Semi-annual Report of "Differential Equations" Department on scientific and social activities for 2019

The department consists of 15 collaborators. There are 13 scientific workers, including 8 doctors of sciences and 4 philosophy doctors. The department conducts 10 research studies on one subject according to the plan in 2019.

Executed scientific works

Theme: "Some problems of Theory of Partial Differential Operators".

Work 1. Asymptotic of the solutions to the initial boundary problem under the condition of acoustic transmission for nonlinear hiperbolic equations. **Executer: d.ph.m.s., prof. A.B.Aliev.**

Suppose that $\Omega \subset R^3$ is a bounded domain with the border Γ_1, $\Omega_2 \subset \Omega$ is a Γ_2 border subdomain and $\Omega_1 = \Omega \setminus \Omega_2$ is a $\Gamma = \Gamma_1 \cup \Gamma_2$ border subdomain, so that $\Gamma_1 \bigcap \Gamma_2 = \varnothing$.

Let's look at the following mixed problem under the condition of acoustic coupling in the domain Ω

$$u_{tt} - \Delta u + \alpha_1 u_t + u + f_1(u) = 0 \quad \text{in} \quad \Omega_1 \times (0,\infty), \tag{1}$$

$$\upsilon_{tt} - \Delta \upsilon + \alpha_2 \upsilon_t + \upsilon + f_2(\upsilon) = 0 \quad \text{in} \quad \Omega_2 \times (0,\infty), \tag{2}$$

$$\delta_{tt} + \beta \delta_t + \delta = -u_t \quad \text{on} \quad \Gamma_2 \times (0,\infty), \tag{3}$$

$$u = 0 \quad \text{on} \quad \Gamma_1 \times (0,\infty), \tag{4}$$

$$u = \upsilon, \ \delta_t = \frac{\partial u}{\partial \nu} - \frac{\partial \upsilon}{\partial \nu} \quad \text{on } \Gamma_2 \times (0,\infty), \tag{5}$$

$$u(x,0) = u_0(x), \ u_t(x,0) = u_1(x), \quad x \in \overline{\Omega}_1, \tag{6}$$

$$\upsilon(x,0) = \upsilon_0(x), \ \upsilon_t(x,0) = \upsilon_1(x), \quad x \in \overline{\Omega}_2, \tag{7}$$

$$\delta(x,0) = \delta_0(x), \ \delta_t(x,0) = \frac{\partial u_0}{\partial \nu} - \frac{\partial \upsilon_0}{\partial \nu} \equiv \delta_1, \ x \in \overline{\Gamma}_2, \tag{8}$$

where ν is a outer normal of Γ; $\alpha_i > 0 \ (i = 1,2)$ and $\beta > 0$ are some constants; $f_i : R \to R \ (i = 1,2)$, $u_0, u_1 : \overline{\Omega}_1 \to R$, $\upsilon_0, \upsilon_1 : \overline{\Omega}_2 \to R$, $\delta_0 : \overline{\Gamma}_2 \to R$ are given functions.

Let's introduce the following functional space

$$V = H^1_{\Gamma_1}(\Omega_1)\times L^2(\Omega_1)\times H^1(\Omega_2)\times L^2(\Omega_2)\times L^2(\Gamma_2)\times L^2(\Gamma_2).$$

V is a Banach space with respect to the following norm:

$$\|w\|^2_V = \|w_1\|^2_{H^1_{\Gamma_1}(\Omega_1)} + \|w_2\|^2_{L^2(\Omega_1)} + \|w_3\|^2_{H^1(\Omega_2)} + \|w_4\|^2_{L^2(\Omega_2)} + \|w_5\|^2_{L^2(\Gamma_2)} + \|w_6\|^2_{L^2(\Gamma_2)},$$

$\forall w = (w_1, w_2, w_3, w_4, w_5, w_6) \in V$.

