

Annual Report of Functional Analysis department on scientific and scientific activity for 2020

The staff of “Functional Analysis” department consists of 21 employees including 18 research associates. 8 of them doctor of sciences, professor.

- 1.Aslanov Hamidulla I. doct. ph.m.s.,prof.sen.r.a.
- 2.Aliyev Soltan A. doct. ph.m.s.,prof.sen.r.a.
- 3.Mirzoyev Sabir S. doct. ph.m.s.,prof.sen.r.a.
- 4.Huseynov Hidayet M. doct. ph.m.s.,prof.sen.r.a.
- 5.Gurbanov Veli M. doct. ph.m.s.,prof.sen.r.a.
- 6.Nabiyev Ibrahim M. doct. ph.m.s.,prof.sen.r.a.
- 7.Aliyev Araz R. doct. ph.m.s.,prof.sen.r.a.
- 8.Eyvazov Elshad H. ph.m.s., lead.r.a.
- 9.Mukhtarov Fekhreddin Sh. c.ph.m.s.,lead.r.a.
- 10.Jabrailova Afet N. c.ph.m.s.,lead.r.a.
- 11.Ibadova Irade A. c.ph.m.s.,lead.r.a.
- 12.Latifova Aygun R. phd in math.,chief.r.a.
- 13.Vahabov Nazim G. chief.r.a.
- 14.Guliyev Namiq I. c.ph.m.s.,r.a.
- 15.Khalilov Vuqar S. phd in math.,chief.r.a.
- 16.Alimardanova Kamilla A. c.ph.m.s.,chief. r.a.
- 17.Osmanli Jalala A. phd in math.,chief. r.a.
- 18.Safarova Aynur N.j.r.a.
- 19.Iskenderli Guller Z.sen.lab.ass.
- 20.Bayramova Aygun F.sen.lab.ass.
- 21.Dadashova Nigar Y.lab.ass.

I. Scientific part.

In 2020, 18 scientific works on “Studying some problems of operator algebras and probability theory” and “Spectral analysis of differential operators” were carried out.

Theme: “Studying some problems of operator algebras and probability theory.”

1. Work: «Studying limit theorems for branching processes by means of generating function». Ex.: doct.ph.m.sc., prof. sen.res.ass. S.A.Aliyev.

In the report period, the immigration case of the sequence of many-dimensional discrete parameter branching random processes was considered. The convergence of the processes under consideration was studied under certain conditions imposed on of the number of derivative particles generated in any particles on the average values matrix. On this topic 3 papers and 2 abstracts were published.

Papers

- 1.S.A.Aliyev. “The optimal irrigation under water use decisions.” Сборник научных трудов Национальный Университета водн.хоз., Ровно ,Украина, вып. 1-2, 2019, с.74-78.
- 2.S.A.Aliyev. “ Calculating steady –state probabilities of the G/M/n/m queueing systems”, Trans.NAS of Azerb.,ser. phis,-tech.and math.sci., Mathematics 39(4), 9-16(2019).
3. Aliyev Soltan, Khalilov Vüqar. “Integral equations for age –dependent branching processes”, Nakhchivan Teachers. Ins., Scientific works, 2019, №3,168-172.

Abstracts

- 1.S.A.Aliyev. “Branching process with special generating function.” Int. konf, “Modern problems of match.and mech.”, 2019,p.94-96.
- 2.S.A.Aliyev. “Convergence of sequence of multidimensional branching random processes,” Республиканская конференция посвящение 97-летию Г.Алиева, Бизнес Университет, Баку, феврал 2020.

2.Work: «Geometry of Banach spaces and operators class Lax-Milgram type theorems». Ex: sen.res.ass. N.G.Vahabov.

The Lax-Milgram theorem is proved by means of the Wintner-Stone theorem on localization of the spectrum of the operator with numerical image in Hilbert

space. Furthermore, the Wintner-Stone theorem was obtained by Berberian expansion of the Hilbert space. By this, the Wintner-Stone theorem is reduced to the classic Toklits theorem on the localization of point spectrum.

