

## REPORT OF “MATHEMATICAL ANALYSIS” DEPARTMENT ON SCIENTIFIC AND SCIENTIFIC ACTIVITY FOR 2020

The staff of “Mathematical Analysis” department consists of 11 research associates, including 4 doctor of sciences, professor, one corresponding member of ANAS.

1. Vagif Guliyev – head of department (doct. phys. math. sci. prof. Corr. member of ANAS).
2. Rovshan Bandaliyev- cand. phys. math. sci., ass. prof., sen. res. ass.
3. Elman Ibrahimov – cand. phys. math. sci., ass. prof., sen. res. ass.
4. Hajibeyov Mubariz- cand. phys. math. sci., ass. prof., sen. res. ass.
5. Zaman Safarov - cand. phys. math. sci., ass. prof., lead. res. ass.
6. Elmira Hajiyeva – cand. phys. math. sci., res. ass.
7. Mehriban Omarova – ph. doctor in math., lead. res. ass.
8. Lala Aliyeva - ph. doctor in math., great res. ass.
9. Fatayi Isayev – cand. phys. math. great res. ass.
10. Aytekin Abdullayeva – cand. phys. math. great res. ass.
11. Aynur Mammadova – cand. phys. math. great res. ass.

### I. SCIENTIFIC PART

In 2020, according to the affirmed plan, six scientific works on the themes of “Modern problems of harmonic analysis” are carried out.

**Work 1:** Regularity of solutions to elliptic equations with BMO coefficients in Orlicz-Morrey-type spaces.

**Executors:** Head of department, corr. member of ANAS, prof. V.S. Guliyev, ph. doctor in math., ass. prof., lead. res. ass. M.N. Omarova

Sufficient conditions were proved for boundedness of parabolic singular and non-singular integral operator and its commutators on generalized parabolic Orlicz-Morrey-type space . Based on the results of researches , global regularity of solution to non-divergent elliptic equations with VMO coefficients was investigated in generalized parabolic Orlicz-Morrey-type spaces. According to this study, the following papers were published:

1. **V.S. Guliyev**, I. Ekincioglu, A.A. Ahmadli, **M.N. Omarova**, Global regularity in Orlicz-Morrey spaces of solutions to parabolic equations with VMO coefficients, **Journal of Pseudo-Differential Operators and Applications** 11 (4) (2020), 1963-1989. (WoS, IF-0.532) Q4
2. **V.S. Guliyev**, R.V. Guliyev, **M.N. Omarova**, M.A. Ragusa, *Schrödinger type operators on local generalized Morrey spaces related to certain nonnegative potentials*, **Discrete and Continuous Dynamical Systems-Series B**, 25 (2) (2020), 671-690. (WoS, IF-1.270) Q2