Let's define the operator $A: D(A) \subset V \to V$ in this space:

$$Aw = (w_2, \Delta w_1 - w_1 - \alpha_1 w_2, w_4, \Delta w_3 - w_3 - \alpha_2 w_4, w_6, -w_2 - w_5 - \beta w_6),$$

$$D(A) = \{ w = (w_1, w_2, w_3, w_4, w_5, w_6) \in V : \Delta w_1 \in L^2(\Omega_1), w_2 \in H^1(\Omega_1), \Delta w_3 \in L^2(\Omega_2), \\ w_4 \in H^1(\Omega_2), \ w_6 = w_{1\nu} - w_{3\nu}|_{\Gamma_2} \}.$$

The condition $w_6 = w_{1\nu} - w_{3\nu}|_{\Gamma_2}$ must be understood in the following weak sense:

$$\int_{\Omega_1} (\Delta w_1 \varphi + \nabla w_1 \nabla \varphi) dx + \int_{\Omega_2} (\Delta w_3 \psi + \nabla w_3 \nabla \psi) dx = \int_{\Gamma_2} w_6 \varphi \, dx$$

$$\forall \varphi \in H^1(\Omega_1), \forall \psi \in H^1(\Omega_2), \quad \varphi = \psi|_{\Gamma_2}.$$

Let's define the function $\Phi: V \to V$;

$\Phi(w) = (0, -f_1(w_1), 0, -f_2(w_3), 0, 0)$, for $\forall w \in V$. Then the problem (1)-(8) can be rewritten in the following form:

$$\begin{cases} w_t = Aw + \Phi(w) \\ w(0) = w_0 \end{cases}$$

$w = (u, u_t, \upsilon, \upsilon_t, \delta, \delta_t)$ and $w_0 = (u_0, u_1, \upsilon_0, \upsilon_1, \delta_0, \delta_1) \in V$.

Suppose that, $w_0 = (u_0, u_1, \upsilon_0, \upsilon_1, \delta_0, \delta_1) \in V$ then the function $w \in C^0([0, \infty); V)$, satisfying the equality

$$w(t) = e^{At} w_0 + \int_0^t e^{A(t-s)} \Phi(w(s)) ds$$

is a weak solution of the problem (1)-(8) . It's proved that the problem (1)-(8) has a weak solution under some conditions. Thus, the problem (1)-(8) poses a strong continuous subgroup.

It's proved that this subgroup has unique minimal global attractor in the space V .

Published works:

1. AKBAR B. ALIEV AND GULSHAN KH. SHAFIYEVA, POTENTIAL WELLS AND GLOBAL SOLVABILITY OF THE CAUCHY PROBLEM FOR SYSTEM OF SEMI-LINEAR KLEIN-GORDON EQUATIONS WITH DISSIPATION, Proceedings of the Institute of Mathematics and Mechanics, National Academy of Sciences of Azerbaijan Volume 45, Number 1, 2019, Pages 119–136.

Submitted works for publication:

1. **А.Б.Алиев,** С.Э. Исаева, Аттракторы для нелинейных волновых уравнений с акустическими условиями сопряжения, Дифференциальные уравнения.

Work 2: On the boundary problem, which the boundary condition depends on the function of Nevanlinna. **Executers: d.ph.m.s., prof. M.Bayramogly, d.m.s., prof. N.M.Aslanova.**

Boundary problem is considered for the operator coefficient differential equation with rational function of the spectral parameter to the boundary condition. The spectrum and trace of a compatible operator are studied.

Accepted works for publication:

1. **Nigar M. Aslanova, Mamed Bayramoglu , Khalig M. Aslanov** "On one class eigenvalue problem with eigenparameter in boundary condition at one endpoint" Filomat 32:19 (2018), 6667–6674 https://doi.org/10.2298/FIL1819667A

Work 3: Asymptotics of differential equations and Wiman-Valiron-type estimations. **Executor: d.ph.m.s., prof. N.M.Suleymanov**.

Published works:

1. Nadir M. Suleymanov, Dunya E. Farajli. On some applications of spectral asymptotics in Wiman-Valiron theory. Spectral theory and its applications. An international Workshop dedicated to the 80th anniversary of an ac.Mirabbas Gasymov. Baku, June 7-8, 2019, pp. 167-170

Work 4: Smoothness of the solutions of nonlinear equations with discontinuous coefficient. **Executor: d.ph.m.s., prof. T.S.Gadjiev.**

In the work, it is considered linear elliptic equations with discontinuous coefficient. The smoothness of the solution of the Dirichlet problem is studied in the non-smooth ends. Coefficients are taken from BMO spaces. Belonging of generalized solutions to the generalized Morrey spaces has been shown.

