Report on scientific and scientific organizational activity of “Functional theory” department of NAS IMM for 2014 year

I. On scientific activity

On the report year 7 works with 7executers on the team “Approximation of functions of many group of variables by Ridge functions, neural networks and non linear super- positions, imbedding theorems for function spaces” were worked out.

Work 1 :Interpolation by Ridge functions on lines

(ex: cand.ph.m.s., doc. V.E.Ismayilov)

Ridge functions are very important in many fields of mathematics. Under the ridge functions we understand a many variable functions on the form $g(\mathbf{a}\cdot\mathbf{x})$. Here g – is an one variable function, $\mathbf{a}=(a_1,\dots,a_n)$ is a non – zero vector (direction), $\mathbf{x}=(x_1,\dots,x_n)$ is an indepedent variable, $\mathbf{a}\cdot\mathbf{x}$ is a scalar product . These functions arise in natural form in various fields of science. Theory of partial differential equations (here ridge functions are called surface waves), computer tomography and mathematical statistics are among these fields.

One of the contemporary fields of science is the theory of neural networks. The neural networks in its turn are used in various fields as medicine, economy, physics, engineering, etc.The ridge functions are in the basic of some principal neural networks models. For example, the most popular model of the theory of neural networks MLP model considers in the simplest form the functions in the form $\sum_{i=1}^{r} c_i\sigma(\mathbf{w}_i\cdot\mathbf{x}-\theta_i)$. It is clear that the $\sigma(\mathbf{w}_i\cdot\mathbf{x}-\theta_i)$ functions are ridge functions. Therefore some theoretical problems of neural networks are closely connected with the probles of ridge functions.(see: A.Pinkus “Approximation theory of the MLP model in neural networks, Acta Numerica. 8 (1999), 143-195").

In spite of the existence of numerical scientific papers devoted to ridge functions , there are no methods from practical point of view have been worked out yet.

In the report year the problems of interpolation by ridge functions on finite member lines given on n – dimentional Euclidean space. Necessary and sufficient condition for possibility of interpolation on two straight lines by the set of sums of ridge functions in two directions were found. It was proved that interpolation by the set of sums of ridge functions in two directions on two or more straight lines is impossible. In should be noted that the appropriate problem on the points of the surface was solved by N.Dyn, M.Light and E.Cheney. But interpolation on straight lines has not been studied yet.

Assume that the a^1 *and* a^2 directions are given on the space R^n. Let' s consider the following set

$$M(a^1, a^2) = \left\{ f_1\left(a^1 \cdot x\right) + f_2\left(a^2 \cdot x\right):\ f_i: R \to R, i = 1,2 \right\}.$$

It is known that $M(a^1, a^2)$ is the set of linear combinations of ridge functions in direstions a^1 *and* a^2. Assume,that we arec given straight lines $\{tb^j + c^j\} b^j \neq 0, j = 1, \dots, k,$ in the space R^n.Under the interpolation on straight lines we understand the finding of the conditions that for any functions $g_j(t), \quad j = 1, \dots, k,$ satisfying the equalities

$$G(tb^j + c^j) = g_j(t), \qquad j = 1, \dots, k,$$

there exists $G \in M(a^1, a^2)$. If there exists the function $G \in M(a^1, a^2)$ satisfying the above equalities, then the on the given lines "the interpolation problem is solvable ", otherwise we say that "the interpolation problem is not solvable".

In the report year necessary ans sufficient conditions for solving interpolation on staraight lines have been found.

Work 2: Some differential properties of functions from generalized Lizorkin-Triebel-Morrey type space.

(ex: doc.ph.m.s., prof. A.M.Najafov)

On the report year the generalized Besov-Morrey and Lizorkin-Triebel – Morrey spaces were introduced and both differential and differential-difference properties of functions from these spaces were studied. Furthermore, uniqueness and existense of the solution of a class of higher fractional order differential equations was indicated.

Work 3. Inequalities between local oscillation characteristics and moothness module of a function and their application to studying the properties of integral operators

(ex: d.ph.m.s., prof. R.M.Rzayev)

Some inequalities between local oscillation of locally summable function and the smoothness modulus in its L^p metric were obtained. By means of these inequalitites appropriase evalutions for a potential type integral operator were obtained.

A Ziqmund type estimations for $R_{\alpha,k}$ operator in the terms of $M_f^k(r)_{pq}$ and $\omega_f^k(\delta)_p$ metric characteristics were obtained.

Work 4: Studying some extremal properties of polynomials given on a line in a complex plane.

(ex: cand. ph.m.s. N.M.Sabziyev)

Studying extremal properties of functions from constructive functions theory is one of the most important problems.The works of some known scientists in this field a great attention.

Work 5. Diliberto – Strauss alqorithm for approximation by Ridge functions.

