

# **ANNUAL REPORT ON SCIENTIFIC, AND SCIENTIFIC ORGANIZATIONAL ACTIVITY OF THE DEPARTMENT OF "EQUATIONS OF MATHEMATICAL PHYSICS" OF INSTITUTE OF MATHEMATICS AND MECHANICS OF NAS OF AZERBAIJAN FOR THE YEAR OF 2020**

In the department of "Equations of Mathematical Physics" 15 workers, 9 of whom are research workers. Of them 3 **doctors of sciences, professors:**

1. Akhundov Adalat Ya. – chief research associate, (full time).
2. Mammadov Farman I. – chief research associate, (full time).
3. Kerimov Nazim B. – chief research associate, (a part time).

## **5 doctors of philosophy in mathematics:**

4. Guliyev Abdurrahim F. – head of department, leading research associate, (full time).
5. Bagirov Shirmail H. – leading research associate, ass. prof., (a part time).
6. Mammadov Elchin M. – leading research associate, ass. prof., (full time).
7. Shukurova Shahla Yu. – senior research associate, (full time).
8. Hasanova Aynur H. – senior research associate, ass. prof., (full time).

## **1 kandidat for a degra:**

9. Mammadli Sayali M. – research associate, (full time).

## **6 laboratory assistants:**

10. Mustafayeva Lala M. – laboratory assistant, (full time).
11. Abdullayeva Aydan J. – laboratory assistant, (full time).
12. Jabrailova (Shafiyeva) Aynur F. – laboratory assistant, (a part time).
13. Asadli Afaq A. – laboratory assistant, (a part time).
14. Gasimov Cesaret C. – laboratory assistant, (a part time).
15. Aliyev Azer A. – laboratory assistant, (a part time).

## **I. SCIENTIFIC PART.**

**In 2020, according to the approved plan, the department conducts 7 research works on the topic "Unambiguous solutions of problems of mathematical physics and qualitative properties of solutions"**

**Work 1: "Some qualitative properties of solutions of second-order parabolic equations".**

**Executor: head of department A.F. Guliyev**

During the reporting period, in special domains, a two-sided estimate was obtained for the fundamental solution of the heat equation. Results are obtained on the equivalence of criteria of the Wiener and Petrovsky type for the regularity of a boundary point with respect to the Dirichlet problem for the heat equation.

**Published papers:**

1. Abdurrahim Quliyev, *On equivalence of Petrovsky and Wiener criteria for the heat equation in symmetrical domains*. 3<sup>rd</sup> International Conference on Mathematical Advances and Applications (ICOMAA 2020), June 24-27 2020, Yildiz Technical University, Turkey.  
<https://icomaa2020.com/wp-content/uploads/2020/06/ICOMAA-2020-ABSTRACT-BOOK-draft.pdf>
2. Guliyev Abdurrahim, Abdullayeva Aydan, *The bilateral estimates of parabolic potentials in special domains*. Respublika Elmi-praktiki konfrans, Lənkəran Dövlət Universiteti, 18 dekabr, 2020.
3. Guliyev Abdurrahim, Asadli Afag, *Wiener's Criterion for the Heat Equation in the symmetric domains*. Respublika Elmi-praktiki konfrans, Lənkəran Dövlət Universiteti, 18 dekabr, 2020.

**Work 2: "The inverse problem of determining the right-hand side of the hyperbolic heat equation".**

**Executor: prof. A.Ya. Akhundov.**

The inverse problem of determining the unknown function on the right-hand side of the heat equation of hyperbolic type is investigated. In addition, the Tikhonov's well-posedness of the inverse problem of determining the unknown coefficients on the right-hand side of a "weak" system of second-order parabolic equations is investigated.

**Published papers:**

1. Ə. Axundov, A. Həbibova, *Bir sinif parabolik tənliklər sistemi üçün tərs məsələ haqqında*. Bakı Mühəndislik Universiteti jurnalı, № 1, 2020.
2. A.Y. Akhundov, N.J. Pashayev, A.Sh. Gabibova, *On an inverse problem for a "weak" system of parabolic equations*. XXXV International Conference

Problems of decision making under uncertainties (PDMU-2020), May 11-15, 2020, Baku-Sheki, Republic of Azerbaijan, p. 10.