3. T.S. Gadjiev, **V.S. Guliyev**, K.G. Suleymanova, *The Dirichlet problem for the uniformly elliptic equation in generalized weighted Morrey spaces*, **Studia Scientiarum Mathematicarum Hungarica** 57 (1) (2020), 68-90. (WoS, IF-0.468) Q4
4. **V.S. Guliyev**, A. Ahmadli, S.E. Ekincioglu, *Oscillatory integrals with variable Calderón-Zygmund kernel on vanishing generalized Morrey spaces*, **Tbilisi Math. J.** 13 (2020), no. 1, 69–82. (WoS, ESCI)
5. **V.S. Guliyev**, H. Armutcu, T. Azeroglu, *Characterizations for the potential operators on Carleson curves in local generalized Morrey spaces*, **Open Mathematics** 18 (1) (2020), 1317-1331. (WoS, IF-0.773) Q3
6. I. Ekincioglu, **V.S. Guliyev**, Esra Kaya,  *$B_n$ -Maximal operator and  $B_n$ -singular integral operators on variable exponent Lebesgue spaces*, **Math. Slovaca** 70(4) (2020), 893-902. (WoS, IF-0.654) Q3
7. **V.S. Guliyev**, A. Kucukaslan, C. Aykol, A. Serbetci, *Riesz potential in the local Morrey–Lorentz spaces and some applications*, **Georgian Math. J.** 27 (2020), no. 4, 557–567. (WoS, IF-0.500) Q4
8. **V.S. Guliyev**, Y.Y. Mammadov, F.A. Muslumova, *Boundedness Characterization of Maximal Commutators on Orlicz Spaces in the Dunkl Setting*, **J. Math. Study**, 53 (1) (2020), 45-65. (WoS, ESCI)
9. A. Eroglu, G.A. Abasova, **V.S. Guliyev**, *Spanne type characterization of parabolic fractional maximal function and its commutators in parabolic generalized Orlicz-Morrey spaces*, Springer Proceedings in Mathematics and Statistics : (Vol. 1) **Operator Theory and Harmonic Analysis**. Vol. 1: New General Trends and Advances of the Theory, 2020, 35-49. (WoS, ESCI)
10. A. Akbulut, V.I.Burenkov, **V.S.Guliyev**, *Anisotropic fractional maximal commutators with BMO functions on anisotropic Morrey-type spaces*, **Trans. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci.** 40 (4), **Mathematics**, 13-32 (2020). (SCOPUS)
11. **M.N. Omarova**, *Characterizations for the commutator of parabolic nonsingular integral operator on parabolic generalized Orlicz-Morrey spaces*, **Tbilisi Math. J.** Vol. 13(1) (2020), pp. 97-111. (WoS, ESCI)
12. **M.N. Omarova**, *Nonsingular integral operator and its commutators on vanishing generalized Orlicz-Morrey spaces*, **Azerbaijan Journal of Mathematics**, Vol. 10(2) (2020), pp. 140-156. (WoS, ESCI)
13. **M.N. Omarova**, *Parabolic non-singular integral operator and its commutators on parabolic vanishing generalized Orlicz-Morrey spaces*, **TWMS J. Pure Appl. Math.**, V.11, N.2, 2020, pp. 213-225. (WoS, ESCI)

**14. M.N. Omarova**, *Global regularity in Orlicz-Morrey spaces of solutions to elliptic and parabolic equations with VMO coefficients*, PROCEEDINGS of the 7 th International Conference on "Control and Optimization with Industrial Applications", Vol. 2 (2020), 293-295. (WoS)

**Work 2: Compactness theorem in Morrey spaces with variable exponents.**

**Executors:** sen. res. ass. (doct. math. sci. prof. of ANAS) Rovshan Bandaliyev., cand. phys. math. sci., ass. prof., lead. res. ass Zaman Safarov

With regards to affirmed plan , analogue of Kolmogorov-Riss compactness theorem was investigated for Morrey space with variable exponent .Based on the results of researches , the following papers were published:

**15. R.A. Bandaliyev**, I.G. Mamedov, M.J. Mardanov, T.K. Melikov, *Fractional optimal control problem for ordinary differential equation in weighted Lebesgue spaces*, **Optimization Letter**, 14(6), 1519–1532 (2020) (WoS, IF-1,502) Q2

**16. R.A. Bandaliyev**, K.H. Safarova, *On boundedness of multidimensional Hausdorff operator on weighted Lebesgue spaces*, **Tbilisi Mathematical Journal**, 13(1) (2020), 39-45 (WoS, ESCI)

**17. Y.Y. Mammadov**, F.A. Muslumova, **Z.V. Safarov**, *Fractional maximal commutator on Orlicz spaces for the Dunkl operator on the real line*, **Trans. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci.** 40 (4), **Mathematics**, 130-144 (2020). (SCOPUS)

**Theses**

**R.A. Bandaliyev**, K.H. Safarova, *Characterization of the Riemann-Liouville fractional integral via the ordinary fractional integro-differential equation*, Ümummilli lider Heydər Əliyevun 97-ci ildönümünə həsr olunmuş “Riyaziyyat, Mexanika və Onların Tətbiqləri” adlı Respublika elmi konfransı. Bakı, BDU, 20-21 may, 2020, 178-179 .

