

ANNUAL REPORT ON SCIENTIFIC, AND SCIENTIFIC ORGANIZATIONAL ACTIVITY OF THE DEPARTMENT OF "EQUATIONS OF MATHEMATICAL PHYSICS" OF INSTITUTE OF MATHEMATICS AND MECHANICS OF NAS OF AZERBAIJAN FOR THE YEAR OF 2018

In the department of "Equations of Mathematical Physics" 13 workers, 11 of whom are research workers. Of them 3 **doctors of sciences, professors, including one member of ANAS:**

1. Huseynov Rauf V. – head of department, chief research associate, corr-member of ANAS, (full time).
2. Akhundov Adalat Ya. – chief research associate, (a part time).
3. Mammadov Farman I. – chief research associate, (a part time).

6 doctors of philosophy in mathematics:

4. Guliyev Abdurrahim F. – leading research associate, (full time).
5. Bagirov Shirmail H. – leading research associate, assistant professor, (a part time).
6. Aliyev Mushfig J. – leading research associate, assistant professor, (full time).
7. Mammadov Elchin M. – senior research associate, (full time).
8. Shukurova Shahla Yu. – senior research associate, (full time).
9. Hasanova Aynur H. – senior research associate, (full time).

1 kandidat for a degru:

10. Mammadli Sayali M. – junior research associate, (full time).

3 laboratory assistants:

11. Mustafayeva Lala M. – laboratory assistant, (full time).
12. Abdullayeva Aydan J. – laboratory assistant, (a part time).
13. Shafiyeva Aynur F. – laboratory assistant, (a part time).

I. SCIENTIFIC PART.

In 2018, according to the approved plan, the department conducts 10 research works on the topic "Unambiguous solutions of problems of mathematical physics and qualitative properties of solutions."

Work 1: "Investigation of the negative spectrum of quasi-elliptic operators".

Executor: corr-member of ANAS, prof. R.V. Huseynov.

During the reporting period, the spectrum for elliptic and some quasi-elliptic equations of high order was investigated. In particular, analogues of the stationary Schrödinger operator of high order are studied. In this case, a negative spectrum was considered for different types of differential operator and a given potential $Q(x)$. We investigated what necessary conditions must be imposed on the potential, that the negative spectrum be finite and infinite.

Work 2: "An approximate solution of an inverse problem for a semilinear elliptic equation".

Executor: prof. A.Ya. Akhundov.

The inverse problem of determining the unknown coefficients on the right-hand side of the system of elliptic equations is investigated. The problem is solved by the method of successive approximations, the convergence of the approximate solution to the exact solution is proved, with the speed of the geometric series, the theorem on the existence, uniqueness, and stability of the solution is proved.

Published papers:

1. Adalat Ya. Akhundov, Aynur Hasanova. [On the existence of a solution of the inverse problem for a system of parabolic equations](#). *Proceedings of the Institute of Mathematics and Mechanics, National Academy of Sciences of Azerbaijan*, 44 (1) (2018), pp. 81-89.
2. Axundov Ə., Paşayev N. Bir tərs məsələnin ümumiləşmiş həllinin varlığı haqqında. *Republican Scientific and Practical Conference on the theme "The unity of science, education and production at the stage of modern development", dedicated to the 95th anniversary of the birth of national leader Heydar Aliyev, Lankaran State University, May 7-8, 2018.*
3. Ахундов А., Селимханов Б. Определение коэффициентов в правой части системы эллиптических уравнений. *Republican Scientific and Practical Conference on the theme "The unity of science, education and production at the stage of modern development", dedicated to the 95th anniversary of the birth of national leader Heydar Aliyev, Lankaran State University, May 7-8, 2018.*

Work 3: "Integral inequalities of Hardy-Sobolev-Poincaré type and their application. Qualitative properties of nonuniformly degenerate elliptic and parabolic partial differential equations".

Executor: prof. F.I. Mammadov.

During the reporting period, one class of unevenly degenerate elliptic equations was investigated. Qualitative properties of the solutions of these equations are studied. An a priori estimate of the Holder norm for these solutions is proved.

