REPUBLIC OF AZERBAIJAN

On the rights of manuscript

ABSTRACT

of the dissertation for the degree of Doctor of Sciences

MIXED PROBLEMS FOR NONSTATIONARY EQUATIONS WITH TRANSMISSION ACOUSTIC CONDITIONS AND MEMORY OPERATORS

Specialty: 1211.01 – Differential equations

Field of science: Mathematics

Applicant: **Sevda Elkhan Isayeva**

Baku-2022

The work was performed at the department of "Differential equations" of the Institute of Mathematics and Mechanics of the National Academy of Sciences of Azerbaijan and at the department of "Differential and integral equations" of the faculty of Mechanics and Mathematics of Baku State University.

Scientific consultant:

doctor of physical and mathematical sciences, professor
Akbar Bayram oglu Aliev

Official opponents:

doctor of physical and mathematical sciences, professor
Tair Sadi oglu Gadjiev

doctor of physical and mathematical sciences, professor
Hamlet Farman oglu Guliyev

doctor of physical and mathematical sciences, professor
Alik Malik oglu Najafov

doctor of mathematical sciences, professor
Mahir Mirzakhan oglu Sabzaliyev

Dissertation council ED 1.04 of Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at the Institute of Mathematics and Mechanics of National Academy of Sciences of Azerbaijan

Chairman of the Dissertation council:

corr.-member of NASA, doctor of phys.-math. sc., prof.

_______________ Misir Jumail oglu Mardanov

Scientific secretary of the Dissertation council:

cand. of phys.-math.sc.
______________ Abdurrahim Farman oglu Guliyev

Chairman of the scientific seminar:

academician of NASA, doc. of phys.-math. sc., prof.

_______________ Yusif Abulfat oglu Mammadov

GENERAL CHARACTERISTICS OF THE WORK

Rationale of the topic and the degree of development. This dissertation is devoted to study mixed problems for the nonlinear wave equations with transmission acoustic conditions and mixed problems for nonlinear hyperbolic and parabolic equations with memory operators.

Some problems in fluid and gas mechanics are reduced to solving mixed problems with acoustic boundary conditions. In this area, the first mathematical research was carried out in the work of J.T.Beale, S.I.Rosencrans. In this work, the authors derived the equations in the domain $\Omega \subset R^3$ with the boundary Γ:

$$u_{tt} - \Delta u = 0 \quad \text{in} \quad \Omega \times (0,\infty),$$

$$\frac{\partial u}{\partial \nu} = \delta_t \quad \text{on} \quad \Gamma \times (0,\infty),$$

$$m\delta_{tt} + d\delta_t + k\delta = -\rho_0 u_t \quad \text{on} \quad \Gamma \times (0,\infty),$$

as a theoretical model to describe the irrotational perturbations of a fluid. Here ρ_0, m, d, k are physical constants, the function $u(x,t)$ is the velocity potential of the fluid and $\delta(x,t)$ models the normal displacement of the point $x \in \Gamma$ at the time t. To obtain this model the authors assumed that each point of the surface Γ acts like a spring in response to the excess pressure and that each point of Γ does not influence each other. Surfaces with this characteristic are called locally reacting.

Numerous works are devoted to the study of mixed problems with acoustic boundary conditions, in which the results similar to those for the case of mixed problems with Dirichlet or Neumann boundary conditions are obtained. In some works acoustic boundary conditions are imposed together with the homogeneous Dirichlet condition on a part of the boundary. In the work of A.T. Cousin, C.L. Frota, N.A. Larkin the existence, uniqueness and exponential decay of global solutions for the mixed problem for a generalized system of equations of the Klein-Gordon type with acoustic boundary conditions on one part of the boundary and a homogeneous Dirichlet condition on the remaining part of the boundary are proved. In the work of C.L.Frota, A.T.Cousin, N.A.Larkin the authors obtained results on the decay of solutions for the nonlinear wave equation when $n=1$; similar results were obtained by J.Y. Park, S.H. Park. C.L.Frota, J.A.Goldstein proved the existence and uniqueness of global solutions for a mixed problem with acoustic boundary conditions on one part of the boundary and a homogeneous Dirichlet condition on the remaining part of the boundary. In some works, acoustic boundary conditions are imposed on an entire boundary. C.L.Frota, L.A.Medeyros, A.Vicente proved the existence, uniqueness and exponential decay of global solutions for a mixed problem in a domain with a nonlocally reacting surface, and also the existence and uniqueness of solutions in the case of a nonmonotone dissipative term. J.M.Jeong, J.Y.Park, Y.H.Kang proved the blow-up of solutions for a quasilinear wave equation with acoustic boundary conditions on one part of the boundary and a homogeneous Dirichlet condition on the remaining part of the boundary. In the work of P.J.Graber, B.Said-Houari results on local existence, global existence, decay and blow-up of solutions of a mixed problem for the wave equation with semilinear acoustic boundary conditions are obtained.

Transmission problems arise in some applications of physics and biology. Transmission problems for hyperbolic equations are investigated in the work of R.Dautray, J.L.Lions, where the uniqueness and regularity of solutions for the linear problem are proved. In the work of J.E. Muñoz Rivera, H. Portillo Oquendo the transmission problem for viscoelastic waves is considered and the exponential decay of solutions is proved. D.Andrade, L.H.Fatori, J.E.Muñoz Rivera proved the existence and exponential decay of global solutions for the transmission problem. In the work of J.J.Bae the transmission problem is studied, where one component is clamped, and the other is in a viscoelastic fluid, and a result on the exponential decay of solutions is obtained. In the work of W.D.Bastos, C.A. Raposo the transmission problem with friction dissipation was studied and a result on the existence and uniqueness of solutions was obtained and the exponential stability of the total energy was established. A.B.Aliev, E.H.Mammadhasanov studied the transmission problem, for which a result on the existence and uniqueness of solutions was established by the method of dynamic regularization of boundary conditions and transmission conditions.

But none of these works considered mixed problems with transmission acoustic conditions, to the solution of which some problems of fluid and gas mechanics are reduced. In the dissertation work such mixed problems with transmission acoustic conditions for nonlinear wave equations are considered.

Differential equations with memory, especially the equations with hysteresis, are of great importance among nonlinear partial differential equations. The concept of a hysteresis operator was first introduced in the work of M.A.Krasnoselsky and A.V.Pokrovsky. Studying the solvability of the Cauchy problem and initial-boundary value problems for partial differential equations with such nonlinearities is a nontrivial problem. Equations with memory are especially difficult if the memory operator is under the time differentiation operator. The global solvability and the absence of global solutions for quasilinear Sobolev equations, when the nonlinear term is under the differentiation operator with respect to t, have been studied in sufficient detail in the monograph by A.G. Sveshnikov, A.B. Alshin, M.O. Korpusov and Yu.D. Pletner. Of the subsequent studies carried out in this direction, one can note the works of M.O.Korpusov and A.G. Sveshnikov. In the works of A. Visintin, M. Hilpert the corresponding problems with a memory operator are investigated applying the results of the theory of nonlinear semigroups. The existence, uniqueness, asymptotic character of periodic solutions of similar problems for semilinear and quasilinear hyperbolic equations were investigated in the works of P.Krejci. In the dissertation work, initial-boundary value problems with memory operators are studied in more detail, in which the memory operator is under the differentiation operator with respect to t.

In the dissertation work two main areas are considered.

1. Mixed problems with transmission acoustic conditions for nonlinear wave equations are studied. It is of great interest to prove the existence of a minimal global attractor for such problems.

2. Initial-boundary value problems for nonlinear hyperbolic and parabolic equations with memory operators under the time differentiation operator are studied.

Object and subject of the research. The main object of the dissertation work is mixed problems with transmission acoustic conditions for nonlinear wave equations and initial-boundary value problems for nonlinear hyperbolic and parabolic equations with memory operators. The subject of this research is the study of the behaviour of solutions of mixed problems with transmission acoustic conditions for wave equations with nonlinear focusing and defocusing sources and initial-boundary value problems for hyperbolic and parabolic equations with memory operators and hysteresis nonlinearities.

Goal and tasks of the research. The main goal and objective of the research is to study initial-boundary value problems with transmission acoustic conditions for nonlinear wave equations, to the solution of which some problems of fluid and gas mechanics are reduced; as well as to study initial boundary value problems for nonlinear hyperbolic and parabolic equations with memory operators and hysteresis nonlinearities, that arise in various biological, technological and chemical processes.

Investigation methods. The dissertation work uses the methods of the theory of differential equations, the theory of functional analysis, the theory of semigroups, including the compactness method, embedding theorems, the principle of contracting mappings and the discretization method.

Key points of the dissertation which will be defended:

• the investigation of the existence and uniqueness of solutions to the initial-boundary value problem with transmission acoustic conditions for nonlinear wave equations with focusing sources;

• the study of the behaviour of solutions of the initial-boundary value problem with transmission acoustic conditions for nonlinear wave equations with focusing sources;
• the investigation of the existence of solutions to the initial-boundary value problem with nonlinear transmission acoustic conditions for nonlinear wave equations with focusing sources;
• the investigation of the existence of solutions to the initial-boundary value problem with transmission acoustic conditions for nonlinear strongly dissipative wave equations with focusing sources;
• the investigation of the existence of solutions to the initial-boundary value problem with transmission acoustic conditions for nonlinear wave equations with defocusing sources;
• the study of the behaviour of solutions of the initial-boundary value problem with transmission acoustic conditions for nonlinear wave equations with defocusing sources;
• the investigation of the existence of a minimal global attractor for an initial-boundary value problem with transmission acoustic conditions for nonlinear wave equations with defocusing sources;
• the investigation of the existence and behaviour of solutions of initial-boundary value problems for semilinear hyperbolic and quasilinear parabolic equations with memory operators;
• the investigation of the existence of solutions to a mixed problem for a system of semilinear hyperbolic equations with memory operators;
• the investigation of the existence of weak solutions of a mixed problem with acoustic boundary conditions for a semilinear hyperbolic equation with a memory operator.

Scientific novelty of the research. The following main results were obtained.
• the existence and uniqueness of local weak and local strong solutions of the initial-boundary value problem with transmission acoustic transmission conditions for nonlinear wave equations with focusing sources were proved;
• if $p \le \min\{q_1, q_2\}$, a result on the existence of global weak solutions for an initial-boundary value problem with transmission

acoustic conditions for nonlinear wave equations with focusing sources was obtained and if $p > \max\{q_1, q_2\}$, a result on the blow-up of weak solutions of this problem in a finite time was obtained;

• a result on the existence and uniqueness of local weak solutions of an initial-boundary value problem with nonlinear transmission acoustic conditions for nonlinear wave equations with focusing sources was obtained;

• the existence of local weak solutions of the initial-boundary value problem with transmission acoustic conditions for nonlinear strongly dissipative wave equations with focusing sources was proved;

• the existence, uniqueness and exponential decay of global strong solutions of the initial-boundary value problem with transmission acoustic conditions for nonlinear wave equations with defocusing sources were proved;

• a result on the existence of a minimal global attractor for an initial-boundary value problem with transmission acoustic conditions for nonlinear wave equations with defocusing sources was obtained;

• the existence and uniqueness of solutions to the initial-boundary value problem for a semilinear hyperbolic equation with a memory operator were proved, a result on the existence of a minimal global attractor for this problem was obtained;

• the existence of solutions to the initial-boundary value problem for a quasilinear parabolic equation with a memory operator was proved and the result on the uniqueness of solutions in the case of hysteresis nonlinearity was obtained;

• the result on the existence of solutions to the initial-boundary value problem for a fourth-order semilinear hyperbolic equation with a memory operator was obtained; the existence of a minimal global attractor for this problem was proved;

• the result on the existence of solutions to the mixed problem for a system of semilinear hyperbolic equations with memory operators was obtained;

• the existence and uniqueness of solutions to the mixed problem for the Timoshenko system with a memory operator were proved;

• a result on the existence of weak solutions of a mixed problem with acoustic boundary conditions for a semilinear hyperbolic equation with a memory operator was obtained.

Theoretical and practical value of the research. The results obtained in the dissertation are new and are of theoretical and applied interest. They can be used in some problems of fluid and gas mechanics, as well as in various biological, technological and chemical processes.

