems and applications of harmonic analysis in local Morrey type spaces" r.a. F. İsayev was on a professional trip at Ankara University, Turkey and together with Dr. A. Kucukaslan

prepared and submitted to print the paper “Two-weighted inequality for  $(p,q)$ -admissible  $B_{k,n}$ -potential operators in weighted Lebesgue spaces”.

- F. Isayev by the Higher Attestation Commission under the President of the Republic of Azerbaijan received the degree of Doctor of Philosophy in Mathematics.

**Work 4:** Boundedness of  $(\Phi, \Psi)$  potential type sublinear operators and their commutators in Orlicz-Morrey spaces.

Executor: cand. ph. m. s. lead. r. a. J.J. Hasanov, r.a. A.N. Mammadova.

The boundedness of potential type sublinear operator and its commutator in generalized vanishing Orlicz-Morrey spaces was considered. Necessary and sufficient condition for bounded action of commutator operator to generalized vanishing Orlicz-Morrey space were found and the obtained results were published in the form of an article in *Collectanea Mathematica* journal.

A.N. Mammadova has continued her research work connected with her dissertation work "Approximation theorems by the generalized Szasz operator" and published one abstract. Now she prepares the paper " Theorem on Approximation by two-variable functions for the generalized Szasz operator".

The obtained results were published in the following paper and abstracts.

1. V.S. Guliyev, F. Deringoz, J.J. Hasanov,  $(\Phi, \Psi)$ -admissible potential operators and their commutators on vanishing Orlicz-Morrey spaces, **Collectanea Mathematica**, 67 (2016), no. 1, 133-153. (Impact Factor 0.843)
2. R. Ayazoglu, J.J. Hasanov, On the boundedness of a B-Riesz potential in the generalized weighted  $B$ -Morrey spaces, **Georgian Math. J.** 23 (2016). no. 2, 143–155. (Impact Factor 0.417) **(TR)**
3. J.J. Hasanov, R. Ayazoglu, L.R. Aliyeva, Construction of Green function for Bessel-Helmholtz equation, **International Journal of Applied Mathematics** 29 (2016), no. 5, 509-522. (Impact Factor 0.100)
4. A. Akbulut, X.A. Badalov, J.J. Hasanov, A. Serbetci,  $p(x)$ -admissible sublinear singular operators in the generalized variable exponent Morrey spaces, *Trans.*

- Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci. 36 (2016), no. 1, Mathematics, 10-17.
5. J.J. Hasanov, L.R. Aliyeva, *Necessary and sufficient conditions for the boundedness of commutators of B-Riesz potentials on  $L_{p,\gamma}$  spaces*, Journal of Contemporary Applied Mathematics-ISSN: 2222-5498, Vol 6, № 1, p.18-41 (2016)
  6. J.J. Hasanov, L.R. Aliyeva, *On the boundedness of fractional B– maximal operator in the generalized weighted B-Morrey spaces*, The Republican Conference urgent problems of mathematics and mechanics devoted to 93 years of general leader HaydarAliyev, Baku, 2016, pp.66-68.
  7. L.R. Aliyeva, X.A. Badalov, J.J. Hasanov,  *$p(x)$ -admissible sublinear singular operators in the generalized variable exponent Morrey spaces*, International Workshop on Non-Harmonic Analysis and Differential Operators, Baku, 25-27 May 2016, pp. 19.
  8. A.E. Abdullayeva, A.N. Mammadova, *Moment operators Szasz and Bernstein-Chlodovsky*, International Workshop on Non-Harmonic Analysis and Differential Operators, Baku 2016, p.1.

- The leader res. ass. of the department J. Hasanov is the scientific secretary of the subject seminar at the doctor dissertation Council of IMM and the institute seminar.

- Citations in MathSciNet Mathematical reviews 99 (86); in Google Scholar 239 (196).

- Doctor's degree dissertation of J. Hasanov was initially discussed and his documents were sent to Higher Certificate Commission.

- A.N. Mammadov is already completing the dissertation work for earning the philosophy doctor degree.

