Annual report of "Applied Mathematics" department of IMM

for 2019

The staff of "Applied Mathematis" department consists of 5 employees: 1 prof.,doct.phys.math.sci., 1 doctor of mech.sci.sen.res.ass.; 1 cand.techn.sci.; sen.res.ass.; 2 laboratory assistants .

In 2019 " Applied Mathematics" department the following researches were carried out around two topics:

TOPIC 1. Bases of viscous fluid hydrodynamic with reqard to physical media in nanosistems.

(doct. phys.math.sci.prof. Aliyev G.G.)

Work A. Mathematical research of the motion of viscous fluid with regard to quantum mechanical effects in nanotubes

(doct. phys.math.sci.prof. Aliyev G.G.)

In the work the determining equations of the motion of viscous fluid in nanotubes

$(10^{-9}m \le d \le 10^{-4}m)$ are offered. The generalization of the Navier boundary condition of fluid sliding with regard to the effect of intensity of physical fieed penetrating deeply into the field existing on the boundary of vessel's wells and fluid is given.

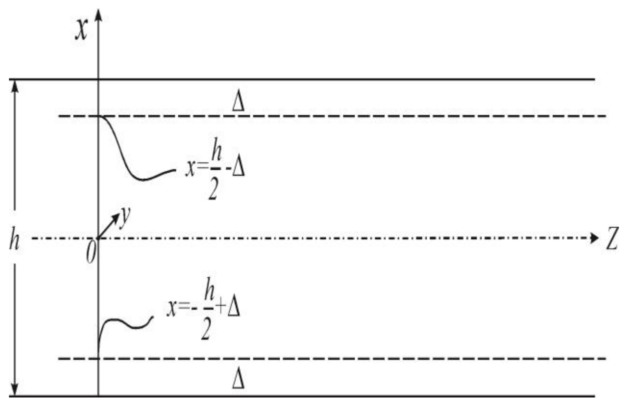

Fig.1

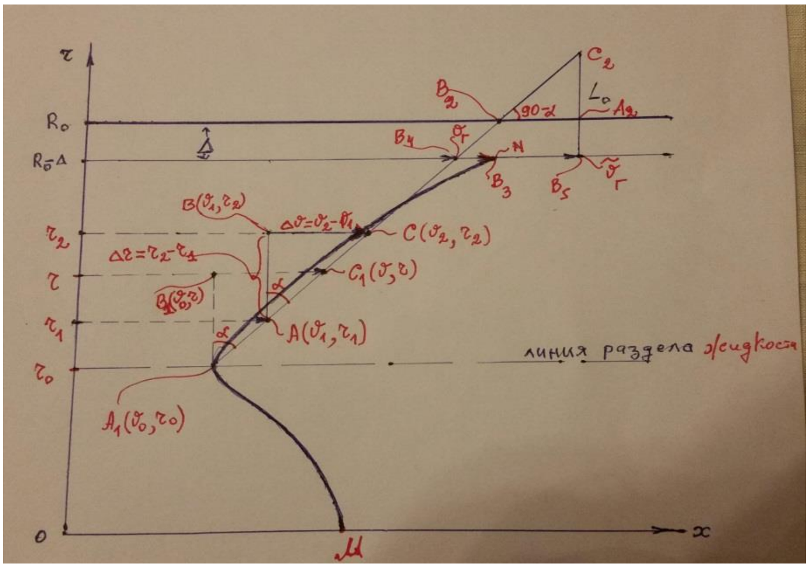

Fig.2

The main equations of the motion of viscous fluid $x_0 \le x \le \frac{h}{2} - \Delta$:

-in the second domain

$$
\begin{cases}
\frac{\partial \upsilon_x}{\partial t} + \upsilon_x \frac{\partial \upsilon_x}{\partial x} + \upsilon_y \frac{\partial \upsilon_x}{\partial y} + \upsilon_z \frac{\partial \upsilon_x}{\partial z} = \\
= X - \frac{1}{\rho_0 (1-\tilde{E})} \cdot \frac{\partial p}{\partial x} + \nu_0 \cdot [\Delta \upsilon_x + \frac{1}{3} \cdot \frac{\partial div \vec{\upsilon}}{\partial x}] + + \frac{2}{3} \cdot \frac{\nu_0}{(1-\tilde{E})} \cdot \frac{\partial \tilde{E}}{\partial x} \cdot (div \vec{\upsilon} - 3 \frac{\partial \upsilon_x}{\partial x}) \\
\\
\frac{\partial \upsilon_y}{\partial t} + \upsilon_x \frac{\partial \upsilon_y}{\partial x} + \upsilon_y \frac{\partial \upsilon_y}{\partial y} + \upsilon_z \frac{\partial \upsilon_y}{\partial z} = \\
= Y - \frac{1}{\rho_0 (1-\tilde{E})} \cdot \frac{\partial p}{\partial y} + \nu_0 \cdot [\Delta \upsilon_y + \frac{1}{3} \cdot \frac{\partial div \vec{\upsilon}}{\partial y}] - \frac{\nu_0}{(1-\tilde{E})} \cdot \frac{\partial \tilde{E}}{\partial x} \cdot (\frac{\partial \upsilon_x}{\partial y} + \frac{\partial \upsilon_y}{\partial x}) \\
\\
\frac{\partial \upsilon_z}{\partial t} + \upsilon_x \frac{\partial \upsilon_z}{\partial x} + \upsilon_y \frac{\partial \upsilon_z}{\partial y} + \upsilon_z \frac{\partial \upsilon_z}{\partial z} = \\
= Z - \frac{1}{\rho_0 (1-\tilde{E})} \frac{\partial p}{\partial z} + \nu_0 \cdot [\Delta \upsilon_z + \frac{1}{3} \frac{\partial div \vec{\upsilon}}{\partial z}] - \frac{\nu_0}{(1-\tilde{E})} \cdot \frac{\partial \tilde{E}}{\partial x} \cdot (\frac{\partial \upsilon_x}{\partial z} + \frac{\partial \upsilon_z}{\partial x})
\end{cases}
$$