Approbation and application. The results of the dissertation were presented at the seminars of the department of the faculty of Mechanics and Mathematics of Baku State University "Differential and integral equations" (doc. of phys.-math. sc. Y.T.Mehraliyev), at the seminars of the faculty of Mechanics and Mathematics of Baku State University (doc. of phys.-math. sc., prof. N.Sh.Isgandarov), at the seminars of the department of the faculty of Applied Mathematics and Cybernetics of Baku State University "Equations of mathematical physics" (doc. of phys.-math. sc., acad. Y.A.Mamedov), at the seminars of the department "Differential equations" (doc. of phys.-math. sc., prof. A.B.Aliev), at the seminars of the Institute of Mathematics and Mechanics of the National Academy of Sciences of Azerbaijan (corr.-member of NASA, doc. of phys.-math. sc., prof. M.J.Mardanov); at the international conference on mathematics and mechanics dedicated to the 50th anniversary of the Institute of Mathematics and Mechanics of the National Academy of Sciences of Azerbaijan (Baku, 2009), at the Third Congress of the World Mathematical Society of Turkish Countries (Almaty, 2009), at the IV international conference "Functional Differential Equations and their applications" (Makhachkala, 2009), at the international conference dedicated to the 80th anniversary of acad. F.G.Maksudov "Spectral theory and its applications" (Baku, 2010), at the international school-seminar "Nonlinear analysis and extremal problems" (Irkutsk, 2010), at the international conference dedicated to the 100th anniversary of acad. Z.I.Khalilov (Baku, 2011), at the VI international scientific conference "Functional differential equations and their applications" (Makhachkala, 2013), at the international conference dedicated to the 55th anniversary of the Institute of Mathematics and Mechanics of the National Academy of Sciences of Azerbaijan (Baku, 2014), at the V International Scientific Conference “International Kolmogorov Readings - XIV, dedicated to the 100th anniversary of prof. Z.A.Skopets” (Koryazhma, 2017), at the XII international conference “Fundamental and applied problems of mathematics and informatics” (Makhachkala, 2017), at the VI congress of the Turkic world mathematical society (Astana, 2017), at the international conference dedicated to 100th anniversary of S.G.Krein (Voronezh, 2017), at the international conference dedicated to the 70th anniversary of prof. G.A.Isakhanly (Baku, 2018), at the international scientific-practical conference ”The latest achievements and successes in the development of technical sciences” (Krasnodar, 2018), at the VI International Conference on Control and Optimization with Industiral Applications (Baku, 2018), at the IX International Conference of the Georgian Mathematical Union (Batumi-Tbilisi, 2018), at the international scientific-practical conference “Contemporary Mathematics and its Applications” (Grozny, 2018), at the V International conference dedicated to the 95th anniversary of L.D.Kudryavtsev "Functional spaces" (Moscow, 2018), at the international conference dedicated to the 90th anniversary of acad. Azad Mirzadzhanyzade (Baku, 2018), at the international conference dedicated to the 80th anniversary of acad. Mirabbas Kasimov (Baku, 2019), VIII International Eurasian Conference on Mathematical Sciences and Applications Dedicated to the 100th Anniversary of Baku State University (IECMSA-2019) (Baku, 2019), at the XIII international conference dedicated to the 55th anniversary of the Faculty of Mathematics and Computer Sciences (Makhachkala, 2019), at the IV Russian Conference with international participation “Mathematical Modeling and Information Technologies” (Syktyvkar, 2020).

Personal contribution of the author. All the results obtained in the dissertation belong to the author.

- A result on the existence of global weak solutions for an initial-boundary value problem with transmission acoustic conditions for nonlinear wave equations with focusing sources was obtained, if $p \le \min\{q_1, q_2\}$ and a result on the blow-up of weak solutions of this problem in a finite time was obtained, if $p > \max\{q_1, q_2\}$;

- A result on the existence of a minimal global attractor for an initial-boundary value problem with transmission acoustic conditions for nonlinear wave equations with defocusing sources was obtained;
- A result on the existence of a minimal global attractor for an initial-boundary value problem for a semilinear hyperbolic equation with a memory operator was obtained.

Publications. 53 scientific works have been published on the topic of the dissertation: publications in editions recommended by HAC under the President of the Republic of Azerbaijan - 21, conference materials - 1, abstracts - 31.

The name of the institution where the dissertation work was performed. Baku State University and Institute of Mathematics and Mechanics of the National Academy of Sciences of Azerbaijan.

Structure and volume of the dissertation (in signs, indicating the volume of each structural subsection separately). The title page consists of 389 signs, contents - 3321 signs, the introduction – 76000 signs, the main content of the dissertation – 388000 signs (I chapter – 136000, II chapter – 76000, III chapter – 108000, IV chapter – 68000), conclusions - 1328 signs and a list of used literature - 21886 signs. The total volume of the dissertation consists of 490924 signs.

THE MAİN CONTENT OF THE DISSERTATION

The dissertation consists of an introduction, four chapters, conclusions and a list of used literature.

The introduction justifies the relevance of the research topic and shows the degree of its development, formulates the purpose and task of the research, provides scientific novelty, notes the theoretical and practical value of the research, and also provides information on the approbation of the work.

First, we introduce some notation and definitions.

R^N – N - dimensional Euclidean space $(R^1 = R)$;
$x = (x_1, \dots, x_N)$ – the point from R^N;
$R^+ = \{t \in R^1 : t > 0\}$;
Z^+ - the set of non-negative integers;
Ω – the domain in R^N;
Γ – the boundary of the domain Ω;
$Q_T = \{(x,t) : x \in \Omega, t \in (0,T)\}$;
$u_x = (u_{x_1}, \dots, u_{x_N})$;
$u_x \upsilon_x = \sum_{i=1}^{N} u_{x_i} \upsilon_{x_i}$;
$D(\Omega)$ –the space of infinitely differentiable functions with compact support in Ω;
$D'(\Omega)$ –the conjugate space of $D(\Omega)$ – the space of distributions in Ω;
$L^p(\Omega)$ –the space of functions summable with the p -th degree in Ω by the measure $dx = dx_1 \dots dx_N$; $\|f\|_{L^p(\Omega)} = \left(\int_\Omega |f(x)|^p \, dx \right)^{1/p}$;
$L^\infty(\Omega)$ –the space of functions bounded almost everywhere in Ω, $\|f\|_{L^\infty(\Omega)} = \operatorname{ess\,sup}_{x \in \Omega} |f(x)|$;

$W^{m,p}(\Omega)=\{\upsilon : D^{\alpha}\upsilon \in L^{p}(\Omega),\ |\alpha| \le m\}$,

$\|\upsilon\|_{W^{m,p}(\Omega)} = \left(\sum_{|\alpha|\le m} \|D^{\alpha}\upsilon\|^{p}_{L^{p}(\Omega)}\right)^{1/p}$, $D^{\alpha}\upsilon = \dfrac{\partial \upsilon^{\alpha_1+...+\alpha_N}}{\partial x_1^{\alpha_1} \ ... \ \partial x_N^{\alpha_N}}$,

$\alpha=(\alpha_1,...,\alpha_N)$, $|\alpha| = \alpha_1+...+\alpha_N$, $\alpha_i \in Z^{+}$, $i=1,...,N$;

$W_0^{m,p}(\Omega)$ – closure of $D(\Omega)$ in $W^{m,p}(\Omega)$;

$H^{m}(\Omega)=W^{m,2}(\Omega)$;

$H_0^{m}(\Omega)=W_0^{m,2}(\Omega)$;

$H^{-m}(\Omega)=\left(H_0^{m}(\Omega)\right)' = W^{-m,2}(\Omega)$;

$H^{s}(\Omega)$ – the Sobolev space of non-integer order s;

$C^{k}(\overline{\Omega})$, $k \in Z^{+}$ – the space of k times continuously differentiable functions in $\overline{\Omega}$;

if X – Banach space, then

$$L^{p}(0,T;X)=\{f : f \text{ - measurable mapping } [0,\mathrm{T}]\to \mathrm{X},$$
$$\left(\int_0^T \|f(t)\|_X^p \, dt\right)^{1/p} < \infty \text{ if } 1 \le p < \infty, \ \sup_{t\in(0,T)} \mathrm{ess} \|f(t)\|_X < \infty \text{ if } p=\infty\};$$

$D((0,T);X)$ – the space of C^{∞}- mappings $(0,\mathrm{T})\to \mathrm{X}$ with compact support in $(0,\mathrm{T})$;

$C^{k}([0,T];X), k\in Z^{+}$ – the space of k times continuously differentiable mappings $[0,T]\to \mathrm{X}$;

$\mathrm{M}(\Omega;C^{0}([0,T]))$ – the space of measurable functions acting from Ω to $C^{0}([0,T])$;

$L(X;Y)$ – the space of continuous linear mappings of the topological vector space X into the topological vector space Y;

$D'((0,T);X)=L(D(0,T);X)$ – the space of distributions on $(0,\mathrm{T})$ with the values in X;

$H(\Delta,\Omega)=\{\, u\in H^1(\Omega):\ \Delta u\in L^2(\Omega)\,\}$,

$\|u\|_{H(\Delta,\Omega)}\equiv\|u\|_{\Delta,\Omega}=\left(\|u\|^2_{H^1(\Omega)}+\|\Delta u\|^2_{L^2(\Omega)}\right)^{1/2}$;

$(u,\upsilon)_i=\int\limits_{\Omega_i}u(x)\upsilon(x)dx,\quad \|u\|_i=\left(\int\limits_{\Omega_i}(u(x))^2\,dx\right)^{1/2}$ – the scalar product and the norm in $L^2(\Omega_i)$, where Ω_i is a domain in R^N $(i=1,2)$;

$(\delta,\theta)_\Gamma=\int\limits_{\Gamma}\delta(x)\theta(x)d\Gamma,\quad \|\delta\|_\Gamma=\left(\int\limits_{\Gamma}(\delta(x))^2\,d\Gamma\right)^{1/2}$ – the scalar product and the norm in $L^2(\Gamma)$, where Γ is a boundary of the domain Ω;

the mapping $\gamma_0:\ H^1(\Omega)\to H^{1/2}(\Gamma)$ – the trace operator of zero order;

the mapping $\gamma_1: H(\Delta,\Omega)\to H^{-1/2}(\Gamma)$ – the Neumann trace operator;

$H^1_{\Gamma_1}(\Omega)=\{\, u\in H^1(\Omega):\ \gamma_0(u)=0$ a.e. on $\Gamma_1\}$, where Γ_1 – is a part of the boundary Γ of the domain Ω;

$$\|u\|_{H^1_{\Gamma_1}(\Omega_1)}=\left(\sum_{i=1}^{N}\int\limits_{\Omega_1}\left(\frac{\partial u}{\partial x_i}\right)^2 dx\right)^{1/2}.$$

We introduce well-known definitions of some concepts.

Definition 1. If $\{S(t)\}_{t\geq 0}$ is a semigroup defined on the metric space (X,d), then the smallest non-empty bounded closed set $A\subset X$ which is invariant under the dynamical system generating the semigroup $\{S(t)\}_{t\geq 0}$ and which satisfies the relation

$$\lim_{t\to\infty}\sup_{\upsilon\in B}\inf_{u\in A}\ d(S(t)\upsilon,\,u)=0$$

for arbitrary bounded set $B\subset X$, is called a minimal global attractor of the semigroup $\{S(t)\}_{t\geq 0}$.

Definition 2. Let $\{S(t)\}_{t\geq 0}$ be a semigroup defined on the metric space (X,d). The set $B_0 \subset X$ is called an absorbing set if for arbitrary bounded set $B \subset X$ there exists the number $t_1(B)$ such that

$$S(t)B \subset B_0$$

for all $t \geq t_1(B)$.

Definition 3. The semigroup $\{S(t)\}_{t\geq 0}$ defined on the metric space (X,d), is called asymptotically compact, if for arbitrary bounded set $B \subset X$ such that $\bigcup_{t\geq 0} S(t)B$ is bounded in (X,d), an arbitrary sequence of the form

$$\{S(t_k)\upsilon_k\}_{k=1}^{\infty}, \text{ where } t_k \to \infty,\ \upsilon_k \in B,$$

has a convergent subsequence.

Definition 4. Let a system which is characterized by two scalar variables $u(t)$ and $w(t)$ (depending on time) in the Banach space X be given, and at each moment of time $t \in [0,T]$ $w(t)$ depends not only on $u(t)$, but also on the previous values of $u(t)$ (that is, on $u|_{[0,t]}$): $w(t) = [\mathsf{F}(u,\xi^0)](t)$ for $\forall t \in [0,T]$, where $\xi^0 \in R^1$ is a given number. Then $\mathsf{F}(u,\xi^0)$ is called a memory operator in X .

Definition 5. For given $\xi^0 \in R^1$, the memory operator $\mathsf{F}(u,\xi^0): u \to w$ is called a hysteresis operator, if this operator satisfies the condition:

$$\begin{cases} [\mathsf{F}(u_1)](\cdot,t) = [\mathsf{F}(u_2)](\cdot,t) \text{ for } \forall u_1,\, u_2 \in \mathrm{Dom}(\mathsf{F}) \\ \text{such that } u_1 = u_2 \text{ on } [0,t] \text{ for } \forall t \in [0,T] \end{cases}$$

and the trajectory of the pair $(u(t), w(t))$ is invariant with respect to arbitrary increasing diffeomorphism $\varphi: [0,T] \to [0,T]$, that is,

$$\mathsf{F}(u \circ \varphi, \xi^0) = \mathsf{F}(u,\xi^0) \circ \varphi \quad \text{on} \quad [0,T],$$

in other words, if $u \to w$, then $u \circ \varphi \to w \circ \varphi$.

Let's move on a summary of the content of the work.

First chapter consists of six sections. In the first section of the first chapter the following mixed problem with focusing nonlinear source terms and transmission acoustic conditions is considered:

$$u_{tt} - \Delta u + |u_t|^{q_1-1} u_t = |u|^{p-1} u \quad \text{in} \quad \Omega_1 \times (0,\infty), \tag{1}$$

$$\upsilon_{tt} - \Delta \upsilon + |\upsilon_t|^{q_2-1} \upsilon_t = |\upsilon|^{p-1} \upsilon \quad \text{in} \quad \Omega_2 \times (0,\infty), \tag{2}$$

$$M\delta_{tt} + D\delta_t + K\delta = -u_t \quad \text{on} \quad \Gamma_2 \times (0,\infty), \tag{3}$$

$$u = 0 \quad \text{on} \quad \Gamma_1 \times (0,\infty), \tag{4}$$

$$u = \upsilon, \; \delta_t = \frac{\partial u}{\partial \nu} - \frac{\partial \upsilon}{\partial \nu} \quad \text{on } \Gamma_2 \times (0,\infty), \tag{5}$$

$$u(x,0) = u_0(x), \; u_t(x,0) = u_1(x), \; x \in \Omega_1, \tag{6}$$

$$\upsilon(x,0) = \upsilon_0(x), \; \upsilon_t(x,0) = \upsilon_1(x), \; x \in \Omega_2, \tag{7}$$

$$\delta(x,0) = \delta_0(x), \delta_t(x,0) = \frac{\partial u_0}{\partial \nu} - \frac{\partial \upsilon_0}{\partial \nu} \equiv \delta_1(x), \; x \in \Gamma_2, \tag{8}$$

where $\Omega \subset R^n \, (n \geq 1)$ is a bounded domain with the smooth boundary Γ_1, $\Omega_2 \subset \Omega$ is a subdomain with the smooth boundary Γ_2 and $\Omega_1 = \Omega \setminus (\Omega_2 \cup \Gamma_2)$ is a subdomain with the boundary $\Gamma = \Gamma_1 \cup \Gamma_2$, such that $\Gamma_1 \cap \Gamma_2 = \varnothing$; $\nu(x)$ – the unit outward normal vector to Γ at the point $x \in \Gamma$; $M, D, K : \Gamma_2 \to R$, $u_0, u_1 : \Omega_1 \to R$, $\upsilon_0, \upsilon_1 : \Omega_2 \to R$, $\delta_0 : \Gamma_2 \to R$ – known functions; $p > 1, \; q_i > 1, \; i = 1,2$.