- A.N. Mammadova gave a talk in "Moment operators of Szasz and Bernstein-Chlodovsky" at the workshop "Nonharmonic analysis and differential operators".

- J. Hasanov has written reviews to the following papers from the foreign journals: 1 paper, Journal of Mathematics and System Science, 1 paper, The

Scientific World Journal, 1 paper, Journal of Inequalities and Applications, 1 paper, Journal of Modern Education Review Academic Star Publishing Company, 1 paper, Integral Transforms and Special Functions, 2 papers, Turkish Journal of Mathematics, 1 paper, Complex Variables and Elliptic Equations, 1 paper, British Journal of Mathematics & Computer Science, 1 paper, Journal of Mathematical Sciences: Advances and Applications, 1 paper, International Journal of Mathematical Analysis, 1 paper, Proceedings of Institute of Mathematics and Mechanics of NAS of Azerbaijan.

**Work 5:** Riesz-Kolmogorov type compactness theorems in variable degree Lebesgue spaces determined in metric space.

Executor: doct. of math. sc. ass. prof., lead. r. a. R.A. Bandaliyev, r.a. A.E. Abdullayeva

Relatively compact sets are studied in variable Lebesgue space given in measurable metric space. This time Kolmogorov-Riesz type theorem for perfect characterization of relations compact sets compact sets is proved. The advantage of the obtained result is that the variable exponent determining the variable space is considered in any positive value. On the other hand, the object that replaces the classic shift operator in variable exponent space is replaced by the mean value of the function.

The paper “Everything you need to know about relatively compact sets in variable Lebesgue spaces” was prepared to be published and at present is in review in impact-factor journal. It should be noted that the mentioned results were obtained in collaboration with professor of Warsaw Technological University Przemyslaw Gorka.

In the case weighted exponent Lebesgue space, the compactness criteria of Kolmogorov-Riesz type was obtained in collaboration with research ass. of the department A.E. Abdullayeva.

A.E. Abdullayeva continues her works on approximation of Bernshtein-Chlodovsky polynomials. The has prepared the paper "Asymptotic estimation of Bernshtein-Chlodovsky polynomials".

Assume that  $(\Omega, \mu)$  -  $\sigma$  is a finite completely measurable space and  $p : \Omega \rightarrow [1, \bar{p}]$   $\mu$  is measurable bounded function. The  $p$  function is a variable exponent and  $\bar{p} = \operatorname{ess\,sup}_{x \in \Omega} p(x) < \infty$ . The denote the class of variable class by  $P(\Omega)$ . When we say  $L_{p(x)}(\Omega, \mu)$ -variable exponent Lebesgue space, one understands a  $\mu$ -measurable class of function  $f : \Omega \rightarrow R$  such that

$$\rho_{p(\cdot)}(f) = \int_{\Omega} |f(x)|^{p(x)} d\mu(x) < \infty.$$

In this space the norm is given as

$$\|f\|_{L_{p(\cdot)}(\Omega, \mu)} = \inf \left\{ \lambda > 0 : \rho_{p(\cdot)} \left( \frac{f}{\lambda} \right) \leq 1 \right\}.$$

Note that the variable exponent Lebesgue space is a Banach space and is a special case of Muserlac-Orlicz space.

Assume that  $(X, \rho)$  is a metric space and  $\Omega \subset X$ . If the condition

$$\exists C_1 > 0 \quad \forall x, y \in \Omega \quad |p(x) - p(y)| \leq \frac{C_1}{\ln \left( e + \frac{1}{\rho(x, y)} \right)}$$

is satisfied, we say that the function ,  $p : \Omega \rightarrow R$  satisfies the local logarithmic Holder condition. If the condition

$$\exists p_{\infty} \in R \quad \exists C_2 > 0 \quad \forall x \in \Omega \quad |p(x) - p_{\infty}| \leq \frac{C_2}{\ln(e + \rho(x, x_0))}$$

is satisfied, then the  $p : \Omega \rightarrow R$  function satisfies the logarithmic Holder condition at the fixed  $x_0 \in X$  point at infinity. If the function  $p : \Omega \rightarrow R$  satisfies both the local logarithmic Holder condition and Holder condition at infinity then we say that this function satisfies the global logarithmic Holder condition and is denoted by  $G_{\log}(\Omega)$ .

