REPUBLIC OF AZERBAIJAN

On the rights of manuscript

ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

DEVELOPMENT AND APPLICATION OF SOFTWARE TO DETERMINE THE PARAMETERS INCLUDED IN DISCRETE NONLINEAR DYNAMIC SYSTEMS

Specialty: 1214.01-Dynamic systems and optimal control

Field of science: Mathematics

Applicant: **Atif Akif oglu Namazov**

Baku-2022

The work was performed at the Scientific Research Institute of Applied Mathematics of Baku State University.

Scientific supervisor:
academician of ANAS, doc. of phys.-math. sc., prof.
Fikret Ahmedali oglu Aliev

Official opponents: doctor of sciences in math., assoc. prof.
Shakir Shykhy oglu Yusubov

doctor of sciences in math., assoc. prof.
İlgar Gurbat oglu Mamedov

cand. of phys.-math. sc., assoc. prof.
Rashad Siraj oglu Mamedov

Dissertation council ED 1.04 of Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at the Institute of Mathematics and Mechanics of the National Academy of Sciences of Azerbaijan.

Chairman of the Dissertation council:
corr.-member of the NASA, doctor of phys.-math. sc., prof.
________________ **Misir Jumail oglu Mardanov**

Scientific secretary of the Dissertation council:
candidate of phys.-math. sc.
________________ **Abdurrahim Farman oglu Guliyev**

Chairman of the scientific seminar:
doctor of phys.-math. science, prof.
________________ **Hamlet Farman oglu Guliyev**

GENERAL CHARACTERISTICS OF THE WORK

Rationale and development degree of the theme.

When operating oil wells, we can show that when using gas-lift method in order to determine optimal mode the program trajectories are built. Though the gas lift method was offered at the late XX century by the Nobel brothers, so far the mathematical model of gas-lift model has not been perfectly developed. V.I.Shurov, A.Mirzajanzade, G.O.Eikrem and other authors have a number of studies in this field. In these research works mainly some technical problems are studied, but in Eikrem’s works the fluid motion is identified by the motion of material point and various problems are considered. Since this identification is not very reasonable, there are shortcomings in finding the optimal mode in the problem and in stabilization problems. From this point of view, for the first time F.A.Aliyev, M.Kh.Ilyasov and M.A.Jamalbeyov in their research works have given a perfect mathematical model that characterizes the motion of fluid-gas mixture during gas-lift process. Afterwards, F.A.Aliyev, M.Kh.Ilyasov and N.V.Nuriyev have reduced the solution of some problems corresponding to this model to the solution of a linear quadratic optimal control problem.

Object and subject of the research: The object of the research work is to solve identification from by means of gaslift method in oil production. In discrete dynamic systems, in oil production, for the first time the hydraulic resistance factor is determined by the fractional differential equations, calculations for solving appropriate practical problems are carried out in a computer and the advantage of this method compared to known models was affirmed.

Objectives of the research. The object of the dissertation work consists of development of the methods for determining hydraulic resistance factor in lifting pipe in creation of automatic control systems for oil recovery in oil industry, description of a new model of motion of plunger in rod pumping units by fractional order oscillatory systems, determination of order of fraction, construction of the discrete Rosser model in the spatial case, and determination of the hydraulic resistance factor in the spatial case by its means.

Research methods. In the dissertation work, partial differential equations, discrete transformations, approximate calculation methods, modern programming and MATLAB program packets were used.

Main hypotheses to be defended.

1. Appropriate identification problem for determining constant indefinite parameters in the domain of definition of dynamic systems was solved and an asymptotic method in the case when small parameters are contained in the system was offered.
2. A method to define the hydraulic resistance factor in all the parts of pumping -compressor pipe in oil production was given.
3. When the domain of definition is divided into appropriate parts, a method to determine the hydraulic resistance factor in one of these parts in oil production was given.
4. In the case when the well depth in oil production is rather large, for the first time accepting the inverse of the distance as a small parameter, an asymptotic method to determine the hydraulic resistance factor in all the parts of pumping-compressor pipe, was offered.
5. For the first time, an identification method for finding fractional order derivative of the oscillatory system written by fractional differential equations (the motion of a plunger in rod-pumping units) by means of statistical data and appropriate algorithm was worked out.
6. In the case when the mass of the oscillatory system is rather large (this case appears when the plunger is filled with oil), the inverse of that mass is accepted as a small parameter and simple asymptotic method for defining the fractional order, is offered.
7. For the first time, the discrete Rosser model in suggested in the spatial form of the gas-lift method and an algorithm for calculating the hydraulic resistance factor, is given.