Accepted works for publication:

1. Proceeding IAM,2019

2.Transaction IMM,2019

3.EJQTDE,2019

4.UKR math.journal.

5.Vestnik Bakinskogo universiteta,2019

Theses:

1. OMTSA,2019 – 3

2. Регулярность решений классов нелинейных эллиптико-параболицеских задач, Современные Методы Теории Краевых Задач, стр. 97, Воронеж.

3. Regularity of solutions of classes nonlinear elliptic-parabolic problems, International Workshop “Spectral Theory and Its Applications.

Work 5: Asymptotic bifurcation of solutions of nonlinear Dirac problems, structural properties of solutions of linear and nonlinear boundary problems for definite and indefinite weighted second and fourth order differential operators. **Executers: prof. Z.S.Aliyev, j.s.w. H.Rzayeva.**

Bifurcation from zero of solutions of eigenvalue problems for indefinite weighted semi-linear elliptic equations,the oscillation properties of eigenvector functions of the linear Dirac problem with spectral parameter contained in boundary condition, local and global bifurcation from zero of nonlinear Dirac problem, basicity properties of eigen and joint functions system of fourth order ordinary differential operator with spectral parameter contained in boundary conditions in the space $L_p, 1 < p < \infty,$ uniformly convergence of the Fourier expansions on this system.

Published works:

1. Z.S. Aliyev S.M. Hasanova, Global bifurcation of positive solutions of semi-linear elliptic partial differential equations of indefinite weight, Zeitschrift für Analysis und ihre Anwendungen, 38(1) (2019), 1-15.
2. Z.S. Aliyev, P.R. Manafova, Oscillation properties for the Dirac equation with spectral parameter in the boundary condition, Bulletin of the Malaysian Mathematical Sciences Society, 2019, 1-15; doi.org/10.1007/s40840-019-00749-1
3. Z.S. Aliyev, F.M. Namazov, On the spectral problem arising in the mathematical model of bending vibrations of a homogeneous rod, Complex Analysis and Operator Theory, 2019, 19 p.; doi.org/10.1007/s11785-019-00924-z

Accepted works for publication:

1. Z.S. Aliyev, P.R. Manafova, Global bifurcation in nonlinear Dirac problems with spectral parameter in boundary condition, Topological Methods in Nonlinear Analysis, 14 p.

2. З.С. Алиев, Н.Б. Керимов, К. Ф. Абдуллаева, О равномерной сходимости спектральных разложений в терминах корневых функций спектральной задачи для уравнения изгибных колебаний стержня, Математические заметки, 6 с.

Submitted works for publication:

1. Z.S. Aliyev, F. M. Namazov, Spectral properties of the equation of a vibrating rod, at both ends of which the masses are concentrated, Electronic Journal of Qualitative Theory of Differential Equations.

2. Z.S. Aliyev, N.B.Kerimov, On the uniform convergence of Fourier series expansions in the system of eigenfunctions of the equation of vibrating rod with the load concentrated on one end, Rocky Montain Journal of Mathematics.

3. Z.S. Aliyev, E.H. Yusifova, Ya.T. Mehraliyev, On one nonlocal inverse boundary problem for partial differential equations of third order, Analysis and Mathematical Physics.

4. З.С. Алиев, Н.Б. Керимов, В.А. Мехрабов, О сходимости разложений по собственным функциям одной краевой задачи со спектральным параметром в граничных условиях. I и II, Дифференциальные уравнения, 14 с. + 12 с.

Work 6. Asymptotic behaviour of eigenvalues of one boundary problem for second-order elliptic differential operator equation contained in spectral parameter both equation itself and boundary condition. **Executor: d.m.s., prof. B.A.Aliyev.**

Published works:

1. **B.Aliyev,** "Solvability of a boundary value problem for second order elliptic differential operators with a complex parameter in the equation and in the boundary condition" Spectral theory and its applications; An İnternational Workshop dedicated to the 80th anniversary of an academician Mirabbas Geogja oglu Gasymov . p.36-37.

2. **Б.Алиев,** "Асимптотическое поведение собственных значений одной краевой задачи для эллиптического дифференциально-операторного уравнения второго порядка со спектральным параметром в уравнении и в граничном условии", «Дифференциальные уравнения», 2019 (in press).