Published papers:

1. Farman Mamedov, Yashar Shukurov. [A Sawyer-type sufficient condition for the weighted Poincaré inequality](#). *Positivity*, 2018, v. 22, № 3, pp. 687–699.
2. Farman Mamedov, Yusuf Zeren and Sayali Mammadli “On solvability of some nonlinear eigenvalue problem in variable exponent Lebesgue spaces”. *International Conference on Mathematical Advances and Applications (ICOMAA), May 11-13, 2018, Istanbul /TURKEY*, p. 21.
3. F.I. Mamedov and S. Mammadli, “A variable exponent Hardy’s inequality approach for some nonlinear eigenvalue problem.” *Caspian Journal of Applied Mathematics, Economics and Ecology*, 2018, v 6, no 1, pp. 17-36.

Work 4: ”Some qualitative properties of solutions of second-order parabolic equations of a non-divergent structure”.

Executor: A.F. Guliyev.

For the Weiestrass-type nucleus in the paraboloid domain, where the parabolic equations have super-solutions, two-sided equivalent estimates were obtained using the value of the nucleus at the pole. Applying the obtained results, a growth theorem is obtained for solutions of second order parabolic equations. The results obtained are of great importance in the study of the qualitative properties of solutions.

Published papers:

1. Abdurrahim Guliyev and Mushfiq Aliyev, The Estimates of Functions Type of Weirstrass Kernel in Special Domains. *International Conference on Mathematical Advances and Applications (ICOMAA) Istanbul/TURKEY, 11-13 May, 2018, p.26.*
2. Elif Deniz, Yusuf Zeren, Abdurrahim Guliyev and Selim Yavuz, An increase theorem for positive solutions of the parabolic equation second order. *International Conference on Mathematical Advances and Applications (ICOMAA), Istanbul/TURKEY, 11-13 May, 2018, p. 43.*
3. Guliyev A.F. The broth Teorem for Positive Solitions of the Heat. *International Scientific Conference on Operators, Functions and Systems of Mathematical Physics, dedicated to the 70th anniversary of Hamlet Isakhanly, Khazar University, Baku, May 21-24, 2018, pp. 90-91.*
4. Гулиев А.Ф. (в соавторстве) «Математика» 11-й класс. *Учебник, 2018, 320 с.*
5. Гулиев А.Ф. (в соавторстве) ГЭЦ, «Математика» 10-й класс. *Сборник тестовых заданий, 2018, 282 с.*
6. Гулиев А.Ф. (в соавторстве) ГЭЦ, «Математика» 11-й класс. *Сборник тестовых заданий, 2018.*
7. Гулиев А.Ф. (в соавторстве) ГЭЦ, «Математика». *Сборник тестовых заданий, 2018, 302 с.*

Work 5: ”Elliptic and parabolic equations with a singular potential and the absence of a global solution of a system of semilinear equations in the exterior domain”.

Executor: ass. prof. Sh.H. Bagirov.

The problem of the existence in the outer domain of positive global solutions of semi-linear elliptic and parabolic equations is studied. The problem of the absence of positive global solutions of a semi-linear elliptic equation with a singular potential and a biharmonic operator in the principal part in the outer domain of the ball is studied and in the case of imposing certain conditions on the solutions on the boundary of the ball, a sufficient condition is found to ensure the absence of a solution. It was shown that if the order of semi-linearity in a power form is greater than one and less than the critical value depending on the dimension of space, then there is no positive global solution. The example showed that the found critical value is accurate, i.e. if the value of semi-linearity is more critical, then we can find an equation in which it will have a positive global solution. A system of weakly coupled semi-linear elliptic equations with a singular potential and a biharmonic operator in the main part was also considered and an exact estimate was found for the absence of a positive global solution to this system. Further, the absence in a cylindrical domain, the basis of which is the outer part of the ball, of positive global solutions of second-order semi-linear parabolic equations and a system of semi-linear parabolic equations with periodic coefficients with respect to time was studied, and in this case, exact sufficient conditions were found under which global solutions do not exist. And in these cases it was shown by example that the obtained sufficient conditions are exact.