- The fluid preservation equation:
- The equations of motion:

$$\frac{\partial\rho_0}{\partial t} + \rho_0 \cdot [div \cdot \vec{\upsilon} - \frac{1}{1-\tilde{E}(x)} \cdot \frac{\partial \tilde{E}(x)}{\partial x} \cdot \upsilon_x] = 0$$

The generalized Navier boundary conditions of transfomiation of homogeneons fluid into inhomogeneons one with reqard to quantum-mechanical effects in nanohidrodinamics are the followings :

$$\upsilon(x|_{x=\frac{h}{2}-\Delta} = \upsilon_0 + L \cdot \frac{\partial \upsilon}{\partial x}$$

Here $\rho = \rho_0 \cdot [1-\tilde{E}(x)]$ и $\mu = \mu_0 \cdot [1-\tilde{E}(x)]$ - are density and viscosity functions along the fluid's depth; $\nu = \frac{\mu_0}{\rho_0}$ is a kinematic viscosity, υ_0 and $L = R_0 - r_0$ are coefficients.

All of these are dependent on the quantum-mechanical effects given from the physical - field intensity $\vec{E} = \frac{E(x)}{E_0}$ penetring deeper into the given fluid. The main quality and quantity results of hydrodynamics of viscous fluid in nano-systems $(10^{-9}m \le d \le 10^{-4}m)$ are the followings:

- formation of empty space between the vessel wall and fluid in the magnitude $\Delta = 0{,}12 \cdot \frac{h}{2}$,

-in the depth of fluid close to the wall the homogeneons fluid will be non – homogeneons :

- depending on the depth of the physical field mechanical propertics of inhomogeneons part (density $\rho(x)$ and viscosity $\mu(x)$) will vary in the form :

$$\rho = \rho_0 \cdot [1-\tilde{E}(x)], \qquad \mu = \mu_0 \cdot [1-\tilde{E}(x)]$$

- along the section of the nanotube , the diagram of viscous fluid flow velocity will not be parabolic , i.e. the gluing effect will be absent . The velocity of flow near the wall will always differ from zero $\upsilon_0 \neq 0$, i.e. the flow will slide becauce of quantum mechanical effects .

The character of sliding velocity in the boundary between the flow and emoli space

Will be composed of 3 types of velocities:

The first velosity $\upsilon_0 = \sqrt{2g \cdot \ell} = 4{,}43 \frac{м}{сек}$ characterizes she velocity of f luid as of a solid ;

The second velosity appears at the expens of nonhomofeneity of the flow at the wall $\upsilon_2 = \upsilon_0 + (\frac{h}{2} - \Delta - x_0) \cdot \frac{d\upsilon}{dr}$;

The their velosity is a velosity that appears at the espense of apparent lengtu of flow sliding in the boundary of flow and empty space . Its length equals the value of the “ forbidden zone “ beetveen the atoms and $\Delta = 0{,}12 \cdot \frac{h}{2}$, $\upsilon_3 = \Delta \cdot \frac{d\upsilon}{dr}$ the fluid in the nanosistems is repeatedey greater than given in theory of classic hydrodynamics.

<u>Work B.</u> Diagnosis of poisoning with toxic substances.

(cand.tech.sci.lead.res.ass. Mirzazade I.H.)

Timely and correct diagnosis of poisonings with toxic substances is basic for selecting tactics of treatment. The character of the poisoning process is carrying out antidate therapy immediately after diagnosis. But in addition to treatment, the result if this treatment should also be taken into account. This time there appears a need for monitoring, i.e. for periodic examination of the patient. The goal and function of monitoring for the problem under consideration is to continue control of the patient after treatment for a certain time and to select tactics of treatment according to the patient’s state, because the complications after the poisoning are closely connected with nervous system, cardiovascular system. One of the tactics for carrying monitoring is determination of time and time intervals. When analyzing time series three components are distinguished: a random component formed as a result of influence of random factors on indicators. The use of time series method for carrying out monitoring of poisonings with carbon dioxide are for the following goals:

- detection of variation of any exponent or a group of exponents in time;
- determination of the cause of variation of exponents;
- prognosis of exponents: to verify the validity of the Mauna – Witney, Wilkoksan, Friedman, Klakson – Wollis criteria being the non – parameter methods of biostatistics.