[http://pdmu.univ.kiev.ua/PDMU\\_2020/PDMU\\_2020\\_Sheki-Baku.pdf](http://pdmu.univ.kiev.ua/PDMU_2020/PDMU_2020_Sheki-Baku.pdf)

3. Axundov Ə., Aslanov H., Hüseynov Z. *İqtisadi göstəricisi periodik xarakterə malik proseslərin stasionar sıralarının harmonik analizi*. Ümummilli Lider Heydər Əliyevin anadan olmasının 97-ci il dönümünə həsr olunmuş “Rəqəmsal iqtisadiyyatda müasir riyazi üsulların tətbiqi” adlı Beynəlxalq elmi-praktiki konfrans, Bakı Biznes Universiteti, 20 fevral, 2020, 5 səh. .
4. Axundov Ədalət, Əliyev Azərin, *Hiperbolik tənlikdə sağ tərəfin tapılması haqqında tərs məsələ*. Respublika Elmi-praktiki konfrans, Lənkəran Dövlət Universiteti, 18 dekabr, 2020.

### **Work 3: ”Qualitative properties of linear and non-linear degenerate elliptic and parabolic equations”.**

**Executors: prof. F.I. Mammadov, Sh.Yu. Shukurova, S.M. Mammadli.**

During the reporting period, a new Sawyer-type sufficient condition was obtained for the weighted Poincaré inequality of fractional order in an irregular domain. In addition, the Dirichlet problem is considered for the class of second-order semi-linear elliptic equations of nondivergence structure in the case when the coefficients of the principal part are continuous and satisfy the Cordes condition. The existence of a strong solution of the Dirichlet problem is investigated for the equations under consideration. For this problem, for a sufficiently small value of the norm of the right-hand side of this equation, the existence of a solution in Sobolev spaces is proved.

#### **Published papers:**

1. Farman Mamedov, Nazira Mammadzade, Lars-Erik Persson, [\*A New Fractional Order Poincare's Inequality With Weights\*](#). [\*Mathematical Inequalities and Applications\*](#), 2020/4/1, Volume 23, Issue 2, pp. 611-624.  
DOI: [10.7153/mia-2020-23-50](https://doi.org/10.7153/mia-2020-23-50) (WoS, Scopus)
2. Farman Mamedov and Yusuf Zeren, *On local properties of degenerated parabolic equations*. 3<sup>rd</sup> International Conference on Mathematical Advances and Applications (ICOMAA 2020), June 24-27 2020, Yildiz Technical University, Turkey, p. 23.  
<https://icomaa2020.com/wp-content/uploads/2020/06/ICOMAA-2020-ABSTRACT-BOOK-draft.pdf>

3. Farman Mamedov and Sayali Memmedli, *On Liouville theorem for degenerated parabolic equations*. 3<sup>rd</sup> International Conference on Mathematical Advances and Applications (ICOMAA 2020), June 24-27 2020, Yildiz Technical University, Turkey, p. 184.  
<https://icomaa2020.com/wp-content/uploads/2020/06/ICOMAA-2020-ABSTRACT-BOOK-draft.pdf>
4. Farman Mamedov and Cesaret Qasimov, *On estimates of the fundamental solution of the degenerate parabolic equations*. 3<sup>rd</sup> International Conference on Mathematical Advances and Applications (ICOMAA 2020), June 24-27 2020, Yildiz Technical University, Turkey, p. 66.  
<https://icomaa2020.com/wp-content/uploads/2020/06/ICOMAA-2020-ABSTRACT-BOOK-draft.pdf>
5. Farman Mamedov and Nazira Mammadzade, *An interpolation inequality for weight cases*. 3<sup>rd</sup> International Conference on Mathematical Advances and Applications (ICOMAA 2020), June 24-27 2020, Yildiz Technical University, Turkey, p. 152.  
<https://icomaa2020.com/wp-content/uploads/2020/06/ICOMAA-2020-ABSTRACT-BOOK-draft.pdf>
6. Sh.Yu. Salmanova, *The dirichlet problem for of semilinear elliptic equations of the second order*. 3<sup>rd</sup> International Conference on Mathematical Advances and Applications (ICOMAA 2020), June 24-27 2020, Yildiz Technical University, Turkey, p. 189.  
<https://icomaa2020.com/wp-content/uploads/2020/06/ICOMAA-2020-ABSTRACT-BOOK-draft.pdf>