Published papers:

1. Ш.Г. Багыров. Отсутствие решений полулинейного бигармонического уравнения с сингулярным потенциалом. *Математические заметки*, 103 (1) (2018), с. 27-37.
2. Shirmayil G. Bagirov. [On Non-Existence of Positive Periodic Solution for Second Order Semilinear Parabolic Equation](#). *Azerbaijan Journal of Mathematics*, 8 (2) (2018), pp. 163-180.
3. Shirmayil Bagirov. Nonexistence of global solutions to the system of semilinear parabolic equations with biharmonic operator and singular potential. *Electronic Journal of Differential Equations*, 2018 (9) (2018), pp. 1-13.
4. Ш.Г. Багыров, Об отсутствии неотрицательных глобальных решений полулинейного параболического уравнения второго порядка с сингулярным потенциалом. *Известия Бакинского университета, серия физико-математических наук*, 2018, №1, с. 70-78.
5. Багиров Ш.Г., Алиев М.Д., Отсутствие глобальных положительных решений слабо связанных систем полулинейных параболических уравнений с периодическими коэффициентами по времени. *International Scientific Conference on Operators, Functions and Systems of Mathematical Physics, dedicated to the 70th anniversary of Hamlet Isakhanly, Khazar University, Baku, May 21-24, 2018*, pp. 194-195.

Work 6: "The boundedness of the Hardy operator in Lebesgue spaces with variable exponent, its application to the qualitative properties of elliptic and parabolic equations".

Executor: ass. prof. M.J. Aliyev.

During the reporting period, some a priori estimates were obtained for linear elliptic equations with discontinuous second-order coefficients, and the boundedness of the Hardy operator in Lebesgue space with a variable exponent was studied, and results were obtained on the boundedness of the maximum Hardy-Littlewood operator in Lebesgue space with a variable exponent. In addition, the existence in the outer domain of global solutions of semilinear parabolic and elliptic equations was investigated. The absence in the cylindrical domain, the base of which is the outer part of the ball, of positive global solutions of a system of second-order semilinear parabolic equations with periodic coefficients over time was studied and exact sufficient conditions were found under which global solutions do not exist. By the example it was shown that the conditions found are exact.

Published papers:

1. Abdurrahim Guliyev and Mushfiq Aliyev, The Estimates of Functions Type of Weirstrass Kernel in Special Domains. *International Conference on Mathematical Advances and Applications (ICOMAA) Istanbul/TURKEY, 11-13 May, 2018, p.26.*
2. Багиров Ш.Г., Алиев М.Д., Отсутствие глобальных положительных решений слабо связанных систем полулинейных параболических уравнений с периодическими коэффициентами по времени. *International Scientific Conference on Operators, Functions and Systems of Mathematical Physics, dedicated to the 70th anniversary of Hamlet Isakhanly, Khazar University, Baku, May 21-24, 2018, pp. 194-195.*

Work 7: "Investigation of the qualitative properties of solutions of a class of nonlinear equations of pseudo-hyperbolic type".

Executor: E.M. Mammadov.

During the reporting period, for the wave equation with strong dissipation and with a nonlinear boundary condition, the problem of stabilization with respect to t was investigated.

The following problem is considered:

$$u_{tt} - \alpha \Delta u - \beta \Delta u_t + f(u) = 0, \quad (x, t) \in \Omega \times [0, T], \quad (1)$$

$$u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x), \quad x \in \Omega, \quad (2)$$

$$\alpha \frac{\partial u}{\partial n} + \beta \frac{\partial u_t}{\partial n} = g(u), \quad (x, t) \in \partial \Omega \times [0, T]. \quad (3)$$

Here, $\Omega \in R^n$, $n \geq 2$, is a bounded domain with a sufficiently smooth boundary $\partial \Omega$, $u_0(x) \in W_2^1(\Omega)$, $u_1(x) \in L_2(\Omega)$ are given functions, $f(u)$ and $g(u)$ are nonlinear functions, α, β are positive numbers, $\frac{\partial}{\partial n}$ is the outward normal to the surface.