1. In Mauna – Witney U-criterion n_1 is the number of the first selection, n_2 is the number of the second selection. The greaters from the sum of two ranks (T_x) is determined.

$$U = n_1 \cdot n_2 + \frac{n_x \cdot (n_x + 1)}{2} - T_x.$$

2. The Wilkokson T – criterion is used for estimating difference between the exponents obtained before and after treatment.

3. The Friedman criterion

$$S = \frac{12n}{k(k+1)} \sum_{j=1}^{k} \left(\frac{1}{n} \sum_{i=1}^{n} r_{ij} - \frac{k+1}{2} \right).$$

If $S < S_\alpha(n,k)$ then the 0-th hypothesis is accepted (tabular value of S_α).

4. the Kruskal – Wollis *H* – criterion $x_1^{n_1} = \{x_{11}.\,....x_{1n_1}\}......x_k^{n_k} = \{x_{k1}.\,....x_{kn_k}\}$ will be as the generalized selection $x = x_1^{n_1}\ U\, x_2^{n_2}\ U ... U\, x_k^{n_k}$. All

$$N = \sum_{i=1}^{k} n_i \qquad\qquad H = \frac{12}{N(N+1)} \cdot \sum_i \frac{T_j^2}{n_j} - 3(N+1)$$

where N is the general amount of N – selection. T_1 is the sum of ranks in each group, n_1 is the number of observations in the j-th group.

If $H \geq H_\alpha$, the 0-th hypothesis is rejected (H_α is tabular value).

Numerous experiments have shown that the monitoring enables to determine variation of indicators in time series, to choose the important ones for verification, to detect the ones subjected to treatment and not to conduct excess poorly analyzes.

TOPIC 2. INTEGRAL MODELING OF FILTRATION PROCESS IN OIL GASPRODUCTION

(doct.phys.math.sci. Aliev G.G., lead.res.ass. Abbasov E.M.)

Work A. Modeling of gas filtration process in the stratum-well system

(lead.res.ass. Abbasov E.M.)

Investigation and study of filtration process in oilgas wells with regard to dynamic relation of stratum-well system is of great scientific and practical value.

In the paper an integral model of the process of nonstationary filtration of gas is constructed and pressure on the wellhead and well bottom and also production rate dynamics are determined by wellhead information.

We consider a flatradial filtration of homogeneous gas in a uniform circular stratum. The equation of a flatradial filtration of homogeneous gas is of the form [1-4]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\theta\, r \frac{\partial P}{\partial r}\right) = \frac{\partial P}{\partial t}, \qquad (1)$$

где $\theta = \dfrac{kP}{\mu m}$.

Boundary and initial conditions

$$P\big|_{r=r_c} = P_c(t),\ t>0,\ (2), \qquad \left.\frac{\partial P}{\partial r}\right|_{r=R_k} = 0, \quad t>0, \tag{3}$$

$$2\pi h r \frac{k}{\mu\beta}\frac{P_c(0)+P_c(T)}{2}\left.\frac{\partial P}{\partial r}\right|_{t=0} = G(r). \tag{4}$$

Allowing for boundary conditions (2) and (3) we will look for the solution of equation (1) in the form [1]:

$$P = P_c(t) + A(t) f(r), \tag{5}$$

where $A(t)$ is an unknown function dependent on time t, $f(r)$ is a function dependent on the coordinate r and satisfying boundary conditions (2) and (3), $P_c(t)$ is pressure at the bottom hole, k is a stratum permeability factor, μ is dynamic viscosity of gas, m is porosity of the stratum rock, $P_c(0)$ is initial pressure at the bottom hole, P is pressure at any point of the stratum. We choose the function $f(r)$ satisfying boundary conditions (2) and (3) as follows (see [1]):

$$f(r) = \ln\frac{r}{r_c} - \frac{r}{R_k} + \frac{r_c}{R_k}. \tag{6}$$

Accepting the process as isothermal, the gas mass G_0 in the stratum at any moment of time may be determined by the formula:

$$G_0 = \frac{2\pi m h}{\beta}\int_{r_c}^{R_k} P\cdot r dr, \tag{7}$$

where $\beta = \frac{P_{\text{atm}}}{\rho_{\text{atm}}}$, h is power of the stratum, P_{atm} is atmosphere pressure, ρ_{atm} is gas density at atmosphere pressure, r_c is a radius of the stratum, R_k is a radius of the stratum contour, r is a coordinate.