**Work 4: "Some basis properties of ordinary differential operators with a spectral parameter in the boundary conditions".**

**Executor: prof. N.B. Kerimov.**

During the reporting period, the following issues were investigated:

1. Investigation of the spectral properties of the eigenvalue problem (in the case when the problem is reduced to an eigenvalue problem for self-adjoint operators in the one-dimensional Pontryagin space) for ordinary differential equations of the fourth order with a spectral parameter in two boundary conditions, as well as determination of sufficient conditions for the formation of a basis for a system of eigenfunctions in the space  $L_p$ ,  $1 < p < \infty$ , after excluding two functions from this system;

2. Uniform convergence of spectral expansions in terms of the system of eigenfunctions of an eigenvalue problem for ordinary differential equations of the fourth order with a spectral parameter in two boundary conditions.

**Published papers:**

1. Z.S. Aliyev, N.B. Kerimov, V.A. Mehrabov, [\*Convergence of Eigenfunction Expansions for a Boundary Value Problem with Spectral Parameter in the Boundary Conditions. I. Differential Equations\*](#), 2020, v. 56, № 2, pp. 143-157.  
Doi: [10.1134/S0012266120020019](https://doi.org/10.1134/S0012266120020019) (WoS, Scopus)
2. Z.S. Aliyev, N.B. Kerimov, V.A. Mehrabov, [\*Convergence of Eigenfunction Expansions for a Boundary Value Problem with Spectral Parameter in the Boundary Conditions. II. Differential Equations\*](#), 2020, v. 56, № 3, pp. 277-289.  
Doi: [10.1134/S0012266120030015](https://doi.org/10.1134/S0012266120030015) (WoS, Scopus)
3. Konul F. Abdullayeva, Ziyatkhan S. Aliyev and Nazim B. Kerimov, [\*On the uniform convergence of Fourier series expansions in the system of eigenfunctions of the equation of a vibrating rod at one end of which the mass is concentrated\*](#). Proceedings of the Institute of Mathematics and Mechanics, National Academy of Sciences of Azerbaijan, 46 (2) (2020).  
<http://proc.imm.az/inpress/pimm0175.pdf> (WoS, Scopus)

**Work 5:** "The problem of the existence of a global solution to a system of semilinear parabolic equations of divergent form with periodic coefficients with respect to the time argument".

**Executor: ass. prof. Sh.H. Bagirov.**

In the outer infinite domain of the ball, a semi-linear parabolic equation with an operator of the Bauandy-Grushin type in the main part is considered, as well as a semi-linear parabolic equation with the participation of lower-order terms with a singular potential, and for this equation the existence of a global solution to the initial value problem is investigated. An exact sufficient condition is found that ensure the absence of a solution. We also consider a system of semi-linear weakly coupled parabolic equations with bounded, measurable, periodic with respect to the time argument coefficients in an infinite cylindrical domain, the base of which is the outer part of the compact, containing the origin, and the problem of the existence of a global solution depending on the nature of nonlinearity is investigated. An exact sufficient condition is found that ensures the absence of such solutions. The example shows that the found sufficient condition is exact.