Then the constant  $C := \max\{C_1, C_2\}$  is called a logarithmic Holder constant with respect to the function  $p$ . Make the following denotations:

$$P_{\log}(\Omega) = \left\{ p \in P(\Omega) : \frac{1}{p} \in G_{\log}(\Omega) \right\}.$$

**Theorem.** Assume that,  $(X, \rho, \mu)$  is a metric space and  $\mu$  is a measure satisfying the double condition. Suppose,  $p \in P_{\log}(X, \mu)$ ,  $0 < \underline{p} \leq \bar{p} < \infty$ . Then necessary and coefficient condition for the family  $\Lambda \subset L_{p(\cdot)}(X, \mu)$  to be completely bounded in the space  $L_{p(\cdot)}(X, \mu)$  is satisfaction of the following conditions:

(a) The family  $\Lambda$  is bounded in the space  $L_{p(\cdot)}(X, \mu)$ , i.e

$$\exists M > 0 \quad \forall f \in \Lambda \quad \int_X |f(x)|^{p(x)} d\mu(x) \leq M;$$

(b)  $\forall 0 < q < \underline{p} \wedge \forall f \in \Lambda$ ,  $l \lim_{r \rightarrow 0} \int_X \left( \frac{1}{|B(x, r)|} \int_{B(x, r)} |f(x) - f(y)|^q d\mu(y) \right)^{\frac{p(x)}{q}} d\mu(x) = 0$ ;

(c) at certain point  $x_0 \in X$

$$\forall f \in \Lambda, \quad l \lim_{R \rightarrow \infty} \int_{X \setminus B(x_0, R)} |f(x)|^{p(x)} d\mu(x) = 0.$$

In the case of weighted and variable exponent Lebesgue space the Kolmogorov. Riesz type compactness criteria was studied together with A.E. Abdullayeva.

A.E. Abdullayeva continues to work on approximation of Bernstein-Khloodovsky polynomials. She has prepared the paper "Asymptotic estimation of Bernstein-Khloodovsky polynomials". In this paper the Bernstein-Khloodovsky polynomial itself and for its generalizations the classic Varanovskaya theorem is proved. The paper will be soon submitted for publication.

As a result of carried out researches two papers and two abstracts were published.

1. Р.А. Бандалиев, Исправление к статьи «Об одном неравенстве в пространстве Лебега со смешанной нормой и с переменным показателем суммируемости, **Матем. заметки**, (99) (2), 2016, 319-320. (Impact Factor 0.425) **(TR)**

2. R.A. Bandaliyev, V. S. Guliyev, I. G. Mamedov and A. B. Sadigov, The optimal control problem in the processes described by the Goursat problem for a hyperbolic equation in variable exponent Sobolev spaces with dominating mixed derivatives, **Journal of Computational and Applied Mathematics**, (305), 2016, 11-17. (Impact Factor 1.266) **(TR)**
3. R.A. Bandaliyev, Compactness criteria in weighted variable Lebesgue spaces, **Miskolc Math. Notes**, (17) (2), 2016 (accepted). (Impact Factor 0.335) **(TR)**
4. R.A. Bandaliyev, Compactness theorem in variable weighted Lebesgue spaces, Abstracts “**Inter. Work. On Non-harmonic analysis and differential operators**”, Baku, 25-27 may, 2016, p. 28.
5. R. A. Bandaliyev and P. Gorka, Hausdorff operator in Lebesgue spaces, VII Inter. Joint Conf. of Georgian Math. Union and Georgian Mech. Union, “**Continuum Mech. and Related Problems of Analysis**”, Batumi, 5-9 September, 2016, p. 93-94.
6. A.E. Abdullayeva, A.N. Mammadova, Moment operators Szasz and Bernstein-Chlodovsky, **International Workshop on Non-Harmonic Analysis and Differential Operators**, Baku 2016., p.1

R.Bandaliyev was invited to international conference “Continuum mechanics and related problems of analysis” held in Sh. Rustaveli Batumi State University on 05.09.2016-09.03.2016 to give a talk on “Boundedness and compactness of Hausdorff operator in weighted and variable Lebesgue spaces”.