(ex.: cand.ph.m.s. I.K.Maharov)

In the work the calculation alqorithm for best approximation by the Ridge functions with respect to the basis directions of continuous two-variable function detwermined on the convex compact subset of R^2 was considered. This alqorithm for the rectangels with the sides parallel to the coordinate axes was given by S.P.Diliberto and E.G.Strauss .

Work 6.: Approximation by interpolation polynomials

(ex. cand.ph.m.s. Arzu M-B.Babayev)

The exact annihilator is structured for the system of n-th order polynomials using the partitioned differences method. By means of this exact annihilator , the exact double sided estimations the best upper and lower approximation are obtained.

At the same time , the definition of A – smothness modulus of the function f was given and upper estimation of the best approximation was obtained.

Work 7. Solution of boundary value problems for a class of operator – differetial equations in some weight spaces.

(ex. doct.ph.m.s. Humbataliyev R.Z.)

In the the report year he was engaged in the problem of “Completeness of the elementary solutions of boundary value problems for a class of higher order operator,differetial equations in weighted spaces”.

For the first time the conditions for the solvability of the problem

$$\left(-\frac{d^2}{dt^2}+A^2\right)^n(t)+\sum_{j=0}^{2n-1}A_{2n-j}u^{(j)}(t)=f(t),t\in R_+=(0,+\infty).,\qquad (1)$$

$$u^{(s_\nu)}(0)=0,\qquad \nu=\overline{0,n-1},\qquad (2)$$

in the weighted space were found. Then this problem is combined with the operator bundle

$$P(\lambda)=\left(-\lambda^2E+A^2\right)^n+\sum_{j=1}^{n}A_{n-j}\lambda^j\ .\qquad (3)$$

and we prove a theorem on completeness of the elementary solution of problem (1), (2) in the weighted space.

II. On **scientific – organizational activity.**

On the report perieod the regular seminars (Thursday at 12.00) were held and the collaborators of the department prof.A.Najafov has given a talk at the institute seminar.

A great majority of the collaborators of the departent have taken an active part in the work of the International Conference devoted to 55 years of IMM NASA. The head of the department cand.ph.m.s. Vuqar Ismayilov has defended doctor's degree dissertation "Appximation by the fixed direction ridge functions".

In the report year 6 papers , 1 book, 2 textbooks and 7 abstracts were published. 2 papers are in print. Two of the papers have been published in Science Citation Index journals "Journal of Appoximation Theory" and "Journal of Mathematical Analysis and Applications" .

The head of the department , cand.ph.m.s. Vuqar Ismayilov has successfully completed the project "The role of double hidden layer neural networks in optimization of oil production" suppposted by SOCAR Science Foundation.

Head of the department cand.ph.m.s. dos. Ismayilov V.E.

Report on scientific-investigations, themes and works planning todo by collaborators of "Functions theory" department of NAS IMM in 201

№	Theme, the name of scientific work, the name of excutor, scientific degree and name	Factual condition, obtaining the main results
1	2	3
1.	Theme: "Approximation of multy variable functions by ridge functions,neural networks,linear and non-linear superpositions , imbedding theorems for function spaces" Work 1. Interpolation by ridge functions on lines ex. cand.ph.m.s., dos. V.E.Ismayilov	Necessary and sufficient conditions for possibility of interpolation on two straight lines by the set of sums of ridge functions were found. It was proved that interpolation by the set of sums of ridge functions in two directions on three or more straight lines is impossible.
2.	Work 2. "Some differential properties of functions from generalized Lizorkin-Triebel – Morrey type space " (ex. doc.ph.m.s., prof. A/M/Najafov)	The generalized Besov-Morrey and Lizorkin –Triebel-Morrey spaces were introduced and both differential and differential-difference properties of functions from these spaces were studied. Furthermore, uniqueness and existence of solution of a class of higher fractional order differential equations was indicated.

3.	Work 3. “Inequalities between local oscillation characteristics and smoothness module of a function and their application to studying the properties of integral operators”. (ex. d.ph.m.s., prof. R.M.Rzayev)	Some inequlities between local oscillation of locally summable function and the smoothness modulus in its Lp metric were obtained . By means of these inequalities appropriase evalutions for a potential type integral operator were obtained.
4	Work 4.”Studying some extrremal properties of polinomials given on a line in a complex plane. (ex: cand.ph.m.s. N.M.Sabziyev)	Inequalities between the norms of linear expressions structured with respect to Legendre polinomials in different metric spaces were proved.
5.	Work 5.”Diliberto-Strauss alqorithm for approximation by ridge functions. (ex: cand.ph.m..s. I.K.Maharov	The calculation alqorithm for the best approximation by the sums ridge functions with repect to thr basis directions of continuous two-variable function determined om the convex compact subset of R^2- was considered.
6.	Work 6.”Approximation by inter-polation polinomials . (ex: cand.ph.m.s. Arzu M-B.Babayev)	The exact annhilator is structured for the system of n-th orders polyno-mials using the partitioned differen-ces method.
7.	Work 7.”Solution of boundary value problems for a class of operator differential equations in some weight spaces” (ex. doct.ph.m.s. Humbataliyev R.Z.)	Completeness of the elementary solutions of boundary value problems for a class of higher order operator-differential equations in weighted spaces was engaged.