The definitions of weak and strong solutions to the problem (1)-(8) are introduced.

Definition 6. A triple of functions $(u(x,t), \upsilon(x,t), \delta(x,t))$, where

$$u : \Omega_1 \times [0,T] \to R, \; \upsilon : \Omega_2 \times [0,T] \to R, \; \delta : \Gamma_2 \times [0,T] \to R$$

is called a weak solution to the problem (1)-(8) if

$$u \in L^{\infty}\left(0,T;H^1_{\Gamma_1}(\Omega_1)\right),\ \upsilon \in L^{\infty}\left(0,T;H^1(\Omega_2)\right),$$
$$\gamma_0(u)=\gamma_0(\upsilon) \text{ a. e. on } \Gamma_2 \times (0,T),$$
$$u_t \in L^{\infty}\left(0,T;L^2(\Omega_1)\right)\cap L^{q_1+1}(\Omega_1 \times (0,T)),$$
$$\upsilon_t \in L^{\infty}\left(0,T;L^2(\Omega_2)\right)\cap L^{q_2+1}(\Omega_2 \times (0,T)),$$
$$\delta,\delta_t \in L^{\infty}\left(0,T;L^2(\Gamma_2)\right)$$
and the following equalities hold:

$$\tfrac{d}{dt}(u_t,\Phi)_1+(\nabla u,\nabla\Phi)_1+\left(|u_t|^{q_1-1}u_t,\Phi\right)_1+\tfrac{d}{dt}(\upsilon_t,\Psi)_2+(\nabla\upsilon,\nabla\Psi)_2+$$
$$+\left(|\upsilon_t|^{q_2-1}\upsilon_t,\Psi\right)_2-(\delta_t,\gamma_0(\Phi))_{\Gamma_2}=\left(|u|^{p-1}u,\Phi\right)_1+\left(|\upsilon|^{p-1}\upsilon,\Psi\right)_2$$
for $\forall\Phi \in H^1_{\Gamma_1}(\Omega_1)$, $\forall\Psi \in H^1(\Omega_2)$ such that $\Phi=\Psi$ on Γ_2, in the sense of distributions in $D'(0,T)$ and
$$\tfrac{d}{dt}(\gamma_0(u)+M\delta_t,e)_{\Gamma_2}+(D\delta_t+K\delta,e)_{\Gamma_2}=0$$
for $\forall e \in L^2(\Gamma_2)$, in the sense of distributions in $D'(0,T)$, and also:
$$u(x,0)=u_0(x),\ u_t(x,0)=u_1(x) \text{ a. e. in } \Omega_1,$$
$$\upsilon(x,0)=\upsilon_0(x),\ \upsilon_t(x,0)=\upsilon_1(x) \text{ a. e. in } \Omega_2,$$
$$\delta(x,0)=\delta_0(x),\ \delta_t(x,0)=\delta_1(x) \text{ a. e. on } \Gamma_2.$$

Definition 7. A triple of functions $(u(x,t),\upsilon(x,t),\delta(x,t))$, where
$$u:\Omega_1\times[0,T]\to R,\ \upsilon:\Omega_2\times[0,T]\to R,\ \delta:\Gamma_2\times[0,T]\to R,$$
is called a strong solution to the problem (1)-(8) if
$$u \in L^{\infty}\left(0,T;H^1_{\Gamma_1}(\Omega_1)\right),\ u_t \in L^{\infty}\left(0,T;H^1_{\Gamma_1}(\Omega_1)\right)\cap L^{q_1+1}(\Omega_1\times[0,T]),$$
$$u_{tt} \in L^{\infty}\left(0,T;L^2(\Omega_1)\right),$$
$$\upsilon \in L^{\infty}\left(0,T;H^1(\Omega_2)\right),\ \upsilon_t \in L^{\infty}\left(0,T;H^1(\Omega_2)\right)\cap L^{q_2+1}(\Omega_2\times[0,T]),$$
$$\upsilon_{tt} \in L^{\infty}\left(0,T;L^2(\Omega_2)\right),$$
$$u(t)\in H(\Delta,\Omega_1),\ \upsilon(t)\in H(\Delta,\Omega_2) \text{ a. e. on } (0,T),$$
$$\delta,\delta_t,\delta_{tt} \in L^{\infty}\left(0,T;L^2(\Gamma_2)\right),$$

and

$$u_{tt}-\Delta u+|u_t|^{q_1-1}u_t=|u|^{p-1}u \quad \text{a. e. in } \Omega_1\times(0,T),$$

$$\upsilon_{tt}-\Delta\upsilon+|\upsilon_t|^{q_2-1}\upsilon_t=|\upsilon|^{p-1}\upsilon \quad \text{a. e. in } \Omega_2\times(0,T),$$

$$\gamma_0(u_t)+M\delta_{tt}+D\delta_t+K\delta=0,\ \gamma_0(u)=\gamma_0(\upsilon) \quad \text{a. e. on } \Gamma_2\times(0,T),$$

$$\langle\gamma_1(u(t)-\upsilon(t)),\gamma_0(\varphi)\rangle_{H^{-1/2}(\Gamma_2)\times H^{1/2}(\Gamma_2)}=(\delta_t(t),\gamma_0(\varphi))_{\Gamma_2}$$

for $\forall\varphi\in H^1_{\Gamma_1}(\Omega_1)$ a. e. on $(0,T)$,

$$u(x,0)=u_0(x),\ u_t(x,0)=u_1(x) \quad \text{a. e. in } \Omega_1,$$

$$\upsilon(x,0)=\upsilon_0(x),\ \upsilon_t(x,0)=\upsilon_1(x) \quad \text{a. e. in } \Omega_2,$$

$$\delta(x,0)=\delta_0(x),\ \delta_t(x,0)=\delta_1(x) \quad \text{a. e. on } \Gamma_2.$$

The following theorem on the existence and uniqueness of local weak solutions for the problem (1)-(8) is proved.

Theorem 1. Let the following conditions be satisfied:

$$M,D,K\in C(\Gamma_2),\ M(x)>0,\ D(x)>0,\ K(x)>0,\ x\in\Gamma_2; \quad (9)$$

$$p>1 \text{ if } n=1,2,\ 1<p\le\frac{n}{n-2} \text{ if } n\ge 3. \quad (10)$$

Then for

$$\left.\begin{aligned}\forall(u_0,\upsilon_0,\delta_0)\in H^1_{\Gamma_1}(\Omega_1)\times H^1(\Omega_2)\times L^2(\Gamma_2)\\ \forall(u_1,\upsilon_1,\delta_1)\in L^{2q_1}(\Omega_1)\times L^{2q_2}(\Omega_2)\times L^2(\Gamma_2)\end{aligned}\right\} \quad (11)$$

there exists the number $T>0$ such that the problem (1)-(8) has a unique weak solution (u,υ,δ), satisfying the conditions:

$$u\in C([0,T];H^1_{\Gamma_1}(\Omega_1)),\ u_t\in C([0,T];L^2(\Omega_1))\cap L^{q_1+1}(\Omega_1\times(0,T)),$$

$$\upsilon\in C([0,T];H^1(\Omega_2)),\ \upsilon_t\in C([0,T];L^2(\Omega_2))\cap L^{q_2+1}(\Omega_2\times(0,T)),$$

$$\delta,\delta_t\in L^\infty(0,T;L^2(\Gamma_2));$$

moreover, if $T_{\max}>0$ – the length of the maximum interval of the existence of the solution (u,υ,δ), then the following alternative is valid: either

$$T_{\max} = +\infty;$$

or

$$\lim_{t \to T_{\max}-0} \left(\|u_t\|_1^2 + \|\upsilon_t\|_2^2 + \|\nabla u\|_1^2 + \|\nabla \upsilon\|_2^2 + \left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2 + \left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2 \right) = +\infty .$$

In the second section of the first chapter the following result on the existence and uniqueness of local strong solutions to the problem (1)-(8) was established.

Theorem 2. Let the conditions (9)-(10) be satisfied and

$$q_i > 1 \text{ if } n = 1,2;\ 1 < q_i \le \frac{n}{n-2} \text{ if } n \ge 3 \ (i = 1,2).$$

Suppose that $p \le \min\{q_1, q_2\}$ and

$$u_0 \in H^1_{\Gamma_1}(\Omega_1) \cap H^2(\Omega_1),\ u_1 \in H^1_{\Gamma_1}(\Omega_1) \cap L^{2q_1}(\Omega_1),$$

$$\upsilon_0 \in H^2(\Omega_2),\ \upsilon_1 \in H^1(\Omega_2) \cap L^{2q_2}(\Omega_2),\ \delta_0, \delta_1 \in L^2(\Gamma_2).$$

Then there exists the number $T > 0$ such that the problem (1)-(8) has a unique strong solution (u, υ, δ).

In the third section of the first chapter the following theorem on the existence of global weak solutions to the problem (1)-(8) is proved under the condition $p \le \min\{q_1, q_2\}$.

Theorem 3. Let the conditions of Theorem 1 and the condition

$$p \le \min\{q_1, q_2\}$$

be satisfied. Then the local weak solution (u, υ, δ) to the problem (1)-(8) is global and T can be taken arbitrarily large.

In the fourth section of the first chapter the following energy function is defined:

$$E(t) = \frac{1}{2}\left[\|u_t\|_1^2 + \|\nabla u\|_1^2 + \|\upsilon_t\|_2^2 + \|\nabla \upsilon\|_2^2 + \left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2 + \left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2 \right] -$$

$$- \frac{1}{p+1}\left(|u|^{p+1}, 1\right)_1 - \frac{1}{p+1}\left(|\upsilon|^{p+1}, 1\right)_2,$$

where (u,υ,δ) is a weak solution to the problem (1)-(8) and the following result on the blow-up of solutions to this problem in a finite time is established.

Theorem 4. Let

$$(u_0, \upsilon_0, \delta_0) \in H^1_{\Gamma_1}(\Omega_1) \times H^1(\Omega_2) \times L^2(\Gamma_2),$$

$$(u_1, \upsilon_1, \delta_1) \in L^{2q_1}(\Omega_1) \times L^{2q_2}(\Omega_2) \times L^2(\Gamma_2)$$

and the conditions (9), (10) be satisfied. Suppose that

$$p > \max\{q_1, q_2\}, \ \ K(x) \ge 1 \ \ (x \in \Gamma_2),$$

$$0 < E(0) < E_1, \ E_1 = \alpha_1^2\left(\frac{1}{2} - \frac{1}{p+1}\right),$$

$$\|\nabla u_0\|_1^2 + \|\nabla \upsilon_0\|_2^2 > \alpha_1^2, \ \alpha_1 = B^{-\frac{1}{p-1}},$$

where B is a positive constant depending on Ω_1, Ω_2, p. Then the weak solution to the problem (1)-(8) blows up in a finite time.

In the fifth section of the first chapter an initial-boundary value problem for nonlinear wave equations with focusing sources with nonlinear transmission acoustic conditions:

$$u = \upsilon, \ \frac{\partial u}{\partial \nu} - \frac{\partial \upsilon}{\partial \nu} + \rho(u_t) = \delta_t \ \text{ on } \ \Gamma_2 \times (0, \infty)$$

is considered. The theorem on the existence and uniqueness of local weak solutions to this problem is proved under the following conditions on nonlinearity ρ:

$$\rho \in C^1(-\infty; +\infty),$$

$$|\rho(s)| \le c_5 |s|^{q_1} \ \ (c_5 > 0);$$

$\rho(s)$ is a monotone increasing function on $(-\infty; +\infty)$.

In the sixth section of the first chapter the existence and uniqueness of local weak solutions of the initial-boundary value problem for nonlinear strongly dissipative wave equations with focusing sources:

$$u_{tt}-\Delta u_t-\Delta u+|u_t|^{q_1-1}u_t=f(u) \text{ in } \Omega_1\times(0,\infty),$$
$$\upsilon_{tt}-\Delta \upsilon_t-\Delta \upsilon+|\upsilon_t|^{q_2-1}\upsilon_t=g(\upsilon) \text{ in } \Omega_2\times(0,\infty),$$
with transmission acoustic conditions:
$$M\delta_{tt}+D\delta_t+K\delta=-u_t,$$
$$u=\upsilon,$$
$$\delta_t=\frac{\partial u}{\partial \nu}-\frac{\partial \upsilon}{\partial \nu}+\frac{\partial u_t}{\partial \nu}-\frac{\partial \upsilon_t}{\partial \nu}$$
on $\Gamma_2\times(0,\infty)$ is proved.