**Work 3: The boundedness of logarithmic potentials on hypergroups.**

**Executors:** cand. phys. math. sci., ass. prof., sen. res. ass Hajibeyov Mubariz. , cand. phys. math. sci., res. ass Elmira Hajiyeva

The boundedness of logarithmic potentials on hypergroups was investigated. Works on this study are carrying out.

**Work 4: The boundedness of maximal and potential operators generated by Gegenbauer differential operator in generalized Morrey spaces.**

**Executors:** cand. phys. math. sci., ass. prof., sen. res. ass Elman Ibrahimov

Studies on the boundedness of maximal and potential operators generated by Gegenbauer differential operator in generalized Morrey spaces is carrying out.

18. **V.S. Guliyev, E.J. Ibrahimov, S.E. Ekincioglu, S.Ar. Jafarova**, *O'Neil inequality for convolutions associated with Gegenbauer differential operator and some applications*, **J. Math. Study**, 53 (1) (2020), 90-124. (WoS, ESCI)
19. **E.J. Ibrahimov, G.A. Dadashova, S.A. Jafarova**, *Boundedness of higher order commutators of  $G$ -fractional integral and  $G$ -fractional maximal operators with  $G$ -BMO functions*, **Trans. A. Razmadze Math. Inst.** Vol. 174 (2020), issue 3, 325-341. (WoS, ESCI)
20. **E.J. Ibrahimov, S.A. Jafarova, S.E. Ekincioglu**, *Maximal and potential operators associated with Gegenbauer differential operator on generalized Morrey spaces*, **Proc. Inst. Math. Mech. Natl. Acad. Sci. Azerb.** 46 (1), 129-143 (2020). (WoS, ESCI)
21. **E.J. Ibrahimov, G.A. Dadashova, S.E. Ekincioglu**, *On the boundedness of the  $G$ -maximal operator and  $G$ -Riesz potential in the generalized  $G$ -Morrey spaces*, **Trans. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci.** 40 (1), **Mathematics**, 111-125 (2020). (SCOPUS)

**Work 5 : Two-weighted inequalities for the commutator of B-Riss potential.**

**Executors:** ph. doctor in math., great res. ass. Lala Aliyeva , cand. phys. math. sci., great ass. Fatayi Isayev

Works on this study are carrying out. Completion of Two-weighted inequalities for the commutator of B-Riss potential is proceeding. Obtained results will be considered to publish in the form of an article. As well, necessary and sufficient conditions were obtained for boundedness of maximal fractional operators give upon Karlseon curves in local Morrey spaces. The boundedness of multilinear operator generated by multilinear Calderon-Zygmund operator was investigated in cartesian product .

22. **H. Armutcu, A. Eroglu, F. Isayev**, *Characterizations for the fractional maximal operators on Carleson curves in local generalized Morrey spaces*, **Tbilisi Math. J.** 13 (1) (2020), 23-38. (WoS, ESCI)
23. **A.F. Ismailova, F.A. Isayev**, *Multi-sublinear operators generated by multilinear Calderon-Zygmund 'operators on product generalized Morrey spaces*, **Trans. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci.** 40 (2), **Mathematics**, (2020), 110-117. (SCOPUS)
24. **A.F. Ismailova, F.A. Isayev**, *Multi-sublinear operators generated by multilinear fractional integral operators on product generalized Morrey spaces*, **Proceedings of IAM** 9 (1) (2020), 18-27.

## **Work 5 : The boundedness of some positive linear operators and approximation theorems on Lebesgue spaces.**

**Executors:** cand. phys. math. great res. ass. Aytekin Abdullayeva, cand. phys. math. sci., great ass. Aynur Mammadova

In this study, properties of weighted modulus of continuity was proved for Lebesgue space. On Lebesgue spaces, the boundedness of some positive linear operators and approximation theorems was proved. In general, an analogue of classic Popovic's theorem for both Szasz operator and Bernstein-Khlovdovsky polynomial and Varenovkaya type asymptotic theorems are investigated in  $L_p(1 < p < \infty)$  space by A.N. Mammadova, A.E. Abdullayeva.