Within the framework of scientific research program "Modern problems and applications of harmonic analysis in local Morrey type spaces", a joint research work was conducted by professor of Ankara University of Turkey Ayhan Sherbetci and one paper was prepared. On April 15-27, 2016 at IMM of ANAS together with professor of Warsaw Technological University Przemyslaw Gorka a workshop was held.

- doct. in math. R. Bandaliyev was an opponent of a doctor of sciences dissertation in physics.

- doct. in math. R. Bandaliyev is a member of the editorial board of "Journal of Mathematics Reshearch" and responsible secretary of "Transactions of Azerbaijan National Academy of Sciences", Issue Mathematics, Series of physical-technical and mathematics science.

- Lead. r.a. of the department R. Bandaliyev is a scientific secretary of D01.111 doctor dissertation Council at IMM.

- Citations in MathSciNet Mathematical reviews 26 (15); Google Scholar 92 (86).

- At the Workshop held at IMM "Nonharmonica analysis and differential operatrs" R.A. Bandaliyev gave a talk on "Compactness theorem in variable weighted Lebesgue spaces", A.E.Abdullayeva on "Moment operators Szasz and Bernstein-Chlodovsky".

A.E. Abdullayev completes the dissertation work for earning the degree of phil. doctor in math.

The reveiws were written to the papers from the following foreign journals: 1 paper, **Linear and Multilinear Algebra**, 1 paper, **Journal of Mathematics research**, 1 paper, **Transactions of Mathematics of National Academy of Sciences of Azerbaijan**, 1 paper, **Proceeding of IMM of NAS of Azerbaijan**.

**Work 6:** Boundedness of rough kernel multilinear fractional integral operators in generalized Morrey spaces.

Executor: cand. ph. m. s. lead. r. a. Z.V. Safarov

The boundedness of rough kernel multilinear fractional integral operators in generalized Morrey spaces is studied. The obtained results were published in the form of a paper.

In 2016 the following papers were published.

1. E.A. Gadjieva, A.A. Hasanov and Z.V. Safarov, Multilinear fractional integral operators with rough kernel on modified Morrey spaces, Proc. IMM of NAS of Azerb., **42**, Issue 1, 2016, 93-105.



2. A. Akbulut, V.H. Hamzayev, Z.V. Safarov, Rough Fractional Multilinear Integral Operators on Generalized Weighted Morrey Space, Azerbaijan Journal of Mathematics, **6** (2), 2016, 128-142.
3. A.A. Hasanov, Z.V. Safarov, Multilinear fractional integral operators with rough kernel on modified Morrey spaces, Abstracts “Inter. Work. On Non-harmonic analysis and differential operators”, Baku, 25-27 may, 2016, p.43.

- He is the technical editor of the journal Transactions of NAS of Azerbaijan (issue mathematics), Transactions of NAS of Azerbaijan (issue mechanics), Proceedings of the institute of mathematics and mechanics.

**Work 7:** Jacobian transformation and its application to approximation theory.

Executor: Cand. ph. m. s., lead. r. a. E.J. Ibrahimov

The work on Jacobian transformation and its application to approximation theory was executed. In the work the order of convergence to zero of Fourier coefficients in arbitrary orthogonal system is given. The exact convergence order of Fourier series was found by means of the obtained results.

The obtained results were reflected in the following papers

1. E.İ. İbrahimov, A. Akbulut, The Hardy-Littlewood-Sobolev theorem for Riezs potential generated by Gegenbauer operator, Transactions of A. Razmadze Mathematical Institute, 170 (2016), no. 2, 166–199.
2. E.I.Ibrahimov, О сходимост рядов Фурье-Якоби в среднем, Владикавказский математический журнал, 18 (3), (2016), 43-60

- Riyaziyyat üzrə elmlər doktoru dissertasiya işinin ilkin müzakirəsi keçirilmiş və sənədlər Ali Attestasiya Komissiyasına göndərilmişdir.