Scientific novelty of research:

1. A new effective method in the lifting pipe is suggested for finding the hydraulic resistance factor.

2. In the case when the lifting pipe is rather large an algorithm for determining the hydraulic resistance factor in different parts, is suggested.
3. In the case when the load is large enough, its inverse is accepted as a small parameter and an asymptotic method is given to determine its fractional order.
4. In oil production process in rod-pumping units, the motion of liquid damper object is written by fractional order differential equations and an inverse problem to define the fractions order is solved.
5. A discrete Rosser model is structured for the gas-lift process and the hydraulic resistance factor is found.

Theoretical and practical importance of the research. The obtained necessary results can be used in studying similar problems, in obtaining more general scientific results. The worked out methods and algorithms are considered to be applied in specific practical problems, including optimal operation problems of oil wells.

Approbation and application. The main results of the dissertation work were reported in the seminars of the Institute of Applied Mathematics of BSU, in the seminars of the departments of "Fluid and gas mechanics" and "Equations of Mathematical Physics" of Institute Mathematics and Mechanics of ANAS and at the following scientific conferences: "Control and optimization with industrial applications" the V International conference COIA-2015 (Baku 2015), the XXI Republican Scientific conference of young doctoral students and researches (Baku 2017), the VI International conference "Control and optimization with industrial applications" COIA-2018 (Baku 2018), Azerbaijan and Turkey University: education, science, technology. Proceed. of the I International scientific practical conference (Baku 2019), the VII International conference "Control and optimization with industrial applications" COIA-2020 (Baku 2020).

Applicant's personal contribution. All the scientific results obtained of the dissertation is the result of the authors activity and direction of the idea of the supervisor and application of the problem statement to specific research object.

Author's publications. On the topic of the dissertation work authors 19 scientific papers were published, including 17 papers, 2 conference materials. The total number of Web of Science index scientific publications is 9.

The name of the organization where the dissertation work was performed. The work was performed at the Scientific Research Institute of Applied Mathematics of Baku State University.

Structure and volume of the dissertation (in signs, indicating the volume of each structural subsection separately) The total volume of the dissertation work consists of 208077 signs (title page - 387, table of contents 2554, introduction – 47136 signs, chapter I – 72000 signs, chapter II – 50000 signs, chapter III– 36000 signs). The dissertation work consists of introduction, 3 chapters, conclusion, a list of references with 81 names. The dissertation work contains 122 page text, 3 figures and 1 table.

THE CONTENT OF THE DISSERTATION

In the **introduction** the rationale of the work is justified and other studies in this field are analyzed.

In chapter **I** an identification problem to determine the parameters of a nonlinear dynamic system in the discrete case is considered.

In **1.1** at first the equation of motion of the object is described by the system of nonlinear difference equations:

$$y(i+1) = f\left(y(i), \alpha\right) \tag{1}$$

$$y_j(0) = y_{0j}\ , \quad j = \overline{1, M} \tag{2}$$

It is clear that the solution of the problem (1)-(2) depends on the vector parameter α. The problem is to define such a vector α that the solution of the Cauchy problem (1)-(2) satisfies the condition

$$y_j(N) = y_{Nj}\, , \quad j = \overline{1, M} \tag{3}$$

Therefore, for solving the problem (1)-(3) we use the quasilinearization method, linearize the equation (1) and get

$$y^k(i+1) = A\left(y^{k-1}(i), \alpha^{k-1}\right) y^k(i) + B\left(y^{k-1}(i), \alpha^{k-1}\right) \alpha^k +$$

$$+C\left(y^{k-1}(i),\alpha^{k-1}\right),\quad i=\overline{0,N-1}. \tag{4}$$

From (4) the solution of $y^k(N)$ can be written as follows:

$$y^k(N)=\Phi^{k-1}(i)y^k(0)+\Phi_1^{k-1}(i)\alpha^k+\Phi_2^{k-1}(i). \tag{5}$$

Then for the k –th iteration we build the following quadratic functional:

$$I^k=\sum_{s=1}^{n}\left(y_s^k(N)-y_{Ns}^k\right)^T A\left(y_s^k(N)-y_{Ns}^k\right), \tag{6}$$