The gas influx from the stratum to the well per a time unit G may be determined by the formula:

$$G = -\frac{dG_0}{dt}. \tag{8}$$

Substituting expression (5) and (6) in formula (7), we get:

$$G_0 = \frac{2\pi m h}{\beta}\left[P_c(t)\frac{R_k^2 - r_c^2}{2} + \frac{R_k^2}{2} D\, A(t)\right], \tag{9}$$

$$D = \ln\frac{R_k}{r_c} - \frac{7}{6} + \frac{1}{2}\left(\frac{r_c}{R_k}\right)^2 + \frac{r_c}{R_k} - \frac{1}{3}\left(\frac{r_c}{R_k}\right)^3.$$

Substituting expression (9) in formula (8) we get:

$$G = -\frac{\pi m h R_k^2}{\beta}\left[\dot{P}_c\left(1 - \frac{r_c^2}{R_k^2}\right) + D\dot{A}(t)\right]. \tag{10}$$

On the other hand, gas influx into the well per a unit time may be determined by the formula [1]:

$$G=\frac{k\left(P_c(0)+P_c(T)\right)}{\mu\,\beta}\pi\,r_c\left.\frac{\partial P}{\partial r}\right|_{r=r_c},\qquad(11)$$

where $P_c(T)$ is pressure at the well bottom at the end of operation period.
Then substituting expression (5) in formula (11), we get:

$$G=\frac{k\left(P_c(0)+P_c(T)\right)}{\mu\,\beta}\pi\,h\,A(t)\left(1-\frac{r_c}{R_k}\right).\qquad(12)$$

Equating equations (10) and (12), we get:

$$\dot{A}+\alpha\,A=-\frac{\dot{P}(t)}{D}.\qquad(13)$$

The solution of differential equation (13) is of the form:

$$A=A_0\exp(-\alpha t)-\frac{1}{D}\int_0^t\dot{P}_c(\tau)\exp\left[-\alpha(t-\tau)\right]d\tau,\qquad(14)$$

where A_0 is an integration constant determined from initial condition (4), $\alpha=\frac{k\,(P_c(0)+P_c(T))}{\mu\,m\,R_k\,D}$. Substituting the obtained expression in formula (5), we get pressure distribution field in the stratum:

$$P=P_c(t)+\left(\ln\frac{r}{r_c}-\frac{r}{R_k}+\frac{r_c}{R_k}\right)\left[A_0\exp(-\alpha t)-\frac{1}{D}\int_0^t\dot{P}_c(\tau)\exp\left[-\alpha(t-\tau)\right]d\tau\right].\qquad(15)$$

We now consider motion of gas in a lifting pipeline. Motion of gas in a pipeline and equation of continuity are described by I.A. Charniy equations [5,6]:

$$-\frac{\partial P}{\partial x}=\frac{\partial Q}{\partial t}+2aQ,\qquad -\frac{\partial P}{\partial t}=c^2\frac{\partial Q}{\partial x},\qquad Q=\rho\upsilon\qquad(16)$$

where c is sound propagation velocity in gas, t is time, x is a coordinate, ρ is gas density under the given pressure, υ a gas flow velocity averaged in cross section of the pipe, a is a resistance coefficient.

Having differentiated the first equation of expression of (16) in time t, and the second one in x, and subtracting one from another, we get:

$$\frac{\partial^2 Q}{\partial t^2}=c^2\frac{\partial^2 Q}{\partial x^2}-2a\frac{\partial Q}{\partial t}.\qquad(17)$$

We represent the velocity of cross-sections of gas column as the sum of two velocities:

$$\upsilon=\upsilon_e+\upsilon_r,\qquad(18)$$

where υ_e is the velocity of the gas column motion as a solid (traveling velocity), υ_r is the velocity of cross-section of gas column from its compressibility (relative velocity).

Substituting expression (18) in the formula

$$Q=\rho\upsilon=\rho\upsilon_e+\rho\upsilon_r\qquad(19)$$

or

$$Q=u_e+u_r,\qquad(20)$$

where $u_e = \rho \upsilon_e$, $u_r = \rho \upsilon_r$.

Then, substituting expression (20) in equation (17), we get:

$$\frac{\partial^2 u_e}{\partial t^2} + \frac{\partial^2 u_r}{\partial t^2} = c^2 \frac{\partial^2 u_r}{\partial x^2} - 2a\left(\frac{\partial u_e}{\partial t} + \frac{\partial u_r}{\partial t}\right). \tag{21}$$

As equation (21) is linear, then it is decomposed into two equations :

$$\frac{\partial^2 u_e}{\partial t^2} + 2a\frac{\partial u_e}{\partial t} = \frac{\dot{P}_c - \dot{P}_{\text{u}}}{l}, \tag{22}$$

where P_{y} is pressure at the bottom head.

$$\frac{\partial^2 u_r}{\partial t^2} = c^2 \frac{\partial^2 u_r}{\partial x^2} - 2a\frac{\partial u_r}{\partial t} + \frac{\dot{P}_{\text{u}} - \dot{P}_c}{l} \tag{23}$$

Having placed the origin of the coordinate axis x in the lower section of the pipe and directing it upwards, for initial and boundary conditions we have:

$$u_e\big|_{t=0} = \frac{G(0)}{f} \quad (24), \qquad \frac{du_e}{dt}\bigg|_{t=0} = 0 \quad (25), \qquad u_r\big|_{t=0} = 0 \quad (26)$$

$$\frac{\partial u_r}{\partial t}\bigg|_{t=0} = 0 \quad (27) \qquad u_r\big|_{x=0} = 0 \quad (28), \qquad \frac{\partial u_r}{\partial x}\bigg|_{x=l} = 0 \quad (29)$$

where f is the area of the flow section of the pipe.