For problem (1) - (3) the following theorem is proved:

Theorem. Let the following conditions be fulfilled:

$$F(u) = \int_0^u f(s)ds, \quad f(0) = 0$$

$$G(u) = \int_0^u g(\tau)d\tau, \quad g(0) = 0. \quad (4)$$

For any $u \in R^1$

$$uf(u) - F(u) \geq 0, \quad ug(u) - G(u) \geq 0. \quad (5)$$

If for function $g(\tau)$

$$G(u) \geq ku^2, \quad \forall u \in R^1, k \geq 2,$$

then the solution $u(x,t)$ stabilizes in t , more precisely, the following relation is true:

$$\|u(x,t)\|_{W_2^1(\Omega)} \rightarrow 0, \quad t \rightarrow \infty.$$

Published papers:

Мамедов Э.М. О поведении решений смешанной задачи для нелинейного волнового уравнения с сильной диссипацией. *Republican Scientific Conference on "Actual problems of mathematics and mechanics", dedicated to the 95th anniversary of the birth of national leader Heydar Aliyev, Baku State University, Baku, May 17-18, 2018, pp. 160-162.*

Work 8: "Investigation of the solution of the boundary value problem for a parabolic equation in a symmetric-geometric domain".

Executor: Sh.Yu. Shukurova.

During the reporting period, the following problem was considered:

$$\sum_{i,j=1}^n a_{ij}(x) u_{x_i x_j} + g(x,u) = f(x), \quad x \in \Omega, \quad (1)$$

$$u_{/\partial\Omega} = 0. \quad (2)$$

Here, the coefficients $a_{ij}(x)$, $i, j = 1, 2, \dots, n$ satisfy the conditions of uniform ellipticity and Cordes, $g(x,u)$ is a Caratheodory function satisfying the following condition: $|g(x)| \leq b_0|u|^q$, $b_0 > 0$.

The following theorem is proved:

Theorem. Let $n > 4$, $1 < q < \frac{n}{n-4}$ and satisfy the above conditions imposed on the coefficients, then there exists a sufficiently small $C > 0$ such that for any $f \in L_2(\Omega)$ there is a solution to the problem (1) - (2) from the class $W_2^2(\Omega)$ and

$$\|f\|_{L_2(\Omega)} \leq C(mes_n \Omega)^{-\frac{n+(n-4)q}{2n(q-1)}}.$$

Work 9: "An approximate solution of an inverse problem for a semilinear parabolic equation".

Executor: A.H. Hasanova.

The work is devoted to the study of an approximate solution of the inverse problem of determining unknown coefficients in the right-hand side of a system of parabolic equations.

In the work for the system of parabolic equations of the reaction-diffusion type, the following problem was considered on determining the unknown coefficients $\{f_k(t), u_k(x, t), k = \overline{1, m}\}$:

$$u_{kt} - \Delta u_k = f_k(t)g_k(x, t, u), \quad (x, t) \in \Omega, \quad (1)$$

$$u_k(x, 0) = \varphi_k(x), \quad x \in \overline{D} = D \cup \partial D, \quad (2)$$

$$\frac{\partial u_k}{\partial N} + b_k(t)u_k = \psi_k(x, t), \quad (x, t) \in S, \quad (3)$$

$$\int_D u_k(x, t)dx = q_k(t), \quad t \in [0, T]. \quad (4)$$

Let the input data of the considered problem with $k = \overline{1, m}$ satisfy the following conditions:

$$1^0. \quad g_k(x, t, p) \in C^{\alpha, \alpha/2}(\overline{\Omega} \times R^m);$$

$$2^0. \quad \varphi_k(x) \in C^{2+\alpha}(\overline{D});$$

$$3^0. \quad \psi_k(x, t) \in C^{\alpha, \alpha/2}(S), \quad b_k(t) \in C^\alpha[0, T];$$

$$4^0. \quad q_k(t) \in C^{1+\alpha}[0, T].$$

If conditions 1⁰–4⁰ are satisfied, then the integral (generalized) solution of problem (1) – (3) can be written as follows:

$$u_k(x, t) = \varphi_k(x) + \int_0^t \int_D \Gamma(x, t; \xi, \tau) [f_k(\tau)g_k(\xi, \tau, u) + \Delta \varphi_k(\xi)] d\xi d\tau + \quad (5)$$

$$+ \int_0^t \int_{\partial D} \Gamma(x, t; \xi, \tau) \rho_k(\xi, \tau) d\xi d\tau, \quad (x, t) \in \Omega,$$

$$f_k(t) = \left[q_{kt}(t) - \int_{\partial D} \psi_k(x, t) dx + b_k(t) \int_{\partial D} u_k(x, t) dx \right] \times \quad (6)$$

$$\times \left(\int_D g_k(x, t, u) dx \right)^{-1}, \quad t \in [0, T], \quad k = \overline{1, m}.$$