After taking into account the expression (5) in the functional (6) we calculate the gradient of the functional with respect to the parameter α^k equate it to zero and for the parameter α^k we get the following expression:

$$\alpha^k=-\frac{1}{2}\sum_{s=1}^{n}\left(\Phi_{1s}^{k-1}A\Phi_{1s}^{k-1}\right)^{-1}\left(y_s^{k^T}(0)\Phi_s^{k-1^T}A\Phi_{1s}^{k-1}+\right.$$

$$+\Phi_{1s}^{k-1^T}A\Phi_s^{k-1}y_s^k(0)+\Phi_{1s}^{k-1^T}A\Phi_{2s}^{k-1}-\Phi_{1s}^{k-1^T}y_{Ns}^{k\;T}+\Phi_{2s}^{k-1^T}\Phi_{1s}^{k-1^T} \tag{7}$$

$$\left.-,y_{Ns}^{k\;T}\Phi_{1s}^{k-1}\right)$$

here it is assumed that $\left(\Phi_{1s}^{k-1}A\Phi_{1s}^{k-1}\right)^{-1}$ exists.

Theorem 1. Assume that the parameter α^k is determined by the formula (7) the function $y^k(i)$ is determined by the relation (4) and $\lim\limits_{k\to\infty}\alpha^k=\alpha,\ \lim\limits_{k\to\infty}y^k(i)=y(i)$ is valid. Then for the parameter α the function $y(i)$ is the solution of the problem (1)-(2) satisfying the condition (3).

In **1.2** the motion of the object is given by the following system of nonlinear-difference equations

$$y(i+1)=f\left(y(i),\alpha,\varepsilon\right),\, i=\overline{1,N-1} \tag{8}$$

and by the certain initial and final values

$$y_j(0)=y_{0j},\; j=\overline{1,M}, \tag{9}$$

$$y_j(N)=y_{Nj}. \tag{10}$$

of n-dimensional phase vector $y(x)$. Here ε is a small parameter.

The problem consists of defining such a vector α from the equation (8) that the solutions of this equation under the given initial conditions (9) of this equation be maximum close to the statistic data.

For the solution of the problem (8)-(10) we use the quasilinearization method and linearize the equation (8):

$$y^k(i+1)=\left(A_0\left(y^{k-1}(i),\alpha^{k-1}\right)+\varepsilon A_1\left(y^{k-1}(i),\alpha^{k-1}\right)\right)y^k(i)+$$
$$+\left(B_0\left(y^{k-1}(i),\alpha^{k-1}\right)+\varepsilon B_1\left(y^{k-1}(i),\alpha^{k-1}\right)\right)\alpha^k+$$
$$+\left(C_0\left(y^{k-1}(i),\alpha^{k-1}\right)+\varepsilon C_1\left(y^{k-1}(i),\alpha^{k-1}\right)\right). \qquad (11)$$

According to the mathematical induction method, for the expression $y^k(N)$ we obtain the following expression:

$$y^k(N)=\left(\Phi_0^{0\,k-1}+\varepsilon\Phi_0^{1\,k-1}\right)y^k(0)+\left(\Phi_1^{0\,k-1}+\varepsilon\Phi_1^{1\,k-1}\right)\alpha^k+$$
$$+\left(\Phi_2^{0\,k-1}+\varepsilon\Phi_2^{1\,k-1}\right). \qquad (12)$$

For the solution of the problem (8)-(10) we build the following quadratic functional:

$$I^{\kappa}=\sum_{s=1}^{n}\left(y_s^k(N)-y_{sN}^k\right)^T A\left(y_s^k(N)-y_{sN}^k\right). \qquad (13)$$