Work 7: Spectral issues for differential and difference equations and their application to the nonlinear equations. **Executor: d.ph.m.s., prof. Agil Kh. Khanmamedov.**

The following perturbed harmonic oscillator

$$-y'' + x^2 y + q(x)y = \lambda y,\ -\infty < x < \infty,\ \ \lambda \in C, \qquad (1)$$

was studied in all axes, here the potential $q(x)$ is a real-valued function which satisfying the following conditions:

$$q(x) \in C^{(1)}(-\infty,\infty), \int_0^{\infty} |x^j q(x)| dx < \infty,\ j = 0,1,2. \qquad (2)$$

The transformation operator satisfying a condition at infinity is established for the equation (3) . The inverse problem was studied in accordance with the eigenvalues of perturbed harmonic oscillator and spectral data consisting of normalizing numbers. Marchenko type main integral equation was derived and unique solvability of main equation was proved. The algorithm of solving an inverse problem was given.

Besides, the Cauchy problem

$$c_n(0) = \hat{c}_n > 0,\ ,\ n \in Z \qquad (3)$$

is considered for the following nonlinear infinite differential equations:

$$\dot{c}_n = c_n\left(\alpha(c_{n+1} - c_{n-1}) - \beta\left((c_{n+1} - c_{n-1})\sum_{k=0}^{2} c_{n+k}\right)\right),\ \ c_n = c_n(t), n \in Z, t \in (0,\infty],\ \cdot = \frac{d}{dt}, \qquad (4)$$

where α, β are real numbers, the sequence $\hat{c}_n$ satisfies the following condition :

$$\sum_{n \in Z} |n||\hat{c}_n - 1| < \infty.$$

We are looking forward such a solution $c(t) = (c_n(t))_{n \in Z}$ of the problem (3)-(4), so that the function $x_n(t) = c_n(t) - 1$ is rapidly decreasing, i.e., for any $T > 0$, the inequality

$$\|M_1(t)\|_{C[0,T]} < \infty, \qquad (5)$$

holds, where $M_1(t) = \sum_{n \in Z} (1 + |n|) x_n(t)$.

The existence and uniqueness of the solution of the problem (3)-(4) has been proved in the class of (5). The algorithm of finding the rapidly decreasing solution of the problem (3)-(4) has been given by the inverse spectral problem method.

Three articles reflecting the obtained results have been published.

1) **А.Ханмамедов,** Операторы преобразования для возмущенного гармонического осциллятора // Математические заметки, 2019, т. 105, № 5, с.740-746.

2) **А.Ханмамедов**, Алгоритм решения задачи Коши для одной бесконечномерной системы нелинейных дифференциальных уравнений // Журнал вычислительной математики и математической физики, 2019, т. 59, № 2, с. 247-252.

3) **A.Khanmamedov**, The inverse spectral problem for the perturbed harmonic oscillator on the entire axis// Proceedings of the Institute of Mathematics and Mechanics, National Academy of Sciences of Azerbaijan Volume 44, Number 2, 2018, Pages 285–294.

Work 8: Gradient estimations for the parabolic equations in the weighted Morrey spaces. **Executor:** ass.prof. Sh.A.Muradova.

1. Sh.Muradova. "Parabolic-fractional integral operators with rough kernels in parabolic local generalized Morrey spaces", OMTSA-2019, Kutahya, TR, 16-20 July, 2019

Work 9: Investigation of multidimensional mixed problem for the one class third order differential equation with a non-linear operator on the right-hand side. **Executor: d.ph.m. ass.prof. A.G.Aliyeva.**

In the work, the existence and uniqueness theorems have been proved for the almost everywhere solution of the multidimensional mixed problem for the one class third order differential equation with a non-linear operator on the right-hand side.

Published works:

1. S.Aliyev, A.Aliyeva,G.Abdullayeva. On the existence of solution to multidimensional third order nonlinear equations. European Journal Pure and Appliet mathematics, vol.12, No2, 2019, p.577-589. (Thompson Reuters).