3. Work: «Studying boundary value problems for random walks described by one order autoregression processes». Ex: cand.ph.m.sc. ass.prof. lead.res.ass. I.A.Ibadova.

The linear boundary value problems for Markov random walks described by the sum of squares of one order autoregression process is studied. So,

$$\tau_a = \inf \{n \geq 1 : S_n \geq a\};$$

the first crossing moment of the Markov random walk $S_n = \sum_{k=1}^n X_{k-1}^2, n \geq 1$ the level a is considered. For (1) type $\tau_a, a > 0$ family, the integral limit theorem and law of large numbers is proved.

Paper

1.I.A. Ibadova A.D. Farhadova .On strong law of large numbers for the family of first passage times for the level in random walk described by a non-linear function of autoregression process of order one (AR (1)) . Caspian Journal of Applied Mathematics, Ecology and Economics V. 7, No 1, 2019, July ISSN 1560-4055 pp.41-45

One paper and two abstracts was submitted to publication.

1.F.H. Rahimov, I.A. Ibadova and A.D. Farhadova. Limit theorem for first passage times in the random walk described by the generalization of the autoregressive process. Uzbek Mathematical Journal

2.F.H. Rahimov , A.D. Farhadova , I.A. Ibadova . On the family of the first passage time of the parabola by a random walk described by the autoregressive process. Национальный Университет Узбекистана им. Мирзо Улугбека Институт Математики им В.И.Романовского Академии Наук Республики Узбекистан республиканская научная конференция «современные проблемы стохастического анализа»

(□, F, P) 11- 12 мая 2020 г. Ташкент

3.F.H. Rahimov S.A.Aliyev. İ.A.İbadova. Limit theorems in the random walk described by the generalization of the autoregressive process. The 7th International Conference on Control and Optimization with Industrial Applications 26-28 August 2020 Baku, Azerbaijan, p.320-322.

4.Work: «Nonlinear boundary value problems for random walks described by autoregression process». Ex: ph.d.in math. ass. prof. lead res. ass. V.S.Xalilov.

In the report year, linear and non-linear boundary value problem for a class of random walks described by one-order autoregression process were studied. Age-dependent branching processes were considered and the results were published.

Paper

1.Aliyev Soltan, Khalilov Vuqar. İnteqral equations for age-dependent branching processes. Naxçıvan Müəllimlər İnstitutu. Elmi əsərlər 2019, №3, səh. 168-172.

Abstract

1.Fada H.Ragimov, Vugar S.Khalilov, Aynura D. Faxradova. On the central limit theorem for the least-squares estimator of the unknown parameter in the autoregressive process of order one (AR(1)). The 7 th International Conference on Control and Optimization With Industrial Application, 26 – 28 august 2020, Baku, Azerbaijan, p.338-340.

5.Work :«Studying eigen-values of magnetic Schrodinger operator with regard to magnetic stress».Ex:doct.ph.m.s., ass.prof. sen.res.ass. E.H.Eyvazov.

In the report year scientific research on the topic were carried out, one paper and 2 abstracts were published.

Paper

1.Elshad H. Eyvazov, Davud H. Orujov.“On negative eigenvalues of the Schrodinger operator” , Proceedings of the Institute of Mathematics and Mechanics, National Academy of Sciences of Azerbaijan Volume 46, Issue 2 , p. 9, 2020, Elektron variant (Online First)

Abstracts

1. Elshad H. Eyvazov. “Differential equation for eigenvalues uncertainties” (PDMU-2020) ABSTRACTS May 11-15, 2020, Baku-Sheki, Republic of Azerbaijan, P.41-42.

2. Эйвазов Эльшад Хатам оглы, “ Модели магнитных Лапласианов с магнитным полем четвертого рода,” международная научная конференция

“Уфимская осенняя математическая школа”, г. Уфа, с 11 по 14 ноября 2020 года., сборник тезисов, часть 1., стр-71.