### Published papers:

1. Sh. G. Bagirov, *[Nonexistence of Global Positive Solutions of Weakly Coupled Systems of Semilinear Parabolic Equations with Time-Periodic Coefficients. DifferentialEquations](#)*, 2020, v. 56, № 6, pp. 721-733.  
Doi: [10.1134/S0012266120060051](#) (WoS, Scopus)
2. Shirmail G. Bagirov, K.A. Guliyev, *Non-existence of global solution of a semi-linear parabolic equation with a singular potential*, Trans. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci. Mathematics, 40 (1), 79-87 (2020).  
<http://trans.imm.az/volumes/40-1/4001-08.pdf> (Scopus)
3. Ş.H. Bağirov, N.Z. Musayeva, *Sinqulyar potensiallı yarım xətti parabolik tənliyin müsbət qlobal həllinin yoxluğu*, Azərbaycan xalqının ümummilli lideri Heydər Əliyevin anadan olmasının 97-ci il dönümünə həsr olunmuş “Riyaziyyat, mexanika və onların tətbiqləri” adlı respublika elmi konfransı, BDU, Bakı, 2020.
4. Bağirov Ş.H., Qasımova X.Ə. *Zaman arqumentin ənəzərən periodik əmsallı ikinci tərtib yarım xətti parabolik tənliyin müsbət qlobal həllinin varlığı*. Azərbaycan xalqının ümummilli lideri Heydər Əliyevin anadan olmasının 97-ci ildönümünə həsr olunmuş “Riyaziyyat, mexanika və onların tətbiqləri” adlı respublika elmi konfransı, BDU, Bakı, 2020.

### **Work 6: ”Investigation of the qualitative properties of solutions of mixed problems for some nonlinear equations of hyperbolic type”.**

**Executor: ass. prof. E.M. Mammadov.**

During the reporting period, the qualitative properties of solutions for one class of hyperbolic equations with nonlinear boundary conditions were investigated.

### Published papers:

Elchin Mamedov, *On a blow up property of solutions some nonlinear problem*. 3<sup>rd</sup> International Conference on Mathematical Advances and Applications (ICOMAA 2020), June 24-27 2020, Yildiz Technical University, Turkey, p. 214.  
<https://icomaa2020.com/wp-content/uploads/2020/06/ICOMAA-2020-ABSTRACT-BOOK-draft.pdf>

### **Work 7: ”Investigation of an inverse problem for an equation of parabolic type with the Neumann boundary condition”.**

**Executor: ass. prof. A.H. Hasanova.**

The work is devoted to the study of an approximate solution of an inverse problem for a second-order semi-linear parabolic equation with a nonlinear Neumann boundary condition. The problem under consideration is reduced to an equivalent problem – to a system of integral equations, and a theorem is proved that ensures the existence of an integral solution to this problem. The algorithm proposed for the approximate solution of the considered inverse problem is substantiated.

### **Published papers:**

1. Misir Mərdanov, Aynur Həsənova, Səbinə Salmanova, *Riyaziyyat bütün elmlərin açarıdır*. Respublika qəzeti, 13 mart, 2020, № 56 (6675), s. 5.  
<http://www.respublica-news.az/index.php/elm-v-t-hsil/item/26263-riyaziyyat-butun-elmlaerin-achar-d-r>
2. Mərdanov M., Həsənova A. *Nüfuzlu elmi jurnal hazırlamağın qaydaları*. “Azərbaycan məktəbi”. (2020), № 1 (690), s. 177-198.  
[https://journal.edu.az/media/files/Misir\\_Mardanov\\_Aynur\\_Hasanova.pdf](https://journal.edu.az/media/files/Misir_Mardanov_Aynur_Hasanova.pdf)  
Doi: 10.29228/edu.101
3. Aynur Hasanova, *On the existence of a integral solution of the inverse problem for equation of parabolic type*. 3<sup>rd</sup> International Conference on Mathematical Advances and Applications (ICOMAA 2020), June 24-27 2020, Yildiz Technical University, Turkey, p. 48.  
<https://icomaa2020.com/wp-content/uploads/2020/06/ICOMAA-2020-ABSTRACT-BOOK-draft.pdf>

## **II. ORGANIZATIONAL ACTIVITY.**

Head of Department, Ph.D. Abdurrahim Guliyev is the scientific secretary of the Scientific-Thematic Seminar in the specialty "Differential Equations", lectures to the master students of the Institute of Mathematics and Mechanics on the subject "Modern problems of mathematics". Head of the department Abdurrahim Guliyev works as a teacher at Baku State University and in lyceum No.1 with a physical and mathematical bias, is an expert in the SEC on the subject of "mathematics".