Work 10: On the solvability of Drichlet problem for the Laplace equation in the weighted Morrey classes. **Executer: d.ph.m. N.R.Ahmedzade.**

In the report period, the solvability of Drichlet problem for the Laplace equation is studied in the weighted Morrey classes. The obtained results were published in 1 article and 1 thesis.

Published works:

1) **N. R. Ahmedzade**, Z. A. Kasumov, On the Dirichlet problem for the Laplace equation with the boundary value in Morrey space, Eurasian Math. J., 2018, Volume 9, Number 4, 9–21

2) **Ahmedzade N.R.**, Kasumov Z.A. Solvability of the Dirichlet problem for the Laplace equation with boundary value from the Morrey space. International Workshop "Spectral Theory and Its Applications" dedicated to the 80th anniversary of the outstanding mathematician, academician Mirabbas Gasymov. Baku, Azerbaijan, 07-08 June, 2019, pp. 61-64

Submitted works for publication

1. T.B. Gasymov, Akhtyamov A., **N.R. Ahmedzade**. On the basicity in weighted Lebesgue spaces of eigenfunctions of a second-order differential operator with a discontinuity point. Proceedings of the Institute of Mathematics and Mechanics, National Academy of Sciences of Azerbaijan.

SOCIAL ACTIVITY OF COLLABORATORS OF THE "DIFFERENTIAL EQUATIONS" DEPARTMENT

The head of the department prof. Akper Aliev is the member of the Expert Commission under the HAC.

Collaborators of the department are the members of Editorial Boards of the following journals of the İnstitute:

Şöbənin əməkdaşları İnstitutun nəzdnində fəaliyyət göstərən aşağıdakı jurnalların və digər xarici jurnalların Redaksiya Heyətlərinin üzvləridirlər:

- Proceedings of IMM - **prof. Akper Aliev, prof. Mammad Bayramogly, prof. Tahir Gadjiev, prof. Ziyatkhan Aliyev;**

- Transactions of IMM - **prof. Akper Aliev, prof. Mammad Bayramogly, prof. Tahir Gadjiev**;

- Azerbaijan Journal of Mathematics - **prof. Akper Aliev**, **prof. Ziyatkhan Aliyev;**

- Caspian Journal of Applied Mathematics, Ecology and Economics - **prof. Akper Aliev, prof. Mammad Bayramogly, prof. Ziyatkhan Aliyev, prof. Agil Khanmamedov, ass.prof. Nigar Aslanova**.

- Balkan Journal of Mathematics **- ass.prof. Nigar Aslanova**.

Participation in the Institute Seminar

All collaborators have been actively participated in the Institute's general works, including the Institute Seminar.

- Agil Khanmamedov

Gave a talk on the theme "On the inverse scattering problem for the the Schredinger Equation, which has an additional linear potential"

- On May 29, 2019, prof. Tahir Hajiyev gave a talk on the theme "The regularity of the solutions of elliptic equations with the discontinuous coefficient on non-smooth domains"**,**

On June 12, 2019, d.m.s., prof. **Nigar Aslanova** gave a talk on the theme "Learning trace issues when parameter contained in both equation and boundary conditions ", in the Institute's Seminar

-Sevda Isaeva

Gave a talk on the theme "Transmission acoustic problems for nonlinear hyperbolic equations with nonlinear dissipation".

Every Wednesday, at 12.00, the scientific seminar named "Modern problems of the theory of differential equations" guided by A.B.Aliev, is conducted. All collaborators of department, including doctoral and post-graduate students participated in the seminar.

During this period, several scientific works and dissertations were discussed at the department.

On June 11, 2019, master of the department Aishen Mammadova had defended her dissertation work entitled "The solvability of non-linear elliptic-parabolic equations".

The collaboratos of the department, prof. Akper Aliev, prof. Mammad Bayramogly, prof. Ziyatkhan Aliyev, prof. Tahir Gadjiev, **prof.** Agil Khanmamedov**, prof. Bakhram Aliev, ass.prof. Nigar Aslanova**, **ass.prof.** Shamsiya Muradova teaches at the universities of the Republic (Azerbaijan Technical University, BSU, ASPU, **AACU**) for bachelors and masters.

Total– 33

Article- 24 (published and prepared for publication)

Thesis – 9.

Head of Department **prof. Akbar B. Aliev**