Considering the expression (12) in the functional (13) we calculate the gradient of the functional with respect to the parameter α^k. Then, introducing the expansion $\alpha^k\approx\alpha_0^k+\varepsilon\alpha_1^k+\ldots$of the parameter α^k, taking this expansion into account in the expression of the gradient and equating to zero, to determine the parameters α_0^k and α_1^k we obtain the following algebraic equations:

$$\sum_{s=1}^{n}\left[\Phi_{1s}^{0\;k-1^T}A\Phi_{0s}^{0\;k-1}y_s^k(0)-\Phi_{1s}^{0\;k-1^T}Ay_{Ns}^k+\Phi_{1s}^{0\;k-1^T}A\Phi_{2s}^{0\;k-1}+\right.$$
$$\left.+\Phi_{1s}^{0\;k-1^T}A\Phi_{1s}^{0\;k-1}\alpha_0^k\right]=0, \qquad (14)$$

$$\sum_{s=1}^{n}\left(\Phi_{1s}^{1\;k-1^T}A\Phi_{0s}^{0\;k-1}y_s^k(0)+\Phi_{1s}^{0\;k-1^T}A\Phi_{0s}^{1\;k-1}y_s^k(0)+\right.$$

$$+\Phi_{1s}^{1\ k-1^T} A\Phi_{2s}^{0\ k-1} + \Phi_{2s}^{1\ k-1^T} A\Phi_{1s}^{0\ k-1} + 2\Phi_{1s}^{0\ k-1^T} A\Phi_{1s}^{1\ k-1} \alpha_0^k +$$
$$+\left(\Phi_{1s}^{0\ k-1^T} A\Phi_{1s}^{0\ k-1} \varepsilon + 2\Phi_{1s}^{0\ k-1^T} A\Phi_{1s}^{0\ k-1}\right)\alpha_1^k = 0. \qquad (15)$$

So, taking into account the expressions (16) and (17) in the expression $\alpha^k \approx \alpha_0^k + \varepsilon\alpha_1^k$, we determine the parameter α^k.

Theorem 2. Assume that the parameters α_0^k and α_1^k were determined by the formulas (14), (15). For $\alpha^k = \alpha_0^k + \varepsilon\alpha_1^k$ and small values of ε the function $y^k(i)$ is found from the relation (11). İf $\lim_{k\to\infty} \alpha^k = \alpha$, $\lim_{k\to\infty} y^k(i) = y(i)$, then for the parameter α the function $y(i)$ is the solution of the problem (8)-(9) satisfying the condition (10).

In **1.3,** the motion of gas and gas-liquid mixture in the gas and gas-lifting pipe in annular space during the gas-lift process, is described by the system of two nonlinear first order ordinary differential equations:

$$\begin{cases} \dot{Q}_i = \dfrac{2a_i(\lambda_c)\rho_i F_i Q_i^2}{c_i^2 \rho_i^2 F_i^2 - Q_i^2}, \quad Q(0) = u, \\ \dot{P}_i = \dfrac{2a_i c_i^2 \rho_i^2 F_i^2 Q_i^2}{c_i^2 \rho_i^2 F_i^2 - Q_i^2}, \quad P(0) = P_0, i = 1, 2. \end{cases} \qquad (16)$$

Here, since the first equation of the system (16) is independent of the second equation, we can solve the first equation by the method of separation of variables.

Now the nonlinear difference equations corresponding to the first equation of the system (16) as follows:

$$Q(i+1) = Q(i) + h\frac{2a_1\rho_1 F_1 Q^2(i)}{c_1^2\rho_1^2 F_1^2 - Q^2(i)}, 0 \le i \le N-1, \qquad (17)$$

$$Q(i+1)=Q(i)+h\,\frac{2a_2\rho_2F_2Q^2(i)}{c_2^2\rho_2^2F_2^2-Q^2(i)} \qquad (18)$$

$$Q(0)=u. \qquad (19)$$

At the point N the equations (17) and (18) are interconnected by the following conditions:

$$Q(N+1)=\gamma Q(N)+(-\delta_3(Q(N)-\delta_2)^2+\delta_1)\overline{Q}, \qquad (20)$$

Assume that any nominal trajectory $Q^0(i)$, the parameter a^0 were selected. Suppose that the $(k-1)$ iteration has already been condusted. Linearizing the equations (17), (18) around these data to within the order $(Q-Q^0,\, a-a^0)$ we obtain :

$$Q^k(i+1)=A_1\left(Q^{k-1}(i),a^{k-1}\right)Q^k(i)+B_1\left(Q^{k-1}(i),a^{k-1}\right)a^k(i)+$$
$$+C_1\left(Q^{k-1}(i),a^{k-1}\right),\quad 0\le i\le N-1, \qquad (21)$$

$$Q^k(i+1)=A_2\left(Q^{k-1}(i),a^{k-1}\right)Q^k(i)+B_2\left(Q^{k-1}(i),a^{k-1}\right)a^k(i)+$$
$$+C_2\left(Q^{k-1}(i),a^{k-1}\right),\quad N\le i\le 2N-1, \qquad (22)$$