Chief research associate of the department prof. Adalat Akhundov is a member of Scientific Council, vice-chairman of Dissertation Board, a member of the editorial staff of the journal «PROCEEDINGS» of Baku University for girls. Professor Adalat Akhundov lectures to the master students of the Institute of Mathematics and Mechanics in the specialties "Differential Equations" and "Equations of Mathematical Physics", works as a professor at Lankaran State University.

Chief research associate of the department prof. Farman Mammadov is a member of the Expert Council of the HAC, a member of editorial board of Azerbaijan and foreign journals, a reviewer of the journal of «Mathematical Reviews of American Mathematical Society».

Chief researcher associate of the department prof. Nazim Kerimov is a professor at the Khazar University.

Leading researcher of the department associate professor Shirmail Bagirov lectures to the master students of the Institute of Mathematics and Mechanics on the subject “Nonlinear differential equations”, works as an associate professor at Baku State University and at the National Aviation Academy.

Leading researcher of the department, ass. prof. Elchin Mammadov is a member of the commission for control of the IMM trade union organization, works as a teacher at Baku State University.

Senior researcher of the department, Shahla Shukurova, works as a teacher at the Khazar University.

Senior research of the department, ass. prof. Aynur Hasanova fellow is a member of a working group set up to use the Thomson Reuters Web of Science network and gather information.

The employee of the department, Aynur Hasanova, is one of the executors of the project “Optimization and application of oil refining methods with a gas lift and deep pump” of the Scientific Fund of the State Oil Company of the Republic of Azerbaijan (SOCAR) (the project was implemented during 2020).

Classes of master students of the department are held online in accordance with programs and schedules. Master and doctoral students continue their scientific research on topics approved by the supervisors. Master student Aynur Jabrailova defended her dissertation online on June 23, 2020 under the guidance of prof. Adalat Akhundov's. The work of the dissertation candidate Sayali Mammadli is progressing according to the plan.

Department employees Abdurrahim Guliyev, Adalat Akhundov, Farman Mammadov, Nazim Kerimov, Shirmail Bagirov, Elchin Mammadov, Shahla Shukurova continue online classes with master's and bachelor's degree students.

Head of the department, Abdurrahim Guliyev and Professor Farman Mammadov continuing their cooperation with Turkish scientists, expanded their scientific ties. Also, Professor Farman Mammadov continues to cooperate with Italian scientists. In addition,



employees of our department cooperate with scientists from Russia, Ukraine, Turkey, America, Sweden and other countries.

Employees of the department, led by Abdurrahim Guliyev, are working on the “English-Azerbaijani-Russian” dictionary of mathematical terms.

Every week, on Wednesdays, under the leadership of department head Abdurrahim Guliyev, and on Mondays, under the leadership of Farman Mammadov, the seminar traditionally carries out its work department on the topic "Modern problems of mathematical physics".

### **KONFERENCES**

Head of the Department, Abdurrahim Guliyev and Chief research associate of the department Professor Farman Mammadov (as a plenary speaker on the topic “On local properties of degenerated parabolic equations”, <https://icomaa2020.com/wp-content/uploads/2020/06/ICOMAA-2020-ABSTRACT-BOOK-draft.pdf>), Elchin Mammadov, Shahla Shukurova, Aynur Hasanova, Sayali Mammadli, master student Jesaret Gasimov took part online in the 3<sup>rd</sup> International Scientific Conference entitled “Mathematical Advances and Applications”, which was held on June 24-27, 2020 at Yildiz Technical University, Istanbul, Turkey.

*Thus, in 2020, employees of the department published 24 papers, of which 7 scientific papers (6 included in journals from the Web of Science and Scopus list (4 papers abroad)), 2 popular science articles, 15 abstracts (8 abroad), 12 papers were submitted for publication.*

**Head of department:**

**Ph. D. Abdurrahim Guliyev**