The second chapter consists of three sections. In the first section of the second chapter the mixed problem (12), (13), (3)-(8) with defocusing nonlinear sources and transmission acoustic conditions is considered:

$$u_{tt}-\Delta u+g_1(u_t)+f_1(u)=0 \quad \text{in} \quad \Omega_1\times(0,\infty), \tag{12}$$
$$\upsilon_{tt}-\Delta \upsilon+g_2(\upsilon_t)+f_2(\upsilon)=0 \quad \text{in} \quad \Omega_2\times(0,\infty). \tag{13}$$

It is assumed that the following conditions are satisfied:
$$g_i\in C^1(R),\ i=1,2,$$
and there exist constants k_{i1}, k_{i2}, $k_{i3}>0$ $(i=1,2)$ such that
$$|g_i(s)|\le k_{i1}|s|^q \text{ if } |s|>1 \text{ and } |g_i(s)|\le k_{i2}|s| \text{ if } |s|\le 1, \tag{14}$$
where q satisfies the inequality $1\le q\le \frac{n+2}{n-2}$ if $n\ge 3$ and $q\ge 1$ if $n\le 2$;
$$g_i'(s)\ge k_{i3}>0 \text{ for } \forall s\in R; \tag{15}$$
$$f_i\in C^1(R),\ i=1,2$$
and there exist constants $c_{i1}, c_{i2}, \bar{c}_{i1}, \bar{c}_{i2}>0$ $(i=1,2)$ such that
$$|f_i(s)|\le c_{i1}|s|^p+c_{i2},\quad |f_i'(s)|\le \bar{c}_{i1}|s|^{p-1}+\bar{c}_{i2}, \tag{16}$$
where p satisfies the inequality $1\le p\le \frac{n}{n-2}$ if $n\ge 3$ and $p\ge 1$ if $n\le 2$;

$$0 \le F_i(s) = \int_0^s f_i(s)ds \le \frac{1}{l_i+1} s f_i(s) \qquad (17)$$

for $\forall s \in R^+$, $l_i > 1, i = 1,2$.

The following theorem on the existence and uniqueness of global strong solutions to the problem (12), (13), (3)-(8) is proved.

Theorem 5. Let the conditions (9), (14)-(17) be satisfied. Suppose that

$$u_0 \in H^1_{\Gamma_1}(\Omega_1) \cap H^2(\Omega_1),\ u_1 \in H^1_{\Gamma_1}(\Omega_1) \cap L^{2q}(\Omega_1),$$
$$\upsilon_0 \in H^2(\Omega_2),\ \upsilon_1 \in H^1(\Omega_2) \cap L^{2q}(\Omega_2),\ \delta_0, \delta_1 \in L^2(\Gamma_2).$$

Then for all $T > 0$ there exists a unique strong solution to the problem (12), (13), (3)-(8), i.e., the solution (u, υ, δ) such that

$$u, u_t \in L^\infty\left(0,T; H^1_{\Gamma_1}(\Omega_1)\right),\ u_{tt} \in L^\infty\left(0,T; L^2(\Omega_1)\right),$$
$$\upsilon, \upsilon_t \in L^\infty\left(0,T; H^1(\Omega_2)\right),\ \upsilon_{tt} \in L^\infty\left(0,T; L^2(\Omega_2)\right),$$
$$u(t,\cdot) \in H(\Delta, \Omega_1),\ \upsilon(t,\cdot) \in H(\Delta, \Omega_2) \text{ a. e. on } (0,T),$$
$$\delta, \delta_t, \delta_{tt} \in L^\infty\left(0,T; L^2(\Gamma_2)\right)$$

and

$$u_{tt} - \Delta u + g_1(u_t) + f_1(u) = 0 \quad \text{a. e. in} \quad \Omega_1 \times (0,T),$$
$$\upsilon_{tt} - \Delta \upsilon + g_2(\upsilon_t) + f_2(\upsilon) = 0 \quad \text{a. e. in} \quad \Omega_2 \times (0,T),$$
$$\gamma_0(u_t) + M\delta_{tt} + D\delta_t + K\delta = 0,\ \gamma_0(u) = \gamma_0(\upsilon) \text{ a. e. on } \Gamma_2 \times (0,T),$$
$$\left\langle \gamma_1(u(t) - \upsilon(t)), \gamma_0(\varphi) \right\rangle_{H^{-1/2}(\Gamma_2) \times H^{1/2}(\Gamma_2)} = \left(\delta_t(t), \gamma_0(\varphi)\right)_{\Gamma_2}$$

for $\forall \varphi \in H^1_{\Gamma_1}(\Omega_1)$ a. e. on $(0,T)$,

$$u(x,0) = u_0(x),\ u_t(x,0) = u_1(x) \text{ a. e. in } \Omega_1,$$
$$\upsilon(x,0) = \upsilon_0(x),\ \upsilon_t(x,0) = \upsilon_1(x) \text{ a. e. in } \Omega_2,$$
$$\delta(x,0) = \delta_0(x),\ \delta_t(x,0) = \delta_1(x) \text{ a. e. on } \Gamma_2.$$

In the second section of the second chapter the following result on the exponential decay of global strong solutions to the problem (12), (13), (3)-(8) was established.

Theorem 6. Suppose that (u,υ,δ) is a global strong solution to the problem (12)-(13), (3)-(8) and

$$E(t)=\frac{1}{2}\Big[\|u_t\|_1^2+\|\nabla u\|_1^2+\|\upsilon_t\|_2^2+\|\nabla\upsilon\|_2^2+$$

$$+(F_1(u),1)_1+(F_2(\upsilon),1)_2+\left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2+\left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2\Big].$$

Then, under the conditions of the Theorem 5, there exist positive constants a and b such that

$$E(t)\le a\exp(-bt)$$

for all $t\ge 0$.

In the third section of the second chapter the following mixed problem with transmission acoustic conditions was considered in the domain $\Omega\subset R^3$:

$$u_{tt}-\Delta u+\alpha_1 u_t+u+f_1(u)=0 \quad \text{in} \quad \Omega_1\times(0,\infty), \qquad (18)$$

$$\upsilon_{tt}-\Delta\upsilon+\alpha_2\upsilon_t+\upsilon+f_2(\upsilon)=0 \quad \text{in} \quad \Omega_2\times(0,\infty), \qquad (19)$$

$$\delta_{tt}+\beta\delta_t+\delta=-u_t \quad \text{on} \quad \Gamma_2\times(0,\infty), \qquad (20)$$

$$u=0 \quad \text{on} \quad \Gamma_1\times(0,\infty), \qquad (21)$$

$$u=\upsilon,\ \delta_t=\frac{\partial u}{\partial\nu}-\frac{\partial\upsilon}{\partial\nu} \quad \text{on} \quad \Gamma_2\times(0,\infty), \qquad (22)$$

$$\left.\begin{aligned}u(x,0)=u_0(x),\ u_t(x,0)=u_1(x),\ x\in\Omega_1,\\ \upsilon(x,0)=\upsilon_0(x),\ \upsilon_t(x,0)=\upsilon_1(x),\ x\in\Omega_2,\end{aligned}\right\} \qquad (23)$$

$$\delta(x,0)=\delta_0(x),\ \delta_t(x,0)=\frac{\partial u_0}{\partial\nu}-\frac{\partial\upsilon_0}{\partial\nu}\equiv\delta_1(x),\quad x\in\Gamma_2, \qquad (24)$$

where $\alpha_i>0$ $(i=1,2)$ and $\beta>0$ – some constants; $f_i:\ R\to R$ $(i=1,2)$ – known functions; it is assumed that $f_i\in C^1(R)$, $i=1,2$ and there exist constants $c_{1i}\ge 0$, $i=1,2$, such that

$$|f_i'(s)| \le c_{1i}\left(1+s^2\right), \tag{25}$$

and

$$\liminf_{|s|\to\infty} \frac{f_i(s)}{s} > -1, \; i=1,2; \tag{26}$$

to prove the smoothness of solutions instead of continuous differentiability of functions f_i and the conditions (25), the stronger assumptions were used: $f_i \in C^2(R)$, $i=1,2$ and there exist constants $c_{2i} \ge 0$, $i=1,2$, for which:

$$|f_i''(s)| \le c_{2i}(1+|s|), \; c_{2i} \ge 0 . \tag{27}$$

It is introduced the phase space:

$$V = \{w=(w_1,w_2,w_3,w_4,w_5,w_6): w_1 \in H^1_{\Gamma_1}(\Omega_1), w_2 \in L^2(\Omega_1),$$
$$w_3 \in H^1(\Omega_2), w_4 \in L^2(\Omega_2), w_5 \in L^2(\Gamma_2), w_6 \in L^2(\Gamma_2), w_1|_{\Gamma_2} = w_3|_{\Gamma_2}\},$$

which is Hilbert space with respect to the norm

$$\|w\|_V^2 = \|w_1\|^2_{H^1_{\Gamma_1}(\Omega_1)} + \|w_2\|^2_{L^2(\Omega_1)} + \|w_3\|^2_{H^1(\Omega_2)} + \|w_4\|^2_{L^2(\Omega_2)} +$$
$$+ \|w_5\|^2_{L^2(\Gamma_2)} + \|w_6\|^2_{L^2(\Gamma_2)}$$

for $\forall w=(w_1,w_2,w_3,w_4,w_5,w_6) \in V$ and the initial-boundary value problem (18)-(24) is formulated in the following form

$$\begin{cases} w_t = Aw + \Phi(w), \\ w(0) = w_0 , \end{cases} \tag{28}$$

where $w=(u,u_t,\upsilon,\upsilon_t,\delta,\delta_t)$, $w_0=(u_0,u_1,\upsilon_0,\upsilon_1,\delta_0,\delta_1)$,

$$A: D(A) \subset V \to V ,$$
$$Aw = (w_2, \Delta w_1 - w_1 - \alpha_1 w_2, w_4, \Delta w_3 - w_3 - \alpha_2 w_4,$$
$$w_6, -w_2|_{\Gamma_2} - w_5 - \beta w_6),$$

where $w_{i\nu}$ $(i=1,2)$ is a derivative of the function w_i along the normal ν (the condition $w_6 = w_{1\nu}\big|_{\Gamma_2} - w_{3\nu}\big|_{\Gamma_2}$ is interpreted in a weak sense - as a fulfillment of the equality

$$\int_{\Omega_1}(\Delta w_1\varphi + \nabla w_1 \nabla \varphi)dx + \int_{\Omega_2}(\Delta w_3\psi + \nabla w_3 \nabla \psi)dx = \int_{\Gamma_2} w_6 \varphi \, d\Gamma_2$$

for $\forall \varphi \in H^1_{\Gamma_1}(\Omega_1)$, $\forall \psi \in H^1(\Omega_2)$ such that $\varphi\big|_{\Gamma_2} = \psi\big|_{\Gamma_2}$),

$$\Phi : V \to V, \quad \Phi(w) = (0, -f_1(w_1), 0, -f_2(w_3), 0, 0) \text{ for } \forall w \in V.$$

To consider strong solutions, it is also introduced the phase space

$$V_1 = \Big\{ w = (w_1, w_2, w_3, w_4, w_5, w_6) \in \left(H^1_{\Gamma_1}(\Omega_1) \cap H^2(\Omega_1)\right) \times$$
$$\times H^1(\Omega_1) \times H^2(\Omega_2) \times H^1(\Omega_2) \times H^{1/2}(\Gamma_2) \times H^{1/2}(\Gamma_2) :$$
$$w_1\big|_{\Gamma_2} = w_3\big|_{\Gamma_2}, w_2\big|_{\Gamma_2} = w_4\big|_{\Gamma_2}, w_6 = w_{1\nu}\big|_{\Gamma_2} - w_{3\nu}\big|_{\Gamma_2} \Big\},$$

which is Hilbert with respect to the norm

$$\|w\|^2_{V_1} = \|w_1\|^2_{H^1_{\Gamma_1}(\Omega_1) \cap H^2(\Omega_1)} + \|w_2\|^2_{H^1(\Omega_1)} + \|w_3\|^2_{H^2(\Omega_2)} + \|w_4\|^2_{H^1(\Omega_2)} +$$
$$+ \|w_5\|^2_{H^{1/2}(\Gamma_2)} + \|w_6\|^2_{H^{1/2}(\Gamma_2)}.$$

The definition of a weak solution to the problem (28) is introduced.

Definition 8. Let $w_0 = (u_0, u_1, \upsilon_0, \upsilon_1, \delta_0, \delta_1) \in V$. The function $w \in C^0([0,\infty); V)$ is called a weak solution to the problem (28) if it satisfies the equality

$$w(t) = e^{At} w_0 + \int_0^t e^{A(t-s)} \Phi(w(s)) ds$$

for all $t \geq 0$.

It is introduced the functional

$$E_w(t)=\frac{1}{2}\|w\|_V^2+\int_{\Omega_1}F_1(w_1)dx\ +\int_{\Omega_2}F_2(w_3)dx\,, \qquad (29)$$

where $F_i(s)=\int_0^s f_i(s)ds$, $i=1,2$.

The main results for the problem (28) or (18)-(24) are established in the following theorems (the method of proof of the existence of a global attractor is based on the use of a suitable energy equation).