We can write the equations at the ends of the intervals $0\le i\le N-1$, $N\le i\le 2N-1$ as follows:

$$Q^k(N)=\Phi_1^{k-1}Q^k(0)+\Phi_{11}^{k-1}a^k+\Phi_{21}^{k-1},$$
$$Q^k(2N)=\Phi_2^{k-1}Q^k(N+1)+\Phi_{12}^{k-1}a^k+\Phi_{22}^{k-1}, \qquad (23)$$

Assume that certain statistical data, i.e. $\tilde{Q}_s(0)$ (gas volume at the well-mouth and $\tilde{Q}_s(2N)$ (volume (debit) of gas-liquid mixture at the well bottom) are known, s is the number of statistical observations. Let us construct the following quadratic functional for the k-th iteration:

$$I^k=\sum_{s=1}^{n}\left[Q_s^k(2N)-\tilde{Q}_s^k(2N)\right]^2. \qquad (24)$$

Then, considering the expression (23) in the functional (24) calculating the gradient of the obtained functional and equating to zero, we obtain:

$$a^k = -\sum_{s=1}^{n}\left[\left(\Phi_{12s}^{k-1}\right)^2\right]^{-1} \times$$

$$\times\sum_{s=1}^{n}\left[\Phi_{2s}^{k-1}\Phi_{12s}^{k-1}Q_s^k(N+1)+\Phi_{22s}^{k-1}\Phi_{12s}^{k-1}-\Phi_{12s}^{k-1}\tilde{Q}_s^k(2N)\right]$$

Here it is assumed that the expression $\left[\left(\Phi_{12s}^{k-1}\right)^2\right]^{-1}$ exists.

Theorem 3. Assume that a^k is a parameter affording a minimum to the functional (24). $Q^k(i)$ is the solution of the system of equations (21)-(22). If $\lim\limits_{k\to\infty} a^k = a,\ \lim\limits_{k\to\infty} Q^k(i) = Q(i)$ then $Q(i)$ is the solution of the system (20)-(22) corresponding to the parameter a.

In **1.4** the equation of motion of the object is described by the system of nonlinear discrete equations with a small parameter in the right hand side:

$$Q(i+1) = Q(i) + h\frac{2a_1\rho_1F_1Q^2(i)}{c_1^2\rho_1^2F_1^2 - Q^2(i)},\ 0 \le i \le N-1, \tag{25}$$

$$Q(i+1) = Q(i) + h\frac{2a_2\rho_2F_2Q^2(i)}{c_2^2\rho_2^2F_2^2 - Q^2(i)},\ N \le i \le 2N-1, \tag{26}$$

$$Q(0) = u, \tag{27}$$

here *h*- is a rather small number.

At the point N the equations (25) and (26) are interconnected with the following condition (20).

The problem consists of defining such a vector a_2 from the equation (26) that the solutions of this equation within the given condition (20) are maximum close to the statistical data $\tilde{Q}_s(2N)$.

Thus, to solve the problem (25)-(27) it is required to build a quadratic functional and to minimize it:

$$I = \sum_{s=1}^{n}\left[Q_s(2N) - \tilde{Q}_s(2N)\right]^2 \to \min. \tag{28}$$

Any nominal trajectory $Q^0(i)$, the parameter a^0 are selected and it is assumed that the $(k-1)$-th iteration has been conducted. Let us linearize the equations (25) and (26) and the condition (20) around these data to within the order $(Q-Q^0,\ a-a^0)$:

$$Q^k(i+1)=\left(A_{10}\left(Q^{k-1}(i),a_1^{\ k-1}\right)+\mu\left(A_{11}\left(Q^{k-1}(i),a_1^{\ k-1}\right)\right)\right)Q^k(i)+$$
$$+\left(B_{10}\left(Q^{k-1}(i),a_1^{\ k-1}\right)+\mu\left(B_{11}\left(Q^{k-1}(i),a_1^{\ k-1}\right)\right)\right)a_1^{\ k}+$$
$$+\left(C_{10}\left(Q^{k-1}(i),a_1^{\ k-1}\right)+\mu\left(C_{11}\left(Q^{k-1}(i),a_1^{\ k-1}\right)\right)\right),\quad 0\le i\le N-1 \qquad (29)$$