Applying the Laplace transform and taking into account the convolution theorem [7-9], allowing for initial conditions (24) and (25) we get:

$$u_e = \frac{G(0)}{f} + \frac{1}{l}\int_0^t P_c(\tau)\exp[-2a(t-\tau)]d\tau - \frac{1}{l}\int_0^t P_{\text{u}}(\tau)\exp[-2a(t-\tau)]d\tau -$$

$$-\frac{1}{2al}\exp(-2at)[P_{\text{u}}(0) - P_c(0)] + \frac{1}{2al}[P_{\text{u}}(0) - P_c(0)]. \tag{30}$$

Allowing for boundary conditions (28) and (29), we will look for the solution of equation (23) in the form: ([5], [6], [7]):

$$u_r = \sum_{i=1}^{n} \varphi_i(t)\left(1 - \cos\frac{i\pi x}{l}\right), \tag{31}$$

where $\varphi_i(t)$ is an unknown function dependent on time t, l is the pipe run depth. Substituting expression (31) in equation (23), multiplying the both hand sides of the obtained expression by $\left(1 - \cos\frac{i\pi x}{l}\right)$ and integrating it from 0 to l, we get the equation:

$$\ddot{\varphi}_i + 2a\dot{\varphi}_i + \frac{c^2 i^2 \pi^2}{3l^2}\varphi_i = \frac{2}{3l}\left(\dot{P}_{\text{u}} - \dot{P}_c\right). \tag{32}$$

Applying the Laplace transform and taking into account the conversion and convolution theorems ([8], [9]), with regard to initial conditions (26) and (27) from equation (32) we get:

$$\varphi_i = \frac{2}{3l}\left[\int_0^t P_{\text{u}}(\tau)\exp[-a(t-\tau)]\cos[\omega_i(t-\tau)]d\tau - \frac{a}{\omega_i}\int_0^t P_{\text{u}}(\tau)\exp[-a(t-\tau)]\times\right.$$

$$\times \sin[\omega_i(t-\tau)]d\tau - \frac{P_u(0)}{\omega_i}\exp(-at)\sin(\omega t) - \int_0^t P_u(\tau)\exp[-a(t-\tau)]\cos[\omega_i(t-\tau)]d\tau +$$

$$+\frac{a}{\omega_i}\int_0^t P_c(\tau)\exp[-a(t-\tau)]\sin[\omega_i(t-\tau)]d\tau + \frac{P_c(0)}{\omega_i}\exp(-at)\sin(\omega t)\Bigg], \qquad (33)$$

$$\omega_i^2 = \frac{c^2 i^2 \pi^2}{3l^2} - a^2 .$$

From the continuity condition allowing for boundary condition (28) and expressions (30), (31) and (33), we get the following integral equation:

$$G(0)\exp(-\alpha t) - \frac{k(P_c(0)+P_k)}{D\mu\beta}\pi h\int_0^t \dot{P}_c(\tau)\exp[-\alpha(t-\tau)]d\tau = G(0) + \frac{f}{2al}[P_u(0) - P_c(0)] +$$

$$+\frac{f}{l}\int_0^t P_c(\tau)\exp[-2a(t-\tau)]d\tau - \frac{f}{l}\int_0^t P_u(\tau)\exp[-2a(t-\tau)]d\tau -$$

$$-\frac{f}{2al}\exp(-2at)[P_u(0) - P_c(0)]. \qquad (34)$$

Using the Laplace transform and taking into consideration the convolution and conversion theorems, from expression (34) we get:

$$P_c = P_c(0)\left[\exp(-\beta_1 t)\frac{2a-\beta_1}{\beta_2-\beta_1} + \exp(-\beta_2 t)\frac{2a-\beta_2}{\beta_1-\beta_2}\right] + \left[\frac{f(P_c(0) - P_u(0))}{2alb} - \frac{G(0)}{b}\right]\times$$

$$\times\left[\frac{2\alpha a}{\beta_1\beta_2} + \frac{(\alpha-\beta_1)(2a-\beta_1)}{\beta_1(\beta_1-\beta_2)}\exp(-\beta_1 t) + \frac{(\alpha-\beta_2)(2a-\beta_2)}{\beta_2(\beta_2-\beta_1)}\exp(-\beta_2 t)\right] +$$