The list of papers of the collaborations of "Functions theory" department published or to be published in 2014

Names,scientific degree and posts of collaborators	Names of scientific works	Published or to be published	Name of publishing house, N, year	page	Coauthors
1	2	3	4	5	6
The papers published in Science Citation Index journals					
1.Ismayilov V.E., cand.ph. m.s., head of the department	1. Interpolation on lines by ridge functions.	paper	J. Approx. Theory 175 (2013), 91--113. Note: the work was included to the report 2013	23	A.Pinkus
	2. On the approximation by neural networks with bounded number of neurons in hidden layers.	paper	J. Math. Anal. Appl. 417 (2014), no. 2, 963--969.	7	
The papers published in other journls					
1.Najafov A.M. doct.ph.m.s., e.a.r.	1. On properties of functions from generalized Besov-Morrey spaces.	paper	Proceedings of Institute of Mathematics and Mechanics, XXXIX, Baku-2013, p.93-104	12	Orujova A.T.
	2.Trace theorems in fractional Sobolev space	paper	Caspian journal of applied math., ecology and economics, vol.1 N2 December 2013 p.89-96	8	
2. Rzayev R.M. doct.,ph.m.s., prof	1. Some embedding theorems and properties of Riesz potentials	paper	American Journal of Mathematics and Statistics, 2013, v.3, №6, p.445-453.	9	Aliyev F.N.

3. Hunbatali-yev R.Z.	1. Fundamentals of economical informatics 2. On solvability of boundary value problems for operatot-differ-ential equations and some spectral problems 3. Probability of theory and mathematical statistics	book book book	Publ.House "Kooperasiya", 2014,Baku Moscow, "Nauka", 2014, 170 pages Publ.House "Kooperasiya", 2014,Baku	335 170 441	F.A.Quliy eva, H.N. Tagıyev F.A.Quli-yeva
4. Babayev Ar.M-B. , cand.ph.m.s.	Approximation of two-varible periodic function by trigonometric function	paper	KhabarlarXXXIV No 1, Bakı-2014, p. 21-29	9	
Abstructs					
1.Najafov A.M. doct.ph.m.s., e.a.r.	1.Interpolation theorems for Besov-Morrey generalized space 2.Some properties of Lizorkin-Triebel-Morrey generalized spaces	abstract abstract	Proc.of the intern.Conf.devo-ted to 55 years of IMM ,Baku,2014, p.280-281 Proc.of the intern.Conf.devo-ted to 55 years of IMM ,Baku,2014, p.282	2 1	Orujova A.T. Khanma-medova H.A.
2. Rzayev R.M. d.ph.m.s.,prof.	Some estimations of approximations of functions by singular integrals.	abstract	Proc.of the intern.Conf.devo-ted to 55 years of IMM ,Baku,2014, p.298-300.	1	Mamedo-va Q.X.

3.Sabziyev N.M., cand.ph.m.s. senior researcher	Analytic representation of the amount of prime numbers and Rieman conjecture	abstract	Proc.of the intern.Conf.devoted to 55 years of IMM ,Baku,2014, 317-319	3	
4.Maharov I.K., cand.ph.m.s., senior researcher	On the alternating algorithm for the appox. by linear superpositions	abstract	Riyaziyyat və mexanika institunun 55 illiyinə həsr olunmuş Beyn.Konfransın materialları, Bakı-2014,s.194	1	İsmayılov V.E.
5.Humbataliyev R.Z. d.ph.m.c., prof.	1.On completeness of system of elementary solutions in weight space 2.On some solvability of boundary value problems in weight space	abstract abstract	Proc.of the intern.Conf.devoted to 55 years of IMM ,Baku,2014, p. 132-134, XXII Inern.Conf.”Mathematics,economy, education”Rus-siya, , p.48- 49, 2014	3 2	
Papers to be published					
1. İsmayılov V.E.,cand.ph. m.s..,ass.prof., head of rhe departrment	Approximation by ridge functions and neural tetüorks üith a bounded number of neurons	In print	Applicable Analysis (Taylor and Francis, USA)		
2. Humbataliyev R.Z. , d.ph.m.s., conducter researcher	On completeness of system of elementary solutions for a class operator-dif. equations of higher order in weight space	In print	Lections of NAS		

Head of the department **cand.ph.m.s.,V.E. İsmayılov**

“Functional department” of NAS IMM by the result 2014 considers the following work more important

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