$$Q(N+1)=(\gamma-2\delta_3(Q^\kappa(N-1)-\delta_2)\overline{Q})Q(N-1)+$$
$$+(-\delta_3(Q^\kappa(N-1)-\delta_2)^2+\delta_1)\overline{Q}+$$
$$+2\delta_3(Q^\kappa(N-1)-\delta_2)\overline{Q}Q^\kappa(N-1),$$

$$Q^k(i+1)=\left(A_{20}\left(Q^{k-1}(i),a_2^{\ k-1}\right)+\mu\left(A_{21}\left(Q^{k-1}(i),a_2^{\ k-1}\right)\right)\right)Q^k(i)+$$
$$+\left(B_{20}\left(Q^{k-1}(i),a_2^{\ k-1}\right)+\mu\left(B_{21}\left(Q^{k-1}(i),a_2^{\ k-1}\right)\right)\right)a_2^{\ k}+$$
$$+\left(C_{20}\left(Q^{k-1}(i),a_2^{\ k-1}\right)+\mu\left(C_{21}\left(Q^{k-1}(i),a_2^{\ k-1}\right)\right)\right),\quad N\le i\le 2N-1, \qquad (30)$$

Using the mathematical induction method, for the solution, $Q^k(2N)$ we obtain the following expression:

$$Q^k(2N)=\left(\Phi_3^{0\,k-1}+\mu\Phi_4^{0\,k-1}\right)Q^k(N+1)+$$
$$\left(\Phi_4^{0\,k-1}+\mu\Phi_4^{1\,k-1}\right)a_2^{\ k}+\left(\Phi_5^{0\,k-1}+\mu\Phi_5^{1\,k-1}\right). \qquad (31)$$

Considering the expression (31) obtained for $Q^k(2N)$ in the functional (28) we find the gradient of the obtained functional with respect to the parameter $a_2^{\ k}$. Then, taking into account in the expansion $a_2^{\ k}=a_{20}^k+\mu a_{21}^k$ of the parameter $a_2^{\ k}$ in this gradient, we equate it to zero. Then, solving the obtained equation with respect to the parameters a_{20}^k and a_{21}^k we obtain:

$$a_{20}^k=-\sum_{s=1}^{n}\left(\left(\Phi_{4s}^{0\ \ k-1}\right)^2\right)^{-1}\left(\Phi_{3s}^{0\ \ k-1}\Phi_{4s}^{0\ \ k-1}Q_s^k(N+1)+\Phi_{5s}^{0\ \ k-1}\Phi_{4s}^{0\ \ k-1}-\right.$$

$$-\tilde{Q}_s^k(2N)\Phi_{4s}^{0\ k-1}\Big) = 0, \tag{32}$$

$$a_{21}^k = -\sum_{s=1}^n \left(\left(\left(\Phi_{4s}^{0\ k-1} \right)^2 + 2\mu\Phi_{4s}^{1\ k-1}\Phi_{4s}^{0\ k-1} \right)^{-1} \times \right.$$

$$\times \Big(\Phi_{3s}^{0\ k-1}\Phi_{4s}^{1\ k-1}Q_s^k(N+1) + \Phi_{5s}^{0\ k-1}\Phi_{4s}^{1\ k-1} -$$

$$-\tilde{Q}_s^k(2N)\Phi_{4s}^{1\ k-1} + \Phi_{3s}^{1\ k-1}\Phi_{4s}^{0\ k-1}Q_s^k(N+1) + \Phi_{5s}^{1\ k-1}(i)\Phi_{4s}^{0\ k-1} -$$

$$-2\Phi_{4s}^{1\ k-1}\Phi_{4s}^{0\ k-1}\left(\left(\Phi_{4s}^{0\ k-1} \right)^2 \right)^{-1} \times$$

$$\left. \times \Big(\Phi_{3s}^{0\ k-1}\Phi_{4s}^{0\ k-1}Q_s^k(N+1) + \Phi_{5s}^{0\ k-1}\Phi_{4s}^{0\ k-1} - \tilde{Q}_s^k(2N)\Phi_{4s}^{0\ k-1} \Big)\Big)\right). \tag{33}$$

Thus, considering the expressions (32) and (33) obtained for the parameters a_{20}^k and a_{21}^k in the expansion $a_2{}^k = a_{20}^k + \mu a_{21}^k$, we determine the parameter $a_2{}^k$.