Theorem 7. Let (25), (26) hold and assume that $w_0\in V$. Then there exists a unique weak solution $w\in C^0([0,\infty);V)$ to the problem (28). Furthermore, if $\overline{w}_1$ and $\overline{w}_2$ - solutions to the problem (28), corresponding to the initial data $\overline{w}_{10}$ and $\overline{w}_{20}$ for which: $\|\overline{w}_{10}\|_V\le r$, $\|\overline{w}_{20}\|_V\le r$ $(r>0)$, then there exists the positive number θ, depending on r such that

$$\|\overline{w}_2(t)-\overline{w}_1(t)\|_V\le e^{\theta t}\|\overline{w}_{20}-\overline{w}_{10}\|_V$$

for all $t\ge 0$. Assuming in addition, that the functions f_i, $i=1,2$ fulfill the conditions (27) and that $w_0\in V_1$, the corresponding weak solution satisfies the regularity property

$$w\in C^1([0,\infty);V)\cap C^0([0,\infty);V_1)$$

(and is called a ”strong” solution).

It is proved that for each weak solution $w=(u,u_t,\upsilon,\upsilon_t,\delta,\delta_t)$ to the problem (28) the energy equation

$$\frac{dE_w}{dt}=-\alpha_1\|u_t\|_1^2-\alpha_2\|\upsilon_t\|_2^2-\beta\|\delta_t\|_{\Gamma_2}^2$$

is valid and $E_w(\cdot)\in C^1([0,\infty))$, where the functional $E_w(t)$ is introduced in (29).

Corollary 1. Under the conditions (25), (26) the system (28) generates a strongly continuous semigroup $S(t)=S_{\alpha_1,\alpha_2\beta}(t)$ in the phase space V, which is defined by the formula

$$S(t)(u_0,u_1,\upsilon_0,\upsilon_1,\delta_0,\delta_1)=(u,u_t,\upsilon,\upsilon_t,\delta,\delta_t),$$

where $(u,u_t,\upsilon,\upsilon_t,\delta,\delta_t)\in C^0([0,\infty);V)$ is a weak solution to the problem (28) corresponding to the initial data $(u_0,u_1,\upsilon_0,\upsilon_1,\delta_0,\delta_1)\in V$.

Theorem 8. Let the conditions (25), (26) be satisfied. Then there exists the constant $R_0>0$ with the following property: for each $R>0$ there exists the number $t_0=t_0(R)>0$ such that the inequality

$$\|S(t)w_0\|_V\le R_0$$

holds for $\forall w_0\in V$ with $\|w_0\|_V\le R$ and for all $t\ge t_0$; therefore, the set

$$B_0=\{z\in V:\|z\|_V\le R_0\}$$

is a bounded absorbing set for a semigroup $S(t)$ in V.

Theorem 9. Let the conditions (25), (26) be satisfied and $\alpha_1=\alpha_2=\alpha$. Then the semigroup $\{S(t)\}_{t\ge0}$ is asymptotically compact.

Theorem 10. Let conditions (25), (26) be satisfied and $\alpha_1=\alpha_2=\alpha$. Then the problem (18)-(24) has a minimal global attractor in the phase space V which is invariant and compact.

The third chapter consists of three sections. In the first section of the third chapter the following mixed problem is considered in the domain $Q=\Omega\times(0,T)$ ($\Omega\subset R^N(N\ge1)$ is a bounded domain with the sufficiently smooth boundary Γ):

$$\frac{\partial^2u}{\partial t^2}+\frac{\partial}{\partial t}[u+\mathsf{F}(u)]-\Delta u+|u|^pu=f\,, \tag{30}$$

$$u=0,\ (x,t)\in\Gamma\times[0,T], \tag{31}$$

$$[u+\mathsf{F}(u)]\big|_{t=0}=u^{(0)}+w^{(0)},\ \ \frac{\partial u}{\partial t}\Big|_{t=0}=u^{(1)}\,, \tag{32}$$

where $p>0$; $f:Q\to R$, $u^{(0)}$, $u^{(1)}$, $w^{(0)}:\ \Omega\to R$ - known functions and the nonlinear operator F acts from the space $\mathrm{M}(\Omega;C^0([0,T]))$ to $\mathrm{M}(\Omega;C^0([0,T]))$. It is assumed that F is a memory operator; i.e.

at any instant t, $[\mathsf{F}(u)](t)$ may depend not only on $u(t)$ but also on the previous evolution of u. It is also assumed that this operator is applied at each point $x \in \Omega$ independently (i.e. the output $[\mathsf{F}(u(x,\cdot))](t)$ depends on $u(x,\cdot)|_{[0,t]}$, but not on $u(y,\cdot)|_{[0,t]}$ for any $y \neq x$) and the following conditions are satisfied:

$$\begin{cases} [\mathsf{F}(\upsilon_1)](\cdot,t) = [\mathsf{F}(\upsilon_2)](\cdot,t) \text{ a.e.in } \Omega, \\ \text{for } \forall \upsilon_1, \upsilon_2 \in \mathrm{M}(\Omega; C^0([0,T])) \text{ such that } \upsilon_1 = \upsilon_2 \\ \text{on } [0,t] \text{ for } \forall t \in [0,T]; \end{cases} \tag{33}$$

$$\begin{cases} \text{if } \upsilon_n \to \upsilon \text{ uniformly in } \mathrm{M}(\Omega; C^0([0,T])), \\ \text{then } \mathsf{F}(\upsilon_n) \to \mathsf{F}(\upsilon) \text{ uniformly on } [0,T], \text{ a.e. in } \Omega; \end{cases} \tag{34}$$

$$\begin{cases} \text{there exist } L > 0 \text{ and } g \in L^2(\Omega) \text{ such that} \\ \|[\mathsf{F}(\upsilon)](x,\cdot)\|_{C^0([0,T])} \leq L \|\upsilon(x,\cdot)\|_{C^0([0,T])} + g(x) \text{ a.e.in } \Omega \\ \text{for } \forall \upsilon \in M(\Omega; C^0([0,T])); \end{cases} \tag{35}$$

$$\begin{cases} \text{if the function } \upsilon \in M(\Omega; C^0([0,T])) \text{ is affine} \\ \text{in } [t_1,t_2] \text{ for } \forall [t_1,t_2] \subset [0,T], \text{ a.e.in } \Omega, \text{ then} \\ \{[\mathsf{F}(\upsilon)](x,t_2) - [\mathsf{F}(\upsilon)](x,t_1)\}[\upsilon(x,t_2) - \upsilon(x,t_1)] \geq 0 \text{ a.e.in } \Omega; \end{cases} \tag{36}$$

$$\begin{cases} \text{there exists } 0 < L_1 < 1 \text{ such that} \\ |[\mathsf{F}(\upsilon)](x,t_2) - [\mathsf{F}(\upsilon)](x,t_1)| \leq L_1 |\upsilon(x,t_2) - \upsilon(x,t_1)| \\ \text{a.e. in } \Omega, \text{for } \forall \upsilon \in M(\Omega; C^0([0,T])) \text{ which is affine} \\ \text{in } [t_1,t_2] \text{ for } \forall [t_1,t_2] \subset [0,T], \text{ a.e. in } \Omega, \end{cases} \tag{37}$$

$$u^{(0)} \in V,\ w^{(0)} \in L^2(\Omega),\ u^{(1)} \in L^2(\Omega), \tag{38}$$

$$f = f_1 + f_2, \quad f_1 \in L^2(Q), \quad f_2 \in W^{1,1}(0,T;V'), \tag{39}$$

where $V = H_0^1(\Omega) \cap L^{p+2}(\Omega)$.

Definition 9. A function $u \in L^2(0,T;V) \cap H^1(0,T;L^2(\Omega))$ such that $\mathsf{F}(u) \in L^2(Q)$ and satisfying the equality

$$\iint_Q \left\{ -\frac{\partial u}{\partial t} \cdot \frac{\partial \upsilon}{\partial t} - [u + \mathsf{F}(u)] \frac{\partial \upsilon}{\partial t} + \nabla u \cdot \nabla \upsilon + |u|^p u \upsilon \right\} dxdt =$$

$$= \int_0^T {}_{V'}\langle f, \upsilon \rangle_V \, dt + \int_\Omega \left[u^{(0)}(x) + w^{(0)}(x) + u^{(1)}(x) \right] \upsilon(x,0) dx$$

for $\forall \upsilon \in L^2(0,T;V) \cap H^1(0,T;L^2(\Omega))$ $(\upsilon(\cdot,T) = 0$ a. e. in $\Omega)$, is called a solution to the problem (30)-(32).

The results on the existence and uniqueness of solutions to the problem (30)-(32) are established in the following theorems.

Theorem 11. Let the conditions (33)-(39) be satisfied. Then the problem (30)-(32) has at least one solution u such that

$$u \in W^{1,\infty}(0,T;L^2(\Omega)) \cap L^\infty(0,T;V), \ \mathsf{F}(u) \in H^1(0,T;L^2(\Omega)).$$

Theorem 12. Let the conditions of Theorem 11 be satisfied. Suppose that

$$p \le \frac{2}{N-2}, \quad N \ge 3 \tag{40}$$

and

$$\{[\mathsf{F}(\upsilon_1)](x,t) - [\mathsf{F}(\upsilon_2)](x,t)\}[\upsilon_1(x,t) - \upsilon_2(x,t)] \ge 0 \text{ a. e. in } \Omega, \tag{41}$$

for $\forall \upsilon_1, \upsilon_2 \in M(\Omega; C^0([0,T]))$ and $\forall t \in (0,T)$. Then the solution to the problem (30)-(32) is unique.

Theorem 13. Let the conditions (33)-(39), (40)-(41) be satisfied. Then the problem (30)-(32) has a unique solution $u \in C^1([0,T];L^2(\Omega)) \cap C([0,T];H_0^1(\Omega))$ for all $T > 0$.

Further, in the same section, the problem (30)-(32) is considered for the case

$$f = h(x), \ h \in L^2(\Omega). \tag{42}$$

Under the condition (40), the equality $V = H_0^1(\Omega) \cap L^{p+2}(\Omega) = H_0^1(\Omega)$ holds. Then, under the conditions of the Theorem 13, there corresponds a semigroup $\{S(t)\}_{t \ge 0}$ to the problem (30)-(32) (with (42)) in the space

$$E = H_0^1(\Omega)\times L^2(\Omega)\times L^2(\Omega)$$

which is defined by the formula

$$S(t)\left(u^{(0)}, u^{(1)}, w^{(0)}\right) = (u, u_t, w),$$

where u is a unique solution of this problem.

The following theorem on the existence of a bounded absorbing set for the semigroup $\{S(t)\}_{t\geq 0}$ generated by the problem (30)-(32) (with $f = h(x)$) is proved using the method of discretization with respect to the variable t.

Theorem 14. The problem (30)-(32) (with $f = h(x)$) has a bounded absorbing set in the space E under the conditions (33)-(38), (40)-(42).

Further, it is proved that the semigroup $\{S(t)\}_{t\geq 0}$ generated by the problem (30)-(32) (with $f = h(x)$) is asymptotically compact in the space E under the conditions of the Theorem 14, and then the following theorem on the existence of a minimal global attractor for this problem is proved.

Theorem 15. Let the conditions of the Theorem 14 be satisfied. Then the problem (30)-(32) (with $f = h(x)$) has a minimal global attractor in the space E, which is invariant and compact.

In the second section of the third chapter, the following mixed problem for a quasilinear parabolic equation is considered in the domain $Q = \Omega\times(0,T)$ ($\Omega\subset R^N(N\geq 1)$ is a bounded domain with a sufficiently smooth boundary Γ):

$$\frac{\partial}{\partial t}[u + \mathsf{F}(u)] - \Delta u + |u|^p u = f\ , \tag{43}$$

$$u = 0,\ (x,t)\in\Gamma\times(0,T), \tag{44}$$

$$[u + \mathsf{F}(u)]\big|_{t=0} = u^0 + w^0, \tag{45}$$

where $0 < p \leq \frac{2}{2-N}$, if $N\geq 3$ and $p > 0$, if $N = 1,2$; $f:\Omega\times(0,T)\to R$ and the nonlinear operator F acts from the space $\mathrm{M}\left(\Omega; C^0([0,T])\right)$ to $\mathrm{M}\left(\Omega; C^0([0,T])\right)$. It is assumed that F is a memory operator, which is applied at each point $x \in \Omega$ independently and satisfies the conditions (33)-(36), and

$$u^0 \in V,\ w^0 \in L^2(\Omega), \tag{46}$$

$$f = f_1 + f_2,\ f_1 \in L^2(Q),\ f_2 \in W^{1,1}(0,T;V'), \tag{47}$$

where $V = H_0^1(\Omega)$.

Definition 10. A function $u \in M(\Omega; C^0([0,T])) \cap \cap L^2(0,T;V)$ such that $\mathsf{F}(u) \in L^2(Q)$ and satisfying the equality

$$\iint_Q \left\{ -[u + \mathsf{F}(u)] \frac{\partial \upsilon}{\partial t} + \nabla u \cdot \nabla \upsilon + |u|^p u \upsilon \right\} dxdt =$$

$$= \int_0^T {}_{V'}\langle f, \upsilon \rangle_V \, dt + \int_\Omega \left[u^0(x) + w^0(x) \right] \upsilon(x,0) dx$$

for $\forall \upsilon \in L^2(0,T;V) \cap H^1(0,T;L^2(\Omega))$ $(\upsilon(\cdot,T) = 0$ a. e. in $\Omega)$, is called a solution to the problem (43)-(45).

Using the method of discretization with respect to the variable t it is proved the following theorem for the problem (43)-(45).

Theorem 16. Let the conditions (33)-(36), (46), (47) be satisfied and

$$0 < p \le \frac{2}{2-N} \text{ if } N \ge 3 \text{ and } p > 0 \text{ if } N = 1, 2.$$

Then the problem (43)-(45) has at least one solution u for which

$$u \in H^1(0,T;L^2(\Omega)) \cap L^\infty(0,T;V),\ \mathsf{F}(u) \in L^2(\Omega; C^0([0,T])). \tag{48}$$

The problem (43)-(45) is also considered under the additional condition that F is a hysteresis nonlinearity of the generalized play type.