**Work 8:** Continuity of logarithmic potentials.

Executor: Cand. ph. m. s., j. r. a. L. Aliyeva.

Studied references on continuity of logarithmic potentials and found sufficient conditions for the existence and continuity of generalized logarithmic potentials in nonhomogeneous spaces

The following paper and abstracts were published.

1. A. Eroglu, M.G. Hajibayov, A. Serbetci, Two weighted inequalities for B-fractional integrals, **J. Inequal. Appl.** 2016, 2016:168. (Impact Factor 0.630) (TR)
2. H. Aslanov, R. Yüzbəyov, L. Əliyeva, A. Həsənova, İnteqral çevirmələri nəzəriyyəsinin elementləri, Dərs vəsaiti, Bakı 2016, 161 s.
3. J.J. Hasanov, R. Ayazoglu, L.R. Aliyeva, *Construction of Green function for Bessel-Helmholtz equation*, **International Journal of Applied Mathematics** 29 (2016), no. 5, 509-522. (Impact Factor 0.100)
4. J.J. Hasanov, L.R. Aliyeva, *Necessary and sufficient conditions for the boundedness of commutators of B-Riesz potentials on  $L_{p,\gamma}$  spaces*, **Journal of Contemporary Applied Mathematics**-ISSN: 2222-5498, Vol 6, № 1, p.18-41 (2016)
5. J.J. Hasanov and L.R. Aliyeva, On the boundedness of fractional B– maximal operator in the generalized weighted B-Morrey spaces, Intern. Conf. devoted to 93 years of the national leader Haydar Aliyev, Baku-2016, pp. 66-68.
6. L.R. Aliyeva, X.A. Badalov and J.J. Hasanov,  $p(x)$ -admissible sublinear singular operators in the generalized variable exponent Morrey spaces, International Workshop on Non-Harmonic Analysis and Differential Operators, Baku, 25-27 May 2016, pp. 19.
7. M.G. Hacibayov, Weighted inequalities for B-fractional integrals, Abstracts “Inter. Work. On Non-harmonic analysis and differential operators”, Baku, 25-27 may, 2016, p.41.
8. A.Eroglu, M.G. Hacibayov, Two weighted inequalities for fractional integrals associated with the Laplace-Bessel differential operator, 2nd International Conference on Analysis and its Applications, July 12-15, 2016 Kırşehir/Turkey, pp. 49

- On May 25-27, 2016 has given a talk on “ $p(x)$ -admissible sublinear singular operators in the generalized variable exponent Morrey spaces” at workshop

“Nonharmonic analysis and differential operators”. Ümuminstitut və şöbənin seminarlarında mütəmadi olaraq iştirak etmişdir.

- Regularly continue at the institute seminars and the department seminar “Actual problems of harmonic analysis” at training seminar held every Tuesday at the department.

- 41 scientific papers 13 abstracts of department collaborators were published, including 28 scientific papers 16 of them were published in Thomson Reuters journals. 16 papers were accepted for publication. 15 paper are in print.

- Citation to department’s collaborators over 400.

- Ass. prof. E. Ibrahimov, ass. prof. M.G. Hajibeyov and ass. prof. C. Hasanov’s dissertation were for earning the doctor of sciences degree in mathematics were discussed and the documents were sent to Higher Certificate Commission.

- On 25.11.2016 candidate for a degree K.R. Rahimova (supervisor prof. V.S. Guliyev) will defend the dissertation work “Boundedness of a clan of anisotropic sublinear operators in Morrey type spaces” for earning phil. Doctor degree in math.

- This year, the department collaborators M.G. Hajibeyov and R.Ch. Mustafayev have given talks at the institute seminars.

**Head of the department**  
**“Mathematical Analysis”**

**Corr. member**  
**of NASA, prof.V.S. GULIYEV**