Theme: “Spectral analyses of differential operators ”

6.Work: «Studying some spectral properties of elliptic type operator equations» Ex: doct.ph.m.sc.prof. sen.res.ass. S.S.Mirzoyev.

The work deals with operator-differential equations. The structure of the spectrum of the problem was studied under some conditions on the coefficients. A theorem on multiple completeness of the system of eigen and associated functions in the space of elementary solutions was proved.

7.Work: «On the existence of generalized solution of a class of partial operator equations in Hilbert space». Ex. doct.ph.m.sc.prof. sen.res.ass. H.I.Aslanov.

Let H – be a separable Hilbert space. In the space $W_2^2(R^n, H)$

$$l(u) = -\sum_{k=1}^n C_k \frac{\partial^2 u}{\partial x_k^2} + \sum_{k=1}^n T_k \frac{\partial u}{\partial x_k} + A^2 u = f(x)$$

we consider the differential equation $x = (x_1, x_2, \dots, x_n)$. Here T_k, A – are self-adjoint operators included in the class of definite operators. The conditions of existence of the generalized solution of the equation in the Sobolev type vector functions were determined

Textbook

1.H.I.Aslanov. “Funksiyalar nəzəriyyəsi və funksional analiz,” Dərs vəsaiti, 726 səhifə.

Conference materials

1.H.İ.Aslanov. “İqtisadi göstəricisi periodik xarakterə malik proseslərin stasionar zaman sıralarının harmonik analizi”, Ümumilli Lider Heydər Əliyevin anadan

olmasının 97-ci ildönümünə həsr olunmuş “Rəqəmsal iqtisadiyyatda müasir riyazi üsulların tətbiqləri”, mövzusunda Respublika elmi-praktik konfransı, Bakı Biznes Universiteti, 20 fevral 2020-ci il.

8.Work: «Direct and inverse scattering problems for discontinuous coefficients Dirac equation» Ex: doct.ph.m.sc. prof. sen.res.as. H.M.Huseynov.

In the work the Dirac system of equations (of second order) is considered on the axis. At any points $a \in (-\infty, -\infty)$ the first component of the solution is continuous, the second components is discontinuous. The lost solutions containing the asymptotics of the problem in positive and negative infinity are structured the existence of transformation operators is shown and scattering data are introduced. Then the main integral equation of the inverse problem is derived and its uniqueness is shown an algorithm for solving the inverse problem is given. One paper was submitted to “Turk.J.Math.”

Conference materials

1. H.M. Hüseyinov, İ.M. Nəbiyev. Çoxfaktorlu istehsal funksiyasının bəzi xassələri . Ümummilli Lider H. Əliyevin anadan olmasının 97-ci ildönümünə həsr olunmuş «Rəqəmsal iqtisadiyyatda müasir riyazi üsulların tətbiqləri» mövzusunda Beynəlxalq elmi-praktik konfransın materialları, Bakı, 20 fevral 2020, Biznes Universiteti nəşriyyatı.

9.Work: «Convergence of spectral expansions corresponding to add order differential operators» Ex: doct.ph.m.sc. prof. sen.res.ass V.M.Kurbanov.

In the report year, convergence of spectral expansions corresponding add order differential operators was studied. Sufficient conditions for absolute and regular convergence of functions included in Sobolev space in eigen functions were found.

Papers

1) V.M.Kurbanov, A.M.Abdullayeva. “ On local uniform equiconvergence rate for the Dirac operator”, Proc. of the IMM, NAS of Azerbaijan, Volume 46, Number 1, 2020, Page 16-31.

2) V.M.Kurbanov, G.R.Gadzhieva. "Bessel inequality and the basis property for $2m \times 2m$ Dirac type system with an integral potential", Differential Equation 2020, Volume 56, Number 5, Page 573-584.