Let for nondecreasing functions $\gamma_l(\sigma), \gamma_r(\sigma) \in C^0(R)$ it is satisfied the condition

$$\gamma_r(\sigma) \le \gamma_l(\sigma) \tag{49}$$

for $\forall \sigma \in R$. Assume that u is a continuous, piecewise linear function on $[0,T]$, namely, u is linear on $[t_{i-1}, t_i]$ for $i = 1, 2, ..., N$.

For given $\xi^0 \in R$, an operator $w = E(u, \xi^0): [0,T] \to R$ is called a hysteresis nonlinearity of the generalized play type if

$$w(t) = \begin{cases} \min\{\gamma_l(u(0)), \max\{\gamma_r(u(0)), \xi^0\}\}, \text{ if } t = 0, \\ \min\{\gamma_l(u(t)), \max\{\gamma_r(u(t)), t_{i-1}\}\}, \text{ if } t \in (t_{i-1}, t_i] \end{cases}$$

$(i = 1, \ldots, N)$ and $w(0) = \xi^0$ only if $\gamma_r(u(0)) \le \xi^0 \le \gamma_l(u(0))$.

It is assumed that

$$[\mathsf{F}(u)](x,t) = \left[E\left(u(x,\cdot), \xi^0(x)\right)\right](t) \text{ a. e. in } \Omega, \qquad (50)$$

for $\forall u \in M(\Omega; C^0([0,T]))$ and for $\forall t \in [0,T]$; where $\xi^0 \in L^1(\Omega)$ and E is a hysteresis operator of the generalized play type.

The following theorems are proved for the problem (43)-(45).

Theorem 17. Let $u_i, \xi_i^0 \in L^2(\Omega)$, $h_i(x) \in L^2(\Omega)$ $(i = 1,2)$; $\gamma_l(\sigma), \gamma_r(\sigma) \in C^0(R)$ - are Lipschitz continuous, affinely bounded functions and satisfy the condition (49). Define F as in (50) and assume that

$$w_i^0 = \min\{\max\{\xi_i^0, \gamma_r(u_i^0)\}, \gamma_l(u_i^0)\} \text{ a. e. on } \Omega \ (i = 1,2).$$

If $u_i \in W^{1,1}(0,T; L^1(\Omega)) \cap L^2(0,T;V)$ are the corresponding solutions to the problem (43)-(45) with $f = h_i$ and $w_i = \mathsf{F}(u_i)$ $(i = 1,2)$, then

$$\int_\Omega \left[(u_1 - u_2)^+(x,t) + (w_1 - w_2)^+(x,t)\right] dx \le$$

$$\le \int_\Omega \left[(u_1^0 - u_2^0)^+(x) + (w_1^0 - w_2^0)^+(x)\right] dx + T \int_\Omega (h_1 - h_2)^+ \, dx$$

for $\forall t \in [0,T]$, where $(\cdot)^+ = \max\{(\cdot), 0\}$.

Theorem 18. Assume that

$$u^0 \in V, \ \xi^0 \in L^2(\Omega), \ h \in L^2(\Omega),$$

the functions $\gamma_l(\sigma), \gamma_r(\sigma) \in C^0(R)$ are Lipschitz continuous, affinely bounded and satisfy the condition (49), and F is defined as in (50).

Then the problem (43)-(45) (with $f = h(x)$) has a unique solution that satisfies the condition (48).

Let $X = H_0^1(\Omega)\times L^2(\Omega)$. Then, under the conditions of the Theorem 18, there corresponds a semigroup $\{S(t)\}_{t\geq 0}$ to the problem (43)-(45) (with $f = h(x)$) in the space X which is defined by the formula

$$S(t)\left(u^{(0)}, w^0\right) = (u, \mathsf{F}(u)),$$

where u is a unique solution to this problem.

Theorem 19. The problem (43)-(45) (with $f = h(x)$) has a bounded absorbing set in the space X under the conditions of Theorem 18.

In the third section of the third chapter the following mixed problem is considered in the domain $Q = \Omega\times(0,T)$ ($\Omega\subset R^N(N\geq 1)$ is a bounded domain with a sufficiently smooth boundary Γ):

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial}{\partial t}[u + \mathsf{F}(u)] + \Delta^2 u + |u|^p u = f\ , \qquad (51)$$

$$u = 0,\ \ \Delta u = 0,\ \ (x,t)\in\Gamma\times[0,T], \qquad (52)$$

$$[u + \mathsf{F}(u)]\big|_{t=0} = u^{(0)} + w^{(0)}\ ,\ \ \frac{\partial u}{\partial t}\Big|_{t=0} = u^{(1)}\ , \qquad (53)$$

where $p > 0$; $f : Q \to R$, $u^{(0)}$, $u^{(1)}$, $w^{(0)}$: $\Omega \to R$ - given functions and the nonlinear operator F acts from the space $\mathrm{M}(\Omega; C^0([0,T]))$ to $\mathrm{M}(\Omega; C^0([0,T]))$. It is assumed that F is a memory operator, which is applied at each point $x\in\Omega$ independently and satisfies the conditions (33)-(37), and

$$u^{(0)} \in V,\ w^{(0)} \in L^2(\Omega),\ u^{(1)} \in L^2(\Omega)\ , \qquad (54)$$

where $V = H_0^2(\Omega)\bigcap L^{p+2}(\Omega)$.

Definition 11. A function $u \in L^2(0,T;V)\bigcap H^1(0,T;L^2(\Omega))$ such that $\mathsf{F}(u)\in L^2(Q)$ and satisfying the equality

$$\iint_Q \left\{ -\frac{\partial u}{\partial t}\cdot\frac{\partial \upsilon}{\partial t} - [u + \mathsf{F}(u)]\frac{\partial \upsilon}{\partial t} + \Delta u \cdot \Delta \upsilon + |u|^p u\upsilon \right\} dxdt =$$

$$= \int_0^T {}_{V'}\langle f, \upsilon\rangle_V \, dt + \int_\Omega \left[u^{(0)}(x) + w^{(0)}(x) + u^{(1)}(x)\right] \upsilon(x,0)dx$$

for $\forall \upsilon \in L^2(0,T;V) \cap H^1(0,T;L^2(\Omega))$ $(\upsilon(\cdot,T) = 0$ a. e. in $\Omega)$, is called a solution to the problem (51)-(53).

The results on the existence and uniqueness of solutions to the problem (51)-(53) are established in the following theorems.

Theorem 20. Let the conditions (33)-(37), (39), (54) be satisfied. Then the problem (51)-(53) has at least one solution u for which

$$u \in W^{1,\infty}(0,T;L^2(\Omega)) \cap L^\infty(0,T;V),\ \mathsf{F}(u) \in H^1(0,T;L^2(\Omega)).$$

Theorem 21. Let the conditions of the Theorem 20 be satisfied. Suppose that

$$p \le \frac{2}{N-2}, \qquad N \ge 3 \tag{55}$$

and

$$\{[\mathsf{F}(\upsilon_1)](x,t) - [\mathsf{F}(\upsilon_2)](x,t)\}[\upsilon_1(x,t) - \upsilon_2(x,t)] \ge 0 \text{ a.e. in } \Omega, \tag{56}$$

for $\forall \upsilon_1, \upsilon_2 \in M(\Omega; C^0([0,T]))$ and $\forall t \in (0,T)$. Then the solution to the problem (51)-(53) is unique.

The following theorems on the existence of a bounded absorbing set and a minimal global attractor for the problem (51)-(53) are proved for the case $f = h(x)$:

$$h \in L^2(\Omega). \tag{57}$$

Theorem 22. The problem (51)-(53) (with $f = h(x)$) has a bounded absorbing set in the space

$$E = H_0^2(\Omega) \times L^2(\Omega) \times L^2(\Omega)$$

under the conditions (33)-(37), (54)-(57).

Theorem 23. Let the conditions of Theorem 22 be satisfied. Then the problem (51)-(53) (with $f = h(x)$) has a minimal global attractor in the space E, which is invariant and compact.

The fourth chapter consists of three sections. In the first section of the fourth chapter the following mixed problem for a system of semilinear hyperbolic equations is considered in the domain $Q=\Omega\times(0,T)$ ($\Omega\subset R^N(N\geq 1)$ is a bounded domain with the sufficiently smooth boundary Γ):

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2}+\dfrac{\partial}{\partial t}\left[u+\mathsf{F}_1(\upsilon)\right]-\Delta u=f_1, \\ \dfrac{\partial^2 \upsilon}{\partial t^2}+\dfrac{\partial}{\partial t}\left[\upsilon+\mathsf{F}_2(u)\right]-\Delta \upsilon=f_2, \end{cases} \tag{58}$$

$$u=0,\ \upsilon=0,\ (x,t)\in\Gamma\times[0,T], \tag{59}$$

$$\left[u+\mathsf{F}_1(\upsilon)\right]\Big|_{t=0}=u^{(0)}+w_1^{(0)},\quad \frac{\partial u}{\partial t}\Big|_{t=0}=u^{(1)}, \tag{60}$$

$$\left[\upsilon+\mathsf{F}_2(u)\right]\Big|_{t=0}=\upsilon^{(0)}+w_2^{(0)},\quad \frac{\partial \upsilon}{\partial t}\Big|_{t=0}=\upsilon^{(1)}, \tag{61}$$

where nonlinear operators F_1, F_2 act from the space $\mathrm{M}(\Omega;C^0([0,T]))$ to $\mathrm{M}(\Omega;C^0([0,T]))$. It is assumed that F_i, $i=1,2$ are memory operators, which are applied at each point $x\in\Omega$ independently and satisfy the conditions (33)-(37) and

$$u^{(0)}\in H_0^1(\Omega),\ w_1^{(0)}\in L^2(\Omega),\ u^{(1)}\in L^2(\Omega),\ f_1\in L^2(Q), \tag{62}$$

$$\upsilon^{(0)}\in H_0^1(\Omega),\ w_2^{(0)}\in L^2(\Omega),\ \upsilon^{(1)}\in L^2(\Omega),\ f_2\in L^2(Q). \tag{63}$$

Definition 12. A pair of functions (u,υ) such that

$$u,\upsilon\in L^2\left(0,T;H_0^1(\Omega)\right)\cap H^1\left(0,T;L^2(\Omega)\right),$$
$$\mathsf{F}_1(\upsilon)\in L^2(Q),\ \mathsf{F}_2(u)\in L^2(Q)$$

and satisfying equalities

$$\iint_Q \left\{ -\frac{\partial u}{\partial t}\cdot\frac{\partial \overline{\upsilon}}{\partial t} - [u + \mathsf{F}_1(\upsilon)]\frac{\partial \overline{\upsilon}}{\partial t} + \nabla u \cdot \nabla \overline{\upsilon} \right\} dxdt =$$

$$= \iint_Q f_1 \overline{\upsilon}\, dxdt + \int_\Omega \left[u^{(0)}(x) + w_1^{(0)}(x) + u^{(1)}(x) \right] \overline{\upsilon}(x,0) dx,$$

$$\iint_Q \left\{ -\frac{\partial \upsilon}{\partial t}\cdot\frac{\partial \overline{\upsilon}}{\partial t} - [\upsilon + \mathsf{F}_2(u)]\frac{\partial \overline{\upsilon}}{\partial t} + \nabla \upsilon \cdot \nabla \overline{\upsilon} \right\} dxdt =$$

$$= \iint_Q f_2 \overline{\upsilon}\, dxdt + \int_\Omega \left[\upsilon^{(0)}(x) + w_2^{(0)}(x) + \upsilon^{(1)}(x) \right] \overline{\upsilon}(x,0) dx,$$

for $\forall \overline{\upsilon} \in L^2(0,T; H_0^1(\Omega)) \cap H^1(0,T; L^2(\Omega))$ $(\overline{\upsilon}(\cdot, T) = 0$ a. e. in $\Omega)$, is called a solution to the problem (58)-(61).

The results on the existence and uniqueness of solutions to the problem (58)-(61) are established in the following theorems.

Theorem 24. Let the conditions (33)-(37), (62)-(63) be satisfied. Then the problem (58)-(61) has at least one solution (u,υ) for which

$$u, \upsilon \in W^{1,\infty}(0,T; L^2(\Omega)) \cap L^\infty(0,T; H_0^1(\Omega)),$$
$$\mathsf{F}_1(\upsilon),\ \mathsf{F}_2(u) \in H^1(0,T; L^2(\Omega)).$$

Theorem 25. Let the conditions of the Theorem 24 be satisfied and

$$\begin{cases} \text{for } \forall r > 0 \text{ there exists } L_{2i}(r) > 0 \text{ such that} \\ \|\mathsf{F}_i(u) - \mathsf{F}_i(\upsilon)\|_{L^2(\Omega; C^0([0,t]))} \le L_{2i}(r) \|u - \upsilon\|_{L^2(\Omega; L^2([0,t]))} \quad \text{a.e. in } \Omega, \\ \text{for } \forall t \in (0,T] \text{ and } \forall u, \upsilon \in \left\{ \tilde{u} \in L^2(Q_t) : \|\tilde{u}\|_{L^2(Q_t)} \le r \right\} \end{cases}$$

$i = 1,2$. Then the solution to the problem (58)-(61) is unique.