Theorem 4. Suppose that the parameters a_{20}^k and a_{21}^k are determined by the relations (32) and (33). For small values of μ the equations $a_2^k = a_{20}^k + \mu a_{21}^k$ and $Q^k(i)$ are found from the equations (29)-(30) if $\lim_{k\to\infty} a_2^k = a_2$, $\lim_{k\to\infty} Q^k(i) = Q(i)$, then for the parameter a_2 the function $Q(i)$ is the solution of the system (20), (25)-(27).

In **2.1** we consider an averaged mathematical model of the gas-lift process with respect to time. Here the motion of equation of the object is given by the following nonlinear difference equations with a small parameter in the right hand side

$$Q(k+1) = Q(k) + h\frac{2a_1\rho_1F_1Q^2(k)}{c_1^2\rho_1^2F_1^2\mu - Q^2(k)}, 0 \le k \le N-1, \tag{34}$$

$$Q(k+1) = Q(k) + h\frac{2a_2\rho_2F_2Q^2(k)}{c_2^2\rho_2^2F_2^2\mu - Q^2(k)}, N \le k \le 2N-1, \tag{35}$$

and by the initial condition

$$Q(0) = u \tag{36}$$

here h is a rather small number.

Assume that in the gaslift well the length l of the pipe in gas lift well was divided by m number parts $[l_i, l_{i+1}]$, i=1, 2, ..., m-1 and in this case, the motion of the gasfluid unit is described in each interval $[l_i, l_{i+1}]$ by the following linear difference equations:

$$Q(k+1) = Q(k) + h\frac{2a_{2,i}\rho_2 F_2 Q^2(k)}{c_2^2\rho_2^2 F_2^2\mu - Q^2(k)},$$

$$Q(N+1) = Q_N,\ N \le k \le 2N-1, \tag{37}$$

here $a_{2,i} = \dfrac{g}{\omega_c} + \dfrac{\lambda_{2,i}\omega_c}{2D}$, i=1, 2, ..., m-1, $\lambda_{2,i}$ is the hydraulic resistance factor in each interval $[l_i, l_{i+1}]$.

Let the n number initial data be known:

$$Q^j(N+1) = \tilde{Q}_N^j,\ \ j = \overline{1,n}. \tag{38}$$

İt is required on each interval $[l_i, l_{i+1}]$ to find such a value of the hydraulic resistance factor λ_i (i=1, 2, ..., m) that at the end of the lifting pipe that the difference between the solution $Q(2N) = Q_{2N}$ of the equation (37) and the final value

$$Q^j(2N) = \tilde{Q}_{2N}^j,\ \ j = \overline{1,n} \tag{39}$$

given to the initial data be minimum.

Using the least square method, we build the quadratic functional

$$I(\lambda_1,\ldots,\lambda_m) = \sum_{j=1}^{n}\left[Q^j(2N) - \tilde{Q}_{2N}^j\right]^2 + \sum_{i=1}^{m}\alpha a_{2,i}^2 \to \min \tag{40}$$

here $\tilde{Q}_{2N}^j$ is the statistical value given to the initial data, $Q^j(2N)$ is the solution of the equation (37).

The solution of the equation (37) of order $O(\mu)$ is as follows :

$$Q(k+1) = Q(k) - 2ha_{2,i}\rho_2 F_2 - h\frac{2a_{2,i}c_2^2\rho_2^3 F_2^3}{Q^2(k)}\mu. \tag{41}$$