**25.** Mammadova A.N., Abdullayeva A.E., *Approximation theorems for two variable Szasz and generalized Bernstein-Khlovdovsky operators*, **Advances and Applications in Mathematical Sciences** 19 (7) (2020), 599-614. (WoS, ESCI)

## **II SCIENTIFIC ORGANIZATIONAL ACTIVITY**

- The main priority of the department is modern problems of harmonic analysis.
- In the department the research works are carried out on 6 themes.
- The department staff consists of 17 collaborators. One of them is a professor (corr. member of ANAS), one doctor of sciences (professor of ANAS), 4 associate professor, 1 senior res. ass., 4 res. ass., 2 engineer programmer, 2 senior laboratory assistants, 3 laboratory assistants.
- Prof. V.S. Guliyev, doc. R.A. Bandaliyev, doc. E.C. İbrahimov, ph.d. F.A. İsayev, ph.d. M.N. Omarova, ph.d. L. Aliyeva, A.E. Abdullayeva, A.N. Mammadova regularly take part in the institute seminars at pre-quarantine period. In the first quarter of annual report, doc. R.A. Bandaliyev and doc. M.Q. Hajibayov delivered lectures in the institute seminar.
- The seminar "Actual problems of harmonic analysis" led by prof. V.S. Guliyev is held every Friday of the week. Prof. V.S. Guliyev, prof. of ANAS R.A. Bandaliyev, ass. prof. M.N. Omarova, A.E. Abdullayeva, A.N. Mammadova have given presentations in these seminars.
- He has continued his activity as a member of the High Attestation Commission under the President of the Republic of Azerbaijan.
- He is a member of the editorial board of "Azerbaijan journal of Mathematics", "Proceedings of Institute of Mathematics and Mechanics of NAS of Azerbaijan", and foreign journals "Eurasian Mathematical Journals", "Communications de la Faculté des Sciences de l'Université d'Ankara. Série A1. Mathematics and Statistics", "Journal of Nonlinear Sciences and Applications (J. Nonlinear Sci. Appl.)", "International Journal of Mathematical Physics" and continues his activity as one of the editor-in-chiefs on "Caspian journal of Applied Mathematics,

Ecology and Economics" , editor-in-chief on mathematics issue of " Transaction of ANAS"(phys.tech.math.ser), assistant of editor-chief on "Applied and Computational Mathematics".Also, he was editor-in-chief on special issue of Tbilisi Mathematical Journal Volume 13, No.1 (2020) (Special issue of the Conference OMTSA – 2019) .

- Prof. V.S. Guliyev also was a reviewer of the papers from "Journal of Mathematical Analysis and Application, Mathematische Nachrichten, Journal of Fourier Analysis and Application, Journal of Mathematical Inequalities , Integral transform and Special Functions , Journal of Function Spaces, Communications in Mathematical Analysis , Complex Variables and Elliptic Equations , Journal of Inequalities and Applications, Collectanea Mathematica " and other impact factor journals.

- On 24-25 August, 2020, in the international virtual conference entitled "Modern Methods, Problems and Applications of Operator Theory and Harmonic Analysis X" hosted by Russia at Southern Federal University , V.S.Guliyev participated as a plenary lecturer with his talk "Characterizations of generalized fractional maximal and integral operators in Orlicz and generalized Orlicz-Morrey spaces on Carnot groups".

- Prof .V.S.Guliyev led "Mathematical Modeling and Simulation" and "Applications of Optimization Procedure in Physical Problems" sections in international virtual COIA2020 conference and he participated with the lecture "Harmonic analysis methods and the regularity problem for PDE's with discontinuous data".