$$+\frac{f}{lb}\left[\frac{\alpha-\beta_1}{\beta_2-\beta_1}\int_0^t P_u(\tau)\exp[-\beta_1(t-\tau)]d\tau + \frac{\alpha-\beta_2}{\beta_1-\beta_2}\int_0^t P_u(\tau)\exp[-\beta_2(t-\tau)]d\tau\right] -$$

$$-\frac{f}{2alb}[P_c(0) - P_u(0)]\left(\frac{\alpha-\beta_1}{\beta_2-\beta_1}\exp(-\beta_1 t) + \frac{\alpha-\beta_2}{\beta_1-\beta_2}\exp(-\beta_2 t)\right) +$$

$$+\frac{G(0)}{b}\left[\exp(-\beta_1 t)\frac{2a-\beta_1}{\beta_2-\beta_1} + \exp(-\beta_2 t)\frac{2a-\beta_2}{\beta_1-\beta_2}\right], \qquad (35)$$

where $b = \frac{k[P_k + P_c(0)]}{\beta\mu}\frac{\pi h}{D}$, P_k pressure on the stratum contour.

β_1 and β_2 are the roots of the equation

$$s^2 + \left(a + \frac{f}{bl}\right)s + \frac{f}{bl}\alpha = 0 . \qquad (36)$$

We consider how pressure changes at the bottom hole when shutting-in it from the well head.

Assume that when the well is shutting-in, gas mass flow on its head decreases by the linear law

$$Q = G_1\left(1 - \frac{t}{T_0}\right), \qquad (37)$$

where T_0 is a time period for which the well shuts-in, G_1 is gas influx in to the well head per a unit time at the beginning of well shut-in operation.

We determine the head and bottom hole pressure. From the continuity condition on the well head, with regard to expressions (30), (31), (33) and (37), using the Laplace transform we get:

$$\frac{G(0)}{fs}+\frac{1}{l}\frac{\dot{\overline{P}}_c}{s(s+2a)}-\frac{1}{l}\frac{\dot{\overline{P}}_\text{u}}{s(s+2a)}+\sum_{i=1}^{n}\frac{2}{3l}\left[\frac{s\overline{P}_\text{u}}{(s+a)^2+\omega_i^2}-\frac{P_\text{u}(0)-P_c(0)}{(s+a)^2+\omega_i^2}-\right.$$

$$\left.-\frac{s\overline{P}_\text{c}}{(s+a)^2+\omega_i^2}\right]=\frac{G_1}{f}\left(\frac{1}{s}-\frac{1}{Ts^2}\right). \quad (38)$$

From expression (38) for $G(0)=G_1$ we will have:

$$\overline{P}_c=P_c(0)\frac{(s+a)^2+\omega^2}{s\left[(s+a)^2+\omega^2+\frac{2}{3}(s+2a)s\right]}+\frac{s\overline{P}_\text{u}-P_\text{u}(0)}{s\left[(s+a)^2+\omega^2+\frac{2}{3}(s+2a)s\right]}+$$

$$+\frac{2}{3}\overline{P}_\text{u}\frac{s(s+2a)}{(s+a)^2+\omega^2+\frac{3}{3}(s+2a)s}-\frac{2}{3}\frac{[P_\text{u}(0)-P_c(0)](s+2a)}{(s+a)^2+\omega^2+\frac{2}{3}(s+2a)s}-$$

$$-\frac{G_1\,l}{f\,Ts^2}\frac{(s+2a)\left((s+a)^2+\omega^2\right)}{(s+a)^2+\omega^2+\frac{2}{3}(s+2a)s}. \quad (39)$$

Now, determining from expression (34) the image of $\overline{P}_c$ and equating it to the right hand side of the expression (39), one can find the image of the well head pressure:

$$\overline{P}_{\text{u}1}=\frac{G(0)\alpha}{s^2}\frac{D\mu\beta}{\pi hk\left(P_T+P_c(0)\right)}+\frac{P_c(0)}{s}-\frac{f\left(P_\text{u}(0)-P_c(0)\right)}{2als^2}\frac{D\mu\beta(s+\alpha)}{\pi hk\left(P_T+P_c(0)\right)}+$$

$$+\frac{f\left(P_\text{u}(0)-P_c(0)\right)}{2als(s+2a)}\frac{D\mu\beta(s+\alpha)}{\pi hk\left(P_T+P_c(0)\right)}+\frac{P_\text{u}(0)-P_c(0)}{s}+$$

$$+\frac{\left(P_\text{u}(0)-P_c(0)\right)}{s^2(s+2a)}\frac{f\,D\mu\beta(s+\alpha)}{\pi hkl\left(P_T+P_c(0)\right)}-\frac{G_1 l}{f\,Ts^2}\frac{(s+a)\left((s+a)^2+\omega_i^2\right)}{\left[\frac{s^2}{3}+\frac{2as}{3}+\omega_i^2+a^2\right]}-$$

$$-\frac{G_1}{f\,Ts^3}\frac{(s+a)\left((s+a)^2+\omega_i^2\right)}{\left[\frac{s^2}{3}+\frac{2as}{3}+\omega_i^2+a^2\right]}\frac{f\,D\mu\beta(s+\alpha)}{\pi hk\left(P_T+P_c(0)\right)}, \quad (40)$$

where $\omega_i^2=\dfrac{c^2i^2\pi^2}{3l^2}-a^2$.