Finding the value of the equation (41) at the points l_1 ,l_2 ,l_3 and using the mathematical induction method, in a first approximation for $Q(l_m) = Q(2N)$, taking the asymptotic expansion in the functional (40) we find the gradient with respect to the parameter $a_{2,i}$ and equating this gradient to zero, we obtain the following system of nonlinear algebraic equations:

$$\frac{\partial I}{\partial a_{2i}} = \sum_{j=1}^{n}\Bigg\{-2\left[Q^j(0) - \tilde{Q}^j_{2N} - \sum_{i=1}^{m} 2a_{2,i}\rho_2 F_2 l_i\right] 2\rho_2 F_2 l_i + 2\alpha a_{2,i} +$$

$$+\left(4\rho_2 F_2 l_i\left(\frac{2a_{2,1}c_2^2\rho_2^3F_2^3 l_1}{\left(Q^j(0)\right)^2} + \sum_{i=2}^{\mathrm{m}}\frac{2a_{2,i}c_2^2\rho_2^3F_2^3 l_i}{\left[Q^j(0) - \sum_{s=1}^{i-1} 2l_s a_{2,s}\rho_2 F_2\right]^2}\right) -\right.$$

$$\left.\left. -2\left[Q^j(0) - \tilde{Q}^j_{2N} - \sum_{i=1}^{m} 2a_{2,i}\rho_2 F_2 l_i\right]\sum_{i=2}^{\mathrm{m}}\frac{2c_2^2\rho_2^3F_2^3 l_i}{\left[Q^j(0) - \sum_{s=1}^{i-1} 2l_s a_{2,s}\rho_2 F_2\right]^2}\right)\mu\right\} =$$

$$= A_i(a_{2,1}, a_{2,2}, \ldots, a_{2,m}) + \mu B_i(a_{2,1}, a_{2,2}, \ldots, a_{2,m}) = 0, \quad (i=1, 2, \ldots, m) \quad (42)$$

We include the expansion of the parameter $a_{2,i}$ (i=1, 2,..., m) in the following form:

$$a_{2,i} = \overset{0}{a}_{2,i} + \mu \overset{1}{a}_{2,i}. \quad (43)$$

Consider this expansion in the expressions $A_i(a_{2,1}, a_{2,2}, \ldots, a_{2,m})$, $B_i(a_{2,1}, a_{2,2}, \ldots, a_{2,m})$ and write in equation the (42) we get a system of linear algebraic equationswith respect to the parameter, $a_{2,i}$, $i = \overline{1,m}$:

$$\overset{0}{A}_i(\overset{0}{a}_{2,1}, \overset{0}{a}_{2,2}, \ldots, \overset{0}{a}_{2,m}) + \mu(\overset{1}{A}_i(\overset{1}{a}_{2,1}, \overset{1}{a}_{2,2}, \ldots, \overset{1}{a}_{2,m}) + \overset{0}{B}_i(\overset{0}{a}_{2,1}, \overset{0}{a}_{2,2}, \ldots, \overset{0}{a}_{2,m})) = 0. \quad (44)$$

It can be easily seen that as the equation (42) is satisfied for any μ from (44), we obtain:

$$\begin{cases} A_i^0(a_{2,1}^0, a_{2,2}^0, \dots, a_{2,m}^0) = 0 \\ A_i^1(a_{2,1}^1, a_{2,2}^1, \dots, a_{2,m}^1) + B_i^0(a_{2,1}^0, a_{2,2}^0, \dots, a_{2,m}^0) = 0, \; i = \overline{1,m} \end{cases}. \tag{45}$$

From the first equation of the system (45) for finding the parameter $a_{2,i}^0$, $i = \overline{1,m}$ we obtain the following system of linear algebraic equations:

$$\begin{bmatrix} \rho_2 F_2 l_1 + \dfrac{\alpha}{n\rho_2 F_2 l_1} & \rho_2 F_2 l_2 & \dots \rho_2 F_2 l_m \\ \rho_2 F_2 l_1 & \rho_2 F_2 l_2 + \dfrac{\alpha}{n\rho_2 F_2 l_2} & \dots \rho_2 F_2 l_m \\ & \dots & \\ \rho_2 F_2 l_1 & \rho_2 F_2 l_2 & \dots \; \rho_2 F_2 l_m + \dfrac{\alpha}{n\rho_2 F_2 l_m} \end{bmatrix} \begin{bmatrix} a_{2,1}^0 \\ a_{2,2}^0 \\ \dots. \\ a_{2,n}^0 \end{bmatrix} =$$

$$= \begin{bmatrix} \dfrac{1}{n} \sum_{j=1}^{n} \left(Q^j(0) - \tilde{Q}_{2N}^