Under the editorial management of V.S.Guliyev vol. 40, no 1, 2020 issue of "Transactions of Azerbaijan National Academy of Science, Issue Mathematics, Series of physical-technical and mathematics science" was published and the next **vol. 40, no 1, 2020** is prepared for publication.

- Prof.V.S.Guliyev and ph. doctor in math., sen. res. ass. Mehriban Omarova's article named " *Non-negative potential related Shrödinger type operators in local generalized Morrey spaces* "(<https://www.aimsciences.org/article/doi/10.3934/dcddb.2019260>) which published in high impact factor 1.008 journal " Discrete and Continuous Dynamical Systems-Series B" (<https://www.aimsciences.org/journal/1531-3492>) for its sufficient high reference ranking about 1% was included in international base "Clarivate Analytics" (Thomson Reuters) consisting of best ranking reference about Mathematics related to publication year and field in 2020. In the article, the boundedness of some Schrödinger type operators on local generalized Morrey spaces related to certain nonnegative potentials belonging to the reverse Hölder class was investigated.

Point out that this case applies to both prof.V.S.Guliyev's article for 2018 published in Q1 level high impact factor 1.188 " Journal of Mathematical Analysis and Applications" (<https://www.journals.elsevier.com/journal-of-mathematical-analysis-and-applications>) journal included in international base "Clarivate Analytics" (Thomson Reuters) and articles

published in high impact factor 1.792 journal Q1" Analysis and Mathematical Physics" (<https://link.springer.com/journal/13324>) for 2019.

- In 2020, the number of citations in MathSciNet mathematical Review to Prof.V.S.Guliyev's papers is 260, in Google Scholar is 631. In general, number of the the citations in MathSciNet mathematical Review is **1196 (936)**; in Google Scholar is **3547 (2916)**.

- In 2020, the number of citations to Sen. res. ass, doct. math. R.A. Bandaliyev's papers is 20 in Google Scholar.

- In 2020, the number of citations to cand. phys. math. sci., ass. prof., lead. res. ass Elman Ibrahimov's papers is 16 in Google Scholar.

- In 2020, the number of citations to ph. doctor in math., sen. res. ass. Mehriban Omarova's papers is 98 in Google Scholar.

- In 2020, the number of citations to cand. phys. math. sci., res. ass Fatayi Isayev's papers is 8 in Google Scholar.

Sen. res. ass, doct. math. R.A. Bandaliyev and ph. doctor in math., sen. res. ass. Mehriban Omarova participated with their talk in virtual Scientific Republic Conference named «Mathematics, Mechanics and their applications » dedicated to the occasion of the 97<sup>th</sup> birth anniversary of Azerbaijan's national leader Heydar Aliyev organizing by Mechanics-Mathematics faculty of BSU in 20-21 may, 2020

- On 24-27 June, 2020 , in the international virtual conference entitled "3rd International E-Conference on Mathematical Advances and Applications" hosted by Turkey , Istanbul Sen. res. ass, doct. math. R.A. Bandaliyev participated with the plenary lecture " On weighted criterion for Hausdorff operator in Lebesgue spaces " . During this conference prof. A.Gogatishvili (Czech Republic), prof. E. Liflyand (Israel), prof. D. Balenau (Turkey) and others took part in conference via online.

- Ph. doctor in math., sen. res. ass. Mehriban Omarova participated with lecture "Global regularity in Orlicz-Morrey spaces of solutions to elliptic and parabolic equations with VMO coefficients" in international virtual COIA2020 conference.

- In the annual report 2020, 25 scientific articles and 2 theses of departament staff were published, 10 paper were accepted for publication, 10 of them are presented to publication. 21 articles which included in Clarivate Analytics' lists (Thomson Reuters). 7 of them were published in high impact factor journals and 4 of them were published in Scopus base adopted journals.

**Head of the department**  
**"Mathematical Analysis"**

**Corr. member of NASA,**  
**prof.V.S. GULIYEV**