For $t=T_0$ from the expression (37) it can be seen that $Q=0$. Then the wellhead pressure for $t\geq T_0$ is determined from expression (40) only with a difference that everywhere instead of t we put $t-T_0$, and instead of $G_1=0$.

Thus, wellhead pressure change after wells shut-in occurs by the formula:

$$P_\text{u}^*=P_{\text{u}1}(t)[\eta(t)-\eta(t-T_0)]+\eta(t-T_0)\cdot P_{\text{u}2}(t), \quad (41)$$

where η is a Heaviside function, $P_{\text{u}1}(t)$ is pressure change at the wellhead for $0<t\leq T$, determined from equation (40), $P_{\text{u}2}(t)$ is the same determined from

expression (40) for $G_1 = 0$. Passing to original from expression (40) with regard to convolution and conversion theorems for the following values of parameters

$c = 300 \text{м} \cdot \text{c}^{-1}$; $\mu = 10^{-5}\ Pa \cdot \text{c}$; $h = 10\text{м}$; $k = 5 \cdot 10^{-14}\ \text{м}^2$; $\rho = 0.668\ \text{кг} \cdot \text{м}^{-3}$;
$l = 3000\ \text{м}$; $P_k = 2.5 \cdot 10^7\ \Pi a$; $P_0 = 24 \cdot 10^6\ \Pi a$; $P_T = 8 \cdot 10^6\ \Pi a$;
$P_c(0) = 24 \cdot 10^6\ \Pi a$; $P_{\text{atm}} = 10^5\ \Pi a$; $T = 5\ c$; $R_k = 300\ \text{м}$; $\pi = 3{,}14$;
$a = 10^{-1} c^{-1}$; $m = 0.2$; $d = 6 \cdot 10^{-2}\ \text{м}$; $r_c = 7.5 \cdot 10^{-2}\ \text{м}$.

we get an expression of the wellhead pressure whose graphs are represented in fig. 1 and 2

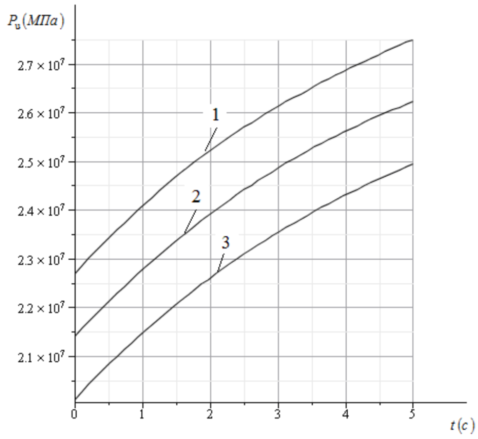

Fig. 1

The graphs of dynamics of pressure at the well head in the time interval of well shut-in depending on the depth of lifting pipes:

$1 - l = 1000\text{м}$, $2 - l = 2000\text{м}$, $3 - l = 3000\text{м}$.

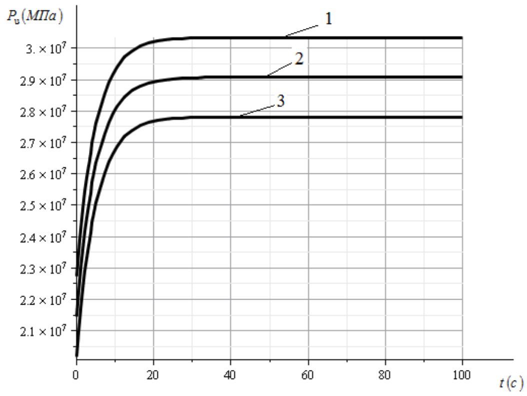

Fig.2

The graphs of dynamics of pressure at the well head after well shut-in depending on the depth of lifting pipes:

$1 - l = 1000\text{м}$, $2 - l = 2000\text{м}$, $3 - l = 3000\text{м}$.

Now we consider the gas production process. Assume that the well head pressure decreases in the course of time by the linear law :

$$P_u(t) = P_u(0) - \frac{P_u(0) - P_u(T)}{T} \cdot t \,, \tag{42}$$

where T is gas production period, $P_u(T)$ is pressure at the well head at the end of production. Then substituting expression (42) in formula (35) we get:

$$\begin{aligned}
P_c &= P_c(0)\left[\exp(-\beta_1 t)\frac{2a-\beta_1}{\beta_2-\beta_1} + \exp(-\beta_2 t)\frac{2a-\beta_2}{\beta_1-\beta_2}\right] + \left[\frac{f\left(P_c(0)-P_u(0)\right)}{2alb} - \frac{G(0)}{b}\right]\times \\
&\times\left[\frac{2\alpha a}{\beta_1\beta_2} + \frac{(\alpha-\beta_1)(2a-\beta_1)}{\beta_1(\beta_1-\beta_2)}\exp(-\beta_1 t) + \frac{(\alpha-\beta_2)(2a-\beta_2)}{\beta_2(\beta_2-\beta_1)}\exp(-\beta_2 t)\right] + \\
&+\frac{f}{lb}\frac{\alpha-\beta_1}{\beta_2-\beta_1}\left\{\frac{P_u(0)}{\beta_1}\left(1-\exp(-\beta_1 t)\right) - \frac{P_u(0)-P_0}{T_0}\left[\frac{t}{\beta_1} - \frac{1}{\beta_1^2}\left(1-\exp(-\beta_1 t)\right)\right]\right\} + \\
&+\frac{f}{lb}\frac{\alpha-\beta_2}{\beta_1-\beta_2}\left\{\frac{P_u(0)}{\beta_2}\left(1-\exp(-\beta_2 t)\right) - \frac{P_u(0)-P_0}{T_0}\left[\frac{t}{\beta_2} - \frac{1}{\beta_2^2}\left(1-\exp(-\beta_2 t)\right)\right]\right\} - \\
&-\frac{\left(P_c(0)-P_u(0)\right)f}{2a\,lb}\left(\frac{\alpha-\beta_1}{\beta_2-\beta_1}\exp(-\beta_1 t) + \frac{\alpha-\beta_2}{\beta_1-\beta_2}\exp(-\beta_2 t)\right) + \\
&+\frac{G(0)}{b}\left[\frac{2a-\beta_1}{\beta_2-\beta_1}\exp(-\beta_1 t) + \frac{2a-\beta_2}{\beta_1-\beta_2}\exp(-\beta_2 t)\right].
\end{aligned} \tag{43}$$

Allowing for expression (14), from expression (12) we get:

$$G = G(0)\exp(-\alpha t) - \frac{k\left(P_c(0)+P_T\right)}{D\mu\beta}\pi h\int_0^t \dot{P}_c(\tau)\exp\left[-\alpha(t-\tau)\right]d\tau \,, \tag{44}$$

where $G(0)$ is gas influx into the well per a unit time at initial time.

Now, allowing for formula (43), for the above values of parameters, from the expression (44) we get:

$$\begin{aligned}
&G = 8.5\cdot 10^{-6}\exp(-2.57\cdot 10^{-6} t) + \\
&+ 0.28\exp(-1.34\cdot 10^{-7} t) - 1.7\exp(-0.01t) + 1.41\,.
\end{aligned} \tag{45}$$

By formulas (43) and (45) the numerical analysis is carried out for the above parameters of the system and $k = 10^{-14}\,м^2$, $k = 5\cdot 10^{-14}\,м^2$, $k = 10^{-13}\,м^2$.

The results of numerical calculations are represented in fig. 3 and fig. 4.

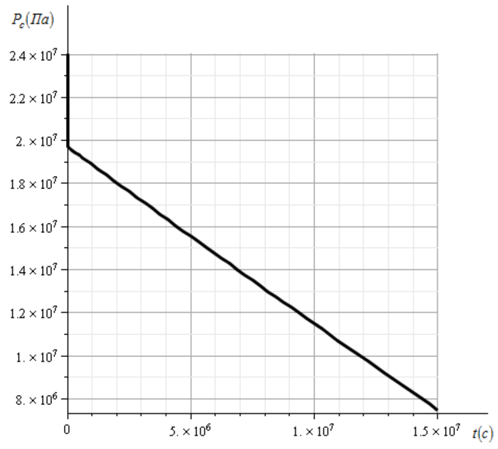

Fig. 3

The graph of pressure dynamics at the well bottom.

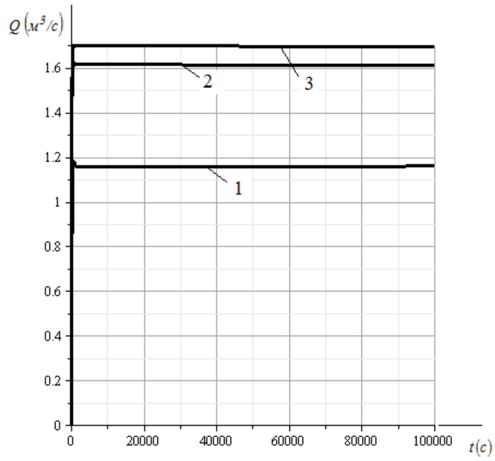

Fig. 4

The graph of gas influx per a unit time at the well head.

$$1-k=10^{-14}м^2,\ \ 2-k=5\cdot10^{-14}м^2,\ \ 3-k=10^{-13}м^2.$$

Thus, we constructed an integral model of nonstationary gas filtration process in the stratum-well system. The analytical expressions allowing to determine well productivity and also bottom head and bottom hole pressure for its not instant termination of gas influx into the well and to study pressure restoration curves.

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