In the second section of the fourth chapter the following mixed problem for the Timoshenko system with a memory operator is considered in the domain $Q = (0,L) \times (0,T)$ $((0,L) \subset R^1)$, which arises in the theory of transverse vibrations of a string:

$$\rho_1\varphi_{tt} - a(\varphi_x + \psi)_x = 0\,, \tag{64}$$

$$\rho_2\psi_{tt} - b\psi_{xx} + a(\varphi_x + \psi) + \mathsf{F}(\psi) = 0\,, \tag{65}$$

$$\varphi|_{x=0} = \varphi|_{x=L} = 0,\ \ \psi\big|_{x=0} = \psi\big|_{x=L} = 0 \text{ on } (0,T), \tag{66}$$

$$\varphi\big|_{t=0} = \varphi^{(0)}, \varphi_t\big|_{t=0} = \varphi^{(1)}, \psi\big|_{t=0} = \psi^{(0)}, \psi_t\big|_{t=0} = \psi^{(1)},\ \mathsf{F}(\psi)\big|_{t=0} = w^{(0)} \tag{67}$$

on $(0, L)$, where t is the time variable, x is the coordinate of the string of length L at its equilibrium position; $\varphi^{(0)}, \varphi^{(1)}, \psi^{(0)}, \psi^{(1)}, w^{(0)}:\ (0, L) \to R$ - given functions, ρ_1, ρ_2, a, b - positive constants. It is assumed that the operator F acts from the space $\mathrm{M}(0, L; C^0([0,T]))$ to $\mathrm{M}(0, L; C^0([0,T]))$, where $\mathrm{M}(0, L; C^0([0,T]))$ is the space of measurable functions acting from $(0, L)$ to $C^0([0,T])$, and that F is a memory operator, which is applied at each point $x \in (0, L)$ independently: $[\mathsf{F}(\psi(x,\cdot))](\mathrm{t})$ depends on $\psi(x,\cdot)\big|_{[0,t]}$ but not on $\psi(y,\cdot)\big|_{[0,t]}$ for any $y \neq x$. It is assmed that the following conditions are satisfied:

$$\begin{cases} [\mathsf{F}(\upsilon_1)](\cdot,t) = [\mathsf{F}(\upsilon_2)](\cdot,t) \text{ a.e. on } (0,L), \\ \text{for } \forall\, \upsilon_1, \upsilon_2 \in \mathrm{M}(0, L; C^0([0,T])) \text{ such that } \upsilon_1 = \upsilon_2 \\ \text{on } [0,t] \text{ for } \forall\, t \in [0,T]; \end{cases} \tag{68}$$

$$\begin{cases} \text{if } \upsilon_{\mathrm{n}} \to \upsilon \text{ uniformly in } \mathrm{M}(0, L; C^0([0,T])), \\ \text{then } \mathsf{F}(\upsilon_n) \to \mathsf{F}(\upsilon) \text{ uniformly on } [0,T], \text{a.e. on } (0,L); \end{cases} \tag{69}$$

$$\begin{cases} \text{there exist } L_1 > 0 \text{ and } g \in L^2(0,L) \text{ such that} \\ \|[\mathsf{F}(\upsilon)](x,\cdot)\|_{C^0([0,T])} \le L_1 \|\upsilon(x,\cdot)\|_{C^0([0,T])} + g(x) \\ \text{for } \forall\, \upsilon \in M(0, L; C^0([0,T])), \text{ a.e. on } (0,L); \end{cases} \tag{70}$$

$$\begin{cases} \text{if the function } \upsilon \in M\left(0, L; C^0\left(\left[0, T\right]\right)\right) \text{ is affine} \\ \text{on } \left[t_1, t_2\right] \text{ for } \forall \left[t_1, t_2\right] \subset \left[0, T\right], \text{ a.e. on } \left(0, L\right), \text{ then} \\ \left\{\left[\mathsf{F}(\upsilon)\right](x, t_2) - \left[\mathsf{F}(\upsilon)\right](x, t_1)\right\}\left[\upsilon(x, t_2) - \upsilon(x, t_1)\right] \geq 0, \text{ a.e. on } \left(0, L\right). \end{cases} \tag{71}$$

$$\varphi^{(0)}, \psi^{(0)} \in H_0^1\left(0, L\right), \ \varphi^{(1)}, \psi^{(1)}, w^{(0)} \in L^2\left(0, L\right). \tag{72}$$

Definition 13. A pair of functions (φ, ψ) such that

$$\varphi, \psi \in L^2\left(0, T; H_0^1(0, L)\right) \cap H^1\left(0, T; L^2(0, L)\right), \ \mathsf{F}(\psi) \in L^2(Q);$$

$$\varphi\big|_{t=0} = \varphi^{(0)}, \ \psi\big|_{t=0} = \psi^{(0)}, \ \mathsf{F}(\psi)\big|_{t=0} = w^{(0)} \ \text{на} \ \left(0, L\right)$$

and satisfying the equalities

$$\iint_Q \left\{-\rho_1 \frac{\partial \varphi}{\partial t} \cdot \frac{\partial \upsilon}{\partial t} + a\varphi_x \upsilon_x - a\psi_x \upsilon\right\} dxdt = -\rho_1 \int_0^L \varphi^{(1)}(x)\upsilon(x, 0)dx,$$

$$\iint_Q \left\{-\rho_2 \frac{\partial \psi}{\partial t} \cdot \frac{\partial \upsilon}{\partial t} + b\psi_x \upsilon_x + a\varphi_x \upsilon + a\psi\upsilon + \mathsf{F}(\psi)\upsilon\right\} dxdt =$$

$$= -\rho_2 \int_0^L \psi^{(1)}(x)\upsilon(x, 0)dx$$

for $\forall \upsilon \in L^2\left(0, T; H_0^1(0, L)\right) \cap H^1\left(0, T; L^2(0, L)\right)$ $\left(\upsilon(\cdot, T) = 0\right.$ a.e. on $(0, L)$), is called a solution to the problem (64)-(67).

The results on the existence and uniqueness of solutions to the problem (64)-(67) are established in the following theorems.

Theorem 26. Let the conditions (68)-(72) be satisfied. Then the problem (64)-(67) has at least one solution (φ, ψ), for which

$$\left. \begin{array}{c} \varphi, \psi \in L^\infty\left(0, T; H_0^1(0, L)\right) \cap H^1\left(0, T; L^2(0, L)\right), \\ \mathsf{F}(\psi) \in L^2\left(0, L; C^0\left(\left[0, T\right]\right)\right) \end{array} \right\} \tag{73}$$

Theorem 27. Let the conditions of the Theorem 26 be satisfied and the operator F also satisfies the condition of global Lipschitz continuity:

$$\exists L_2 > 0: \ \forall t \in \left(0, T\right], \ \forall \upsilon_1, \upsilon_2 \in L^2\left(0, L; C^0\left(\left[0, t\right]\right)\right),$$

$$\left\| \mathsf{F}(\upsilon_1) - \mathsf{F}(\upsilon_2) \right\|_{L^2\left(0, L; C^0([0, t])\right)} \leq L_2 \left\| \upsilon_1 - \upsilon_2 \right\|_{L^2\left(0, L; C^0([0, t])\right)}.$$

Then the solution to the problem (64)-(67) (which satisfies the condition (73)) is unique.

In the third section of the fourth chapter a quasilinear hyperbolic equation is considered in the domain $Q = \Omega \times (0,T)$:

$$u_{tt} + [u + \mathsf{F}(u)]_t - \Delta u = f \tag{74}$$

with boundary conditions

$$u = 0, \quad (x,t) \in \Gamma_0 \times [0,T], \tag{75}$$

$$u_t + pz_{tt} + lz_t + rz = 0, \quad (x,t) \in \Gamma_1 \times [0,T], \tag{76}$$

$$\frac{\partial u}{\partial \nu} = z_t, \quad (x,t) \in \Gamma_1 \times [0,T] \tag{77}$$

and with the initial conditions

$$[u + \mathsf{F}(u)]\big|_{t=0} = u^{(0)} + w^{(0)}, \;\; u_t\big|_{t=0} = u^{(1)}, \; x \in \Omega, \tag{78}$$

$$z\big|_{t=0} = z^{(0)}, \; z_t\big|_{t=0} = \frac{\partial u^{(0)}}{\partial \nu} \equiv z^{(1)}, \; x \in \Gamma_1, \tag{79}$$

where $\Omega \subset R^N \; (N \geq 1)$ is a bounded domain with the sufficiently smooth boundary $\Gamma = \Gamma_0 \bigcup \Gamma_1$, and $\Gamma_0 \bigcap \Gamma_1 = \varnothing$; $p,l,r\colon \Gamma_1 \to R$ are known functions, $\nu(x)$ – the unit outward normal to Γ at the point $x \in \Gamma$ and the nonlinear operator F acts from the space $\mathrm{M}(\Omega; C^0([0,T]))$ to $\mathrm{M}(\Omega; C^0([0,T]))$. It is assumed that F is a memory operator which acts independently at each point $x \in \Omega$ and the conditions (33)-(37) are satisfied. It is also assumed that

$$u^{(0)} \in \widehat{H}, \; w^{(0)} \in L^2(\Omega), \; u^{(1)} \in L^2(\Omega), \; z^{(0)}, z^{(1)} \in L^2(\Gamma_1), \tag{80}$$

$$f \in L^2(Q), \tag{81}$$

where $\widehat{H} = \{ u \in H^1(\Omega) : \gamma_0(u) = 0 \text{ a.e. on } \Gamma_0 \}$.

The concept of a weak solution to the problem (74)-(79) is introduced.

Definition 14. A pair of functions $(u(x,t), z(x,t))$ is called a weak solution to the problem (74)-(79), if

$$u \in L^2\left(0,T;\widehat{H}\right),\ u_t \in L^2(Q),\ z \in L^2(\Gamma_1),\ z_t \in L^2(\Gamma_1),$$
$$\mathsf{F}(u) \in L^2(Q)$$

and the following equalities hold:

$$\tfrac{d}{dt}(u_t,\upsilon)_{L^2(\Omega)} + \tfrac{d}{dt}(u+\mathsf{F}(u),\upsilon)_{L^2(\Omega)} + (\nabla u,\nabla\upsilon)_{L^2(\Omega)} -$$
$$-(z_t,\gamma_0(\upsilon))_{L^2(\Gamma_1)} = (f,\upsilon)_{L^2(\Omega)}$$

for $\forall\upsilon \in \widehat{H}$, in the sense of distributions in $D'(0,T)$, and

$$\tfrac{d}{dt}(\gamma_0(u) + pz_t, e)_{L^2(\Omega)} + (lz_t + rz, e)_{L^2(\Gamma_1)} = 0$$

for $\forall e \in L^2(\Gamma_1)$, in the sense of distributions in $D'(0,T)$; as well as:

$$[u+\mathsf{F}(u)]\big|_{t=0} = u^{(0)} + w^{(0)},\ \ u_t\big|_{t=0} = u^{(1)} \ \text{ a. e. in } \Omega,$$
$$z\big|_{t=0} = z^{(0)},\ z_t\big|_{t=0} = z^{(1)} \ \text{ a. e. on } \Gamma_1.$$

The result on the existence of weak solutions to the problem (74)-(79) is established in the following theorem using the method of discretization with respect to the variable t.

Theorem 28. Let the conditions (33)-(37), (80)-(81) be satisfied and

$$p,l,r \in C(\Gamma_1),$$
$$p(x) \ge 0,\ l(x) > 0,\ r(x) \ge 0 \text{ for } x \in \Gamma_1.$$

Then the problem (74)-(79) has a weak solution (u,z).

CONCLUSIONS

The dissertation work is devoted to the study of mixed problems with transmission acoustic conditions for nonlinear wave equations, as well as initial-boundary value problems for nonlinear hyperbolic and parabolic equations with memory operators and hysteresis nonlinearities.

The following main results are obtained in the dissertation:

- a result on the existence of global weak solutions for a mixed problem with transmission acoustic conditions for nonlinear wave equations with focusing sources is obtained, if $p \le \min\{q_1, q_2\}$ and a result on the blow-up of weak solutions of this problem in a finite time, if $p > \max\{q_1, q_2\}$;
- the existence, uniqueness and exponential decay of global strong solutions of a mixed problem with transmission acoustic conditions for nonlinear wave equations with defocusing sources is proved; a result on the existence of a minimal global attractor for such a problem is obtained;
- the existence and uniqueness of solutions to the initial-boundary value problem for a semilinear hyperbolic equation with a memory operator is proved, a result on the existence of a minimal global attractor is obtained;
- results on the existence of solutions to mixed problems for a system of semilinear hyperbolic equations with memory operators and for the Timoshenko system with memory operator are obtained; a result on the existence of weak solutions of a mixed problem with acoustic boundary conditions for a semilinear hyperbolic equation with a memory operator is obtained.

The main results of the dissertation were published in the following works:

1. Исаева, С.Э. Смешанная задача для одного гиперболического уравнения четвертого порядка с запоминающим оператором // – Баку: Известия БГУ, серия физ.-мат. наук, – 2015. №3, – с. 88-98.
2. Исаева, С.Э. Смешанная задача для одного полулинейного гиперболического уравнения с запоминающим оператором с акустическими граничными условиями // – Баку: Известия БГУ, серия физ.-мат. наук, – 2017. №1, – с. 34-43.
3. Исаева, С.Э. Смешанная задача для одной системы полулинейных гиперболических уравнений с запоминающими операторами // – Елец: CONTINUUM Математика, Информатика, Образование, – 2018. №2 (10), – с. 35-40.
4. Исаева, С.Э. Смешанная задача для одного квазилинейного параболического уравнения с гистерезисом // – Екатеринбург: Международный научно-исследовательский журнал, часть I, – 2018. №9 (75), – р. 24-31.
5. Исаева, С.Э. Смешанная задача для одной системы с акустическими условиями сопряжения // – Сыктывкар: Вестник Сыктывкарского Университета, Серия 1: Математика, Механика, Информатика, Образование, – 2018. №4 (29), – с. 34-42.
6. Исаева, С.Э. Существование глобального аттрактора для одной задачи с запоминающим оператором // Тезисы Международной конференции по математике и механике, посвященной 50-летию института математики и механики НАН Азербайджана, – Баку: – 2009. – с. 150-151.
7. Исаева, С.Э. Смешанная задача для одного квазилинейного гиперболического уравнения с запоминающим оператором // Материалы четвертой международной научной конференции “Функционально - дифференциальные уравнения и их приложения“, – Махачкала: – 21 – 24 сентября, – 2009. – с. 127-128.