10.Work: «Spectral properties of Sturm-Liouville operators with a quadratic function of the parameter in the boundary condition». Ex: doct.ph.m.sc prof. sen.res.as. I.M.Nabiyev.

In the report period (2020) some spectral properties of Sturm-Liouville operator with nonseparated boundary condition were studied one of the boundary conditions contain a quadratic function of the parameter. A condition for the validity of the spectrum was found, asymptotics of eigenvalues was studied. Furthermore, the representation of the characteristic function by means of the eigenvalues in the form of infinite product was obtained and it was proved that the considered operator has no functions associated to the eigen-functions.

Papers

1.A.G. Ferzullazadeh, I.M. Nabiev. Some properties of the spectrum of t Dirac operator with a spectral parameter in the boundary condition

// Proceedings of IMM of NAS of Azerbaijan, 2020, v. 46, № 2., p.189-196. Web of Science (ESCI), Scopus 0.28 <http://proc.imm.az/inpress/pimm0172.pdf>

2. Л.И. Маммадова, И.М. Набиев. Спектральные свойства оператора Штурма-Лиувилля со спектральным параметром, квадратично входящим в граничное условие // Вестник Удмуртского университета. Математика. Механика. Компьютерные науки. 2020. Т. 30. Вып. 2. С. 237–248. Web of Science (ESCI), Scopus 0.401 http://vst.ics.org.ru/uploads/vestnik/2_2020/20-02-07.pdf

Conference materials

1. H.M. Hüseyinov, İ.M. Nəbiyev. Çoxfaktorlu istehsal funksiyasının bəzi xassələri / Ümummilli Lider H. Əliyevin anadan olmasının 97-ci ildönümünə həsr olunmuş «Rəqəmsal iqtisadiyyatda müasir riyazi üsulların tətbiqləri» mövzusunda Beynəlxalq elmi-praktik konfransın materialları, Bakı, 20 fevral 2020, Biznes Universiteti nəşriyyatı.

11. Work: «On radiation conditions of many-dimensional magnetic Helmholtz equation». Ex: doct.ph.m.s. prof. sen.res.ass. A.R.Aliyev.

In the work it is shown that for the existence of the unique solution of the magnetic Helmholtz equation it is sufficient to give weaker radiation conditions than Ikabe-Saito conditions. Alongside with this it was proved that the Helmholtz operator is self-adjoint. The existence of the solution of inhomogeneous Helmholtz equation satisfying the radiation condition was justified. These results were given in [3].

Papers

1. Aliev A.R., Rzayev R.R. About one approach to the description of semi-structured indicators on a given data sample / In: 10th International Conference on Theory and Application of Soft Computing, Computing with Words and Perceptions - ICSCCW-2019. Advances in Intelligent Systems and Computing, vol. 1095. Springer Nature Switzerland AG, 2020, p.p. 436-444. (Scopus) https://link.springer.com/chapter/10.1007/978-3-030-35249-3_56#citeas

2. Aliev A.R., Mamedov V.M., Seyidov M.I. About one mathematical model of reliability and safety of complex systems / In: 10th International Conference on Theory and Application of Soft Computing, Computing with Words and Perceptions - ICSCCW-2019. Advances in Intelligent Systems and Computing, vol. 1095. Springer Nature Switzerland AG, 2020, p.p. 712-719 (Scopus) https://link.springer.com/chapter/10.1007/978-3-030-35249-3_92#citeas

3. Aliev A.R., Rajabov S.S. Radiation conditions for the magnetic Helmholtz equation // Mathematical Notes, 2020, vol. 108, no. 1-2, p.p. 155–161. (Web of Science və Scopus) <https://link.springer.com/article/10.1134/S0001434620070160>

12. Work: «On frames approximated by multiplication operator». Ex: cand.ph.m.sc. ass.prof. lead r.a. A.N.Jabrailova.

In the work it is shown that for the function $f(t)$ summable with the measurable function $\varphi(t)$ and square the approximations $T_\varphi f(t) = \varphi(t)f(t)$ obtained from the multiplication operator $\{T_\varphi^n\}_{n=0}^\infty$ may be frame in $L_2(a,b)$. In the

special case this fact shows that the systems in the form $\{\varphi^n(t)\}_{n=0}^{\infty}$ can not be frame in $L_2(a,b)$ for any measurable $\varphi(t)$ function.