The following theorem is proved:

Theorem. Let conditions 1⁰–4⁰ be fulfilled. Then there is such $T^*(0 < T^* \leq T)$ that for $(x, t) \in \overline{D} \times [0, T^*]$ the system of integral equations (5) – (6) has the solution and $f_k(t) \in C[0, T^*]$, $u_k(x, t) \in C(\overline{D} \times [0, T^*])$.

Published papers:

1. Adalat Ya. Akhundov, Aynur Hasanova. [On the existence of a solution of the inverse problem for a system of parabolic equations.](#) *Proceedings of the Institute of Mathematics and Mechanics, National Academy of Sciences of Azerbaijan*, 44 (1) (2018), pp. 81-89.
2. A.H. Hasanova. Existence of a generalized solution of the inverse problem for a system of parabolic equations. *The 5th International Conference "Functional Spaces. Differential operators. Problems of Mathematical Education" dedicated to the 95th anniversary of the birth of n.-k. RAS, Acad. European Academy of Sciences Kudryavtseva, Moscow, 2018, pp. 219-221.*
3. M.Ə. Şahverdiyev, A.H. Həsənova, L.R. Əliyeva, S.Ə. Həsənova. Riyazi analiz. Sıralar nəzəriyyəsinin elementləri. Учебное пособие (часть III), Баку, 2018, 144 с.

Work 10: "Integral inequalities of Hardy-Sobolev-Poincaré type and their application."

Executor: S.M. Mammadli.

During the reporting period, using the Poincare-Sobolev-type inequalities obtained for differential equations with a variable exponent, the existence and uniqueness of the solution of the Dirichlet problem were studied.

Published papers:

1. Farman Mamedov, Yusuf Zeren and Sayali Mammadli, "On solvability of some nonlinear eigenvalue problem in variable exponent Lebesgue spaces". *International Conference on Mathematical Advances and Applications (ICOMAA), May 11-13, 2018, Istanbul /TURKEY, p. 21.*
2. F.I. Mamedov and S. Mammadli, "A variable exponent Hardy's inequality approach for some nonlinear eigenvalue problem." *Caspian Journal of Applied Mathematics, Economics and Ecology, 2018, v 6, no 1, pp. 17-36.*

II. ORGANIZATIONAL ACTIVITY.

Thad of the department, corr. member of ANAS, prof. Rauf Huseynov is a member of Scientific Council, Dissertation Board and a member of editorial staff of scientific journals «TRANSACTIONS» and «PROCEEDINGS» of ANAS. Professor Rauf Huseynov lectures to the masters of the Institute of Mathematics and Mechanics on the subject "Modern problems of mathematics".

Cheif research associate of the department prof. Adalat Akhundov is a member of Scientific Council, vice-chairman of Dissertation Board, a member of the editorial staff of the journal «PROCEEDINGS» of Baku University for girls. Professor Adalat Akhundov

lectures to the masters of the Institute of Mathematics and Mechanics in the specialties "Differential Equations" and "Equations of Mathematical Physics", works as a professor at Lankaran State University.

Chief research associate of the department prof. Farman Mammadov is a member of the Expert Council of the HAC, a member of editorial board of Azerbaijan and foreign journals, a reviewer of the journal of «Mathematical Reviews of American Mathematical Society», Works as a leading researcher at the State Oil Company of the Republic of Azerbaijan (SOCAR).

The leading researcher of the department, associate professor Shirmail Bagirov lectures to the masters of the Institute of Mathematics and Mechanics on the subject "Nonlinear differential equations", works as an associate professor at Baku State University.