8. Исаева, С.Э. Смешанная задача для одного квазилинейного гиперболического уравнения с запоминающим оператором // Тезисы Международной конференции, посвященной 80-летнему юбилею академика Ф. Г. Максудова “Спектральная теория и ее приложения“, – Баку: – 2010. – с. 173-175.
9. Исаева, С.Э. Смешанная задача для квазилинейного гиперболического уравнения // Тезисы II Международной школы-семинар “Нелинейный анализ и экстремальные задачи“, – Иркутск: – 28 июня – 4 июля, – 2010. – с. 32-33.
10. Исаева, С.Э. Существование ограниченного поглощающего множества для одной смешанной задачи с запоминающим оператором // Функциональный анализ и его приложения. Материалы Международной конференции, посвященной 100 летнему юбилею академика З.И.Халилова, – Баку: – 2011. – с. 156-159.
11. Исаева, С.Э. Существование ограниченного поглощающего множества для одной проблемы с гистеризисом // Материалы VI Международной научной конференции “Функционально-дифференциальные уравнения и их приложения“, – Махачкала: – 23 – 26 сентября, – 2013. – с. 135-137.
12. Исаева, С.Э. Смешанная задача для одного полулинейного гиперболического уравнения с запоминающим оператором // Актуальные проблемы математики и механики, Материалы Международной конференции, посвященной 55-летию института математики и механики НАН Азербайджана, – Баку: – 2014. – с. 180-182.
13. Исаева, С.Э. Асимптотическая компактность полугруппы, порожденной смешанной задачей для полулинейного гиперболического уравнения с запоминающим оператором // Материалы международной конференции “Прикладные отрасли математики и ИКТ. Новые учебные технологии”, – Гянджа: – 2014. – с. 94-101.

14. Исаева, С.Э. Смешанная задача для одного гиперболического уравнения четвертого порядка с запоминающим оператором // Материалы республиканской научной конференции “Актуальные проблемы математики и механики”, – Баку: – 20 – 21 мая, – 2015. – с. 48-49.
15. Исаева, С.Э. Смешанная задача для одной системы полулинейных гиперболических уравнений с запоминающим оператором // Материалы республикан-ской научной конференции “Актуальные проблемы математики и механики”, – Баку: – 17 – 18 мая, – 2017. – с. 175-177.
16. Исаева, С.Э. Смешанная задача для одного полулинейного гиперболического уравнения с запоминающим оператором с акустическими граничными условиями // Материалы V Международной научной конференции “Международные Колмогоровские чтения - XIV, посвященные 100-летию проф. З.А.Скопеца“, – Коряжма: – 12 – 15 сентября, – 2017. – с. 40-43.
17. Исаева, С.Э. Смешанная задача для одной системы с акустическими граничными условиями // Материалы XII Международной конференции “Фундаментальные и прикладные проблемы математики и информатики“, – Махачкала: – 19 – 22 сентября, – 2017. – с. 118-119.
18. Исаева, С.Э. Система полулинейных гиперболических уравнений с запоминающим оператором // Материалы республиканской научной конференции “Актуальные проблемы математики и механики”, – Баку: – 2 – 3 ноября, – 2017. – с. 192-194.
19. Исаева, С.Э. Смешанная задача для одной системы полулинейных гиперболических уравнений с запоминающим оператором // Материалы Международ-ной Конференции посвященной 100-летию со дня рождения С. Г. Крейна, – Воронеж: – 13 – 19 ноября, – 2017. – с. 103-105.
20. Исаева, С.Э. Смешанная задача для одного квазилинейного параболического уравнения с гистерезисом // Новейшие достижения и успехи развития технических наук,

Выпуск III, Сборник научных трудов по итогам международной научно-практической конференции, – Краснодар: июнь, – 2018. – с. 41-43.
21. Исаева, С.Э. Существование локальных решений смешанной задачи для волновых уравнений с акустическими условиями сопряжения // Материалы Международной научно-практической конференции “Современная математика и ее приложения“, Чеченский Государственный Педагогический Университет, – Грозный: – 21 – 23 октября, – 2018. – с. 70-72.
22. Исаева, С.Э. Существование локальных решений для нелинейных волновых уравнений с акустическими условиями сопряжения // Материалы республиканской научной конференции “Актуальные проблемы математики и механики”, – Баку: – 17 – 18 мая, – 2019. – с. 97-98.
23. Исаева, С.Э. Существование локальных решений для нелинейных гиперболических уравнений с акустическими условиями сопряжения // “Фундаментальные и прикладные проблемы математики и информатики“, Материалы XIII Международной конференции, приуроченной к 55-летию факультета математики и компьютерных наук, – Махачкала: – 16 – 20 сентября, – 2019. – с. 74-76.
24. Исаева, С.Э. Существование локальных решений для одного нелинейного уравнения с акустическими условиями сопряжения // Материалы республиканской научной конференции “Актуальные проблемы математики и механики”, – Баку: – 20 – 21 мая, – 2020. – с. 135-136.
25. Исаева, С.Э. Существование локальных решений для одного нелинейного уравнения с акустическими условиями сопряжения // IV Всероссийская конференция с международным участием “Математическое моделирование и информационные технологии”, Сыктывкарский государственный университет им. Питирима Сорокина, – Сыктывкар: – 12 – 14 ноября, – 2020. – с. 53.
26. Aliev, A.B., Isayeva, S.E. The existence and behavior of global solutions to a mixed problem with acoustic transmission

conditions for nonlinear hyperbolic equations with nonlinear dissipation // – Moscow: Doklady Mathematics, – 2018. 98 (3), – p. 555-558.
27. Aliev, A.B., Isayeva, S.E. Existence and nonexistence of global solutions for nonlinear transmission acoustic problem // Turkish Journal of Mathematics, – 2018. 42 (6), – p. 3211-3231.
28. Aliev, A.B., Isayeva, S.E. A qlobal attractor for one semilinear hyperbolic equation with memory operator // – Moscow: Computational Mathematics and Mathematical Physics, – 2015. vol. 55, №11, – p. 1823-1835.
29. Aliev, A.B., Isayeva, S.E. Exponential stability of the nonlinear transmission acoustic problem // Mathematical Methods in the Applied Sciences, – 2018. 41 (16), – p. 7055-7073.
30. Aliev, A.B., Isayeva, S.E. Attractors for Semilinear Wave Equations with Acoustic Transmission Conditions // – Moscow: Differential Equations, – 2020. 56, – p. 447-461.
31. Aliev, A.B., Isayeva, S.E. Existence and nonexistence of global solutions for nonlinear transmission acoustic problem // Proceedings of the 6th International Conference on Control and Optimization with Industiral Applications, Volume II, – Baku: – 11 – 13 july, – 2018. – p. 56-58.
32. Aliev, A.B., Isayeva, S.E. Existence of local solutions for nonlinear wave equations with transmission acoustic conditions // "Modern problems of innovative technologies in oiland gas production and applied mathematics", Proceedings of the International Conference dedicated to 90th anniversary of acad. Azad Mirzajanzade, ANAS, Az. State Oil and Industry University, – Baku: – 13 – 14 december, – 2018. – p. 123-125.
33. Aliev, A.B., Isayeva, S.E. Attractor for nonlinear transmission acoustic problem // Spectral Theory and its Applications, International Workshop dedicated to 80th anniversary of acad. Mirabbas Geogja oglu Gasymov, IMM of NAS of Azerbaijan, – Baku: – 7 – 8 june, – 2019. – p. 31-33.
34. Aliev, A.B., Isayeva, S.E. Attractor for nonlinear transmission acoustic problem // 8th International Eurasian

Conference on Mathematical Sciences and Applications Dedicated to the 100th Anniversary of Baku State University (IECMSA-2019), Book of Abstracts, – Baku: – 27 – 30 august, – 2019. – p. 99-100.
35. Isayeva, S.E. The mixed problem for one semilinear hyperbolic equation with memory // – Baku: Transactions of NAS of Azerbaijan, – 2010. vol. XXX, № 1, – p. 105-112.
36. Isayeva, S.E. The mixed problem for quasilinear parabolic equation with memory // – Baku: Proceedings of IMM of NAS of Azerbaijan, – 2010. vol. XXXII, №1, – p. 111-118.
37. Isayeva, S.E. The existence of an absorbing set for one mixed problem with memory operator // – Baku: Transactions issue mathematics and mechanics series of physical-technical and mathematical sciences, – 2011. vol. XXXI, №4, – p. 85-94.
38. Isayeva, S.E. The existence of an absorbing set for one mixed problem with hysteresis // – Baku: Transactions of NAS of Azerbaijan, – 2013. vol. XXXIII, № 1, – p. 27-34.
39. Isayeva, S.E., Şukurova G. D. The initial-boundary value problem for one fourth order hyperbolic equation with memory operator // – Baku: Transactions of NAS of Azerbaijan, Issue Mathematics, Series of Physical-Technical and Mathematical Sciences, – 2016. vol. XXXVI, № 1, – p. 64-73.
40. Isayeva, S.E. The initial-boundary value problem for one system of semilinear hyperbolic equations with memory operator // – Baku: News of BSU, series of phus.-math. sciences, – 2018. №2, – p. 26-36.
41. Isayeva, S.E. Existence of local solutions for nonlinear wave equations with transmission acoustic conditions // – Baku: News of BSU, series of phus.-math. sciences, – 2018. №3, – p. 28-44.
42. Isayeva, S.E. The existence of a global attractor for one fourth order hyperbolic equation with memory operator // Universal Journal of Mathematics and Applications, – 2019. vol. II, Issue I, ISSN 2619-9653, – p. 36-41.
43. Isayeva, S.E., Rzayeva, N.A. Timoshenko systems with memory in external force // Applied Mathematical Sciences, HIKARI Ltd, – 2019. vol. 13, №2, – p. 87-101.

44. Isayeva, S.E. Existence of solutions of nonlinear strongly dissipative wave equations with acoustic transmission conditions // – Moscow: Computational Mathematics and Mathematical Physics, – 2020. 60, – p. 286-301.
45. Isayeva, S.E. Nonlinear wave equations with nonlinear transmission acoustic condition // – Baku: Proceedings of IMM of NAS of Azerbaijan, – 2021. vol. 47, №2, – p. 232-249.
46. Isayeva, S.E. Existence of a bounded adsorbing set for one problem with hysteresis // Тезисы Международной конференции по физике, математике и технических наук, – Нахичевань: – 2008. – с. 99-100.
47. Isayeva, S.E. Existence of a bonded absorbing set for one problem with hysteresis // Abstracts of the Third Congress of the World Mathematical Society of Turkic Countries, – Almaty: – june 30 – july 4, vol. 1, – 2009. – p. 217.
48. Isayeva, S.E., Rzayeva N.A. Timoshenko systems with memory operator in shear force // Материалы международной научной конференции “Теоретические и прикладные проблемы математики”, – Сумгаит: – 25 – 26 мая, – 2017. – с. 119-120.
49. Isayeva, S.E. The existence of a global attractor for one fourth order hyperbolic equation with memory operator // Abstracts of VI congress of the Turkic world mathematical society, – Astana: – 2 – 5 october, – 2017. – p. 71.
50. Isayeva, S.E. The existence and uniqueness of global solutions for the nonlinear transmission acoustic problem // Материалы республиканской научной конференции “Актуальные проблемы математики и механики”, – Баку: – 17 – 18 мая, – 2018. – с. 55-58.
51. Isayeva, S.E. Existence of global solutions of one nonlinear transmission acoustic problem // Operators, Functions, and Systems of Mathematical Physics Conference, International conference dedicated to Hamlet Isaxanli’s 70th anniversary, Khazar University, – Baku: – 21 – 24 may, – 2018. – p. 103-105.
52. Isayeva, S.E. Asymptotic behaviour of solutions of one fourth order hyperbolic equation with memory operator // IX International

Conference of the Georgian Mathematical Union, – Batumi – Tbilisi: – 3 – 8 september, – 2018. – p. 135.
53. Isayeva, S.E. The existence of a global attractor for one fourth order hyperbolic equation with memory operator // Материалы Пятой Международной конференции, посвященной 95-летию со дня рождения Л.Д.Кудрявцева “Функциональные пространства“, Российский Университет Дружбы Народов (РУДН), – Москва: – 26 – 29 ноября, – 2018. – с. 97-98.

The defense will be held on **6 May 2022** **at** **14^{00}** at the meeting of the Dissertation council ED 1.04 of Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at the Institute of Mathematics and Mechanics of National Academy of Sciences of Azerbaijan.

Address: AZ 1141, Baku, B.Vahabzadeh, 9.

Dissertation is accessible at the Institute of Mathematics and Mechanics of National Academy of Sciences of Azerbaijan Library

Electronic versions of dissertation and its abstract are available on the official website of the Institute of Mathematics and Mechanics of National Academy of Sciences of Azerbaijan.

Abstract was sent to the required addresses on **30 March 2022.**

Signed for print: 25.02.2022
Paper format: 60x84 1/16
Volume: 76060
Number of hard copies: 20