1. Afet Jabrailova, Aydin Shukurov. "On frames that are iterates of a multiplication operator" Revista Colombiana de Matematicas, 2020 (was submitted to publication.)

2. Джабраилова А.Н. «О кратной полноте системы собственных и присоединенных элементов операторного пучка в гильбертовом пространстве. Междун. Научная конф. «Уфимская осенняя матем. школа» 23-26 сентября 2020 г. Башкирский Гос. Унив. Г. Уфа

13.Work: «Spectral properties of two-interval anti-periodic Sturm-Liouville problem». Ex: cand.ph.m.s.ass prof. lead r.a. F.Sh.Mukhtarov.

In the work in the domain $[-1,0) \cup (0,1]$ some spectral properties of the problem determined by the expression $Lu = -u''(x) + q(x)u(x) = \lambda u(x)$ anti-periodic conditions $u(-1) = -u(1), u'(-1) = -u'(1)$ and the conditions $u(+0) = Ku(-0), u'(+0) = \frac{1}{k}u'(-0)$ at the discontinuity point were studied. It was proved that the given problem has numerous eigen-values. These eigenvalues are real numbers. The eigenfunctions associated with different eigenvalues are orthogonal in the space $L_2(-1,0) \oplus L_2(0,1)$. The asymptotic formulas of eigen values was proved.

1. Serdar PAŞ, Kadriye Aydemir, Fahreddin Muhtarov. Spectral Properties of the Anti-Periodic Boundary-Value-Transition problems. Journal of New Theory. ISSN:2149-1402(was submitted to publication.)

14.Work: «Direct problems of spectral analysis for a Bessel operator with a spectral parameter in the boundary condition». Ex: cand.ph.m.s. lead.r.a. N.J.Guliyev.

In the report period, some direct problems of spectral analysis for a Bessel operator with a spectral parameter in the boundary condition were studied. The paper was submitted to publication.

Papers

1. Guliyev N.J. [*On extensions of symmetric operators*](#), Oper. Matrices 14 (2020), no. 1, 71–75. (2019 impact-factor: **0.417**)
2. Guliyev N.J. [*A Riesz basis criterion for Schrödinger operators with boundary conditions dependent on the eigenvalue parameter*](#), Anal. Math. Phys. 10 (2020), no. 1, Paper No. 2, 8 pp. (2019 impact-factor: **2.056**)
3. Guliyev N.J. [*Essentially isospectral transformations and their applications*](#), Ann. Mat. Pura Appl., 199 (2020), no. 4, 1621–1648. (2019 impact-factor: **0.959**)
4. Guliyev N.J. [*On two-spectra inverse problems*](#), Proc. Amer. Math. Soc., 148 (2020), no. 10, 4491–4502. (2019 impact-factor: **0.927**)

15.Work: «Direct and inverse scattering problems for the system of ordinary differential equations». Ex: cand.ph.m.s. sen.r.a. K.I.Alimardanova

In the report period in the general and special cases the direct and inverse scattering problems were studied for six ordinary differential equations on a semi-axis $x > 0$. One paper was prepared for publication.

16.Work: «Representation of integral solution for the system of Dirac equation when there is a discontinuity condition inside the interval at two points». Ex: ph.d. in math. sen.res.ass. A.R. Latifova.

Paper

1. Latifova, A. R., and A. K. Khanmamedov. “Inverse Spectral Problem for the One-Dimensional Stark Operator on the Half-Axis”. Ukrains’kyi Matematychnyi Zhurnal, Vol. 72, no. 4, Mar. 2020, pp. 494-08, doi:10.37863/umzh.v72i4.2302.