Leading research associates of the department Abdurrahim Guliyev – scientific secretary, ass. prof. Shirmail Bagirov and senior research associate Aynur Hasanova are the member of the Scientific Subject Seminar.

The leading researcher of the department Abdurrahim Guliyev works as a teacher at Baku State University and in lyceum No. 1 with a physical and mathematical bias.

The leading researcher of the department Mushfig Aliyev works as an associate professor at Baku State University.

The senior researcher of the department Elchin Mammadov is a member of the commission for control of the IMM trade union organization, works as a teacher at Baku State University.

A senior research of the department Aynur Hasanova fellow is a member of a working group set up to use the Thomson Reuters Web of Science network and gather information.

Employees of the department RV. Huseynov, A.Ya. Akhundov, F.I. Mammadov, A.F. Guliyev, Sh.H. Bagirov, E.M. Mammadov gave scientific reviews on dissertations, as well as were official opponents of dissertations.

The employee of the department Abdurrahim Guliyev (in co-authorship) became the winner of the tender conducted for the implementation of the draft textbook on mathematics for the 11th grade, compiled on the basis of a new curriculum. Also, Abdurrahim Guliyev, one of the co-authors of the compilations of test tasks in mathematics for the 10th and 11th grades, a textbook on mathematics for the 11th grades of general education schools of the country, compiled for the first time under a new curriculum and is a science editor a textbook on mathematics for the 7th grade. Department employee Aynur Hasanova is the editor of the 11th grade mathematics textbook published in Russian.

Professor Farman Mammadov and Abdurrahim Guliyev, continuing their cooperation with Turkish scientists, expanded their scientific ties. Also, Professor Farman Mammadov continues to cooperate with Italian scientists.

Leading researcher of the department, Associate Professor Sh. Bagirov, in the general seminar of the Institute June 13, 2018 made a report on the topic "Lack of global positive solutions of loosely coupled systems of semilinear parabolic equations with periodic coefficients in time".

On December 12, 2018, at the Institute-wide seminar, Lead Researcher of the Department Abdurrahim Guliyev made a presentation on the topic "Two-sided estimation of the fundamental solution of the heat equation in special domains and its applications".

Every week on Wednesdays, under the guidance of member corr. ANAS, prof. R.W. Huseynov traditionally carries out its work the seminar of the department on the topic "Qualitative properties of differential equations".

KONFERENCES

1. The employees of the department, Professor Farman Mamedov, with a plenary report on the topic "On qualitative properties of some class elliptic equations with nonuniformly degeneration" (<http://icomaa2018.com/invited-speakers/>), and Abdurrahim Guliyev, with plenary report on the topic "The Estimates of Functions Type of Weirstrass Kernel in Special Domains", took part in the International Scientific Conference ICOMAA, held May 11-13, 2018 in Turkey, in the city of Istanbul.
2. The employees of the department Professor Farman Mammadov with a plenary report on the topic "On study of regularity properties for some class elliptic equations with nonuniformly degeneration" (<https://sites.google.com/view/ofsmmp2018/invited-speakers>), Abdurrahim Guliyev, Shirmail Bagirov and Mushfig Aliyev took part in the International Scientific Conference on "Operators, Functions and Systems of Mathematical Physics" dedicated to the 70th anniversary of Hamlet Isakhanli, held May 21-24, 2018 in Baku at the Khazar University.
3. The employee of the department, Professor Adalat Akhundov, took part in the Republican Scientific and Practical Conference on the theme "Unity of science, education and production at the stage of modern development", dedicated to the 95th anniversary of the birth of the national leader Heydar Aliyev, held May 7-8, 2018 in the Lankaran State University.
4. The employee of the department Elchin Mammadov took part in the Republican conference on "Actual problems of mathematics and mechanics" dedicated to the 95th anniversary of the birth of national leader Heydar Aliyev, held May 17-18, 2018 in Baku State University.

Thus, in 2018, employees of the department published 16 scientific papers, of which 7 papers (3 papers abroad included in journals from the Thomson Reuters list), 9 abstracts (4 abroad), 2 textbooks, 3 collections of test items, 12 papers are presented for print.

Head of department:

**corr-member of ANAS,
d.f.-m.s., prof. R.V. Huseynov.**