17.Work: «Direct problem for Sturm-Liouville operator in the impedance form». Ex: ph.d.m.s. J.A.Osmanly.

In the report year the spectral solutions of the equation were studied. Integral representation for them were obtained and spectral data were introduced.

18.Work: «Direct and inverse scattering problems for the system of hyperbolic equations on a semi-axis in the case of 2 incident wave». Ex: sen.r.a.a A.N.Safarova.

In the work for the equation

$$u_{tt}(x,t) - \beta u_{ttxx}(x,t) - du_{xx}(x,t) = d(t)u(x,t) + (x,t) \quad (1)$$

in the rectangular $D_T = \{(x,t): 0 \leq x \leq 1, 0 \leq t \leq T\}$ we study a boundary value problem with nonlocal first wind integral condition

$$\sum_0^T W_i(t)u(x,t)dt = h_i(x) \quad (i = 1,2; 0 \leq x \leq 1) \quad (2)$$

with respect to time and Neumann boundary condition

$$u_x(0,t) = 0, \quad u_x(1,t) = 0 \quad (0 \leq t \leq T) \quad (3).$$

1. А.Н.Сафарова., Г.М.Назарова. Об одно задаче с нелокальным по переменной времени условием для уравнения распространения продольных волн. Riyaziyya, Mexanika və onların tətbiqləri adlı Respublika Elmi Konfransının materialları, Bdu, 2020, səh 159-160.

II. Scientific-social activity.

doct. ph.m.s.,prof. H.I. Aslanov was a chairman of the State Attestation Commission in the mechanics-mathematics of BSU. He supervises 3 doctoral students.

doct. ph.m.s.,prof.sen.r.a. S.A. Aliyev is a member of the editorial board of six scientific journals. Has supervised one ph.d. dissertation Engaged in pedagogical activity at BSU.

doct. ph.m.s.,prof.sen.r.a. A.R. Aliyev is an editor-in-chief of the international scientific journal “Azerbaijan Journal of High performance computing” (Azerbaijan), manager editor of the international scientific journal “Azerbaijan journal of Mathematics” and a member of the journals: «Proceedings of the Institute Mathematics and Mechanics, National Academy of Sciences of Azerbaijan» (Azərbaycan), «Transactions of Azerbaijan Institutes of Technology» (Azərbaycan).

doct. ph.m.s.,prof.sen.r.a. I.M. Nabiyev has participated at 1 international conference. Has published 3 scientific papers (including 2 papers, one conference material the papers were published in Web of Science, Scopus basis journals). He

was a chairman of State Attestation Commission at Azerb. State Oil and Industry University (on the Masters degree).

doct. ph.m.s.,prof.sen.r.a. H.M. Huseynov is a member of editorial board of a lot of international and republican scientific journals. He is an adviser of magisters and doctoral students.

doct. ph.m.s.,prof.sen.r.a. S.S. Mirzoyev's two cand. for degrees have submitted their dissertation works to the Scientific Council of IMM.

doct. ph.m.s.,prof.sen.r.a. V.M. Gurbanov is a member of editorial board of a lot of international and republican scientific journals. He is an adviser of magisters and doctoral students.

Many of the department collaborators are engaged in pedagogical activity in different higher education institutions of the Republic.

PARTICIPATION IN SCIENTIFIC SEMINARS

All the collaborators have participated in the institute and department seminars.

PUBLISHED SCIENTIFIC PAPERS

In the report period the employees of our department have published 1 textbook, 17 papers, 6 abstracts, 2 proceedings of conference, 4 papers and 3 abstracts were submitted for publication. 12 of these papers are in journals Web of Science and Scopus base and Thomson Reuters lists.

Head of department:

d.ph.m.s.prof. H.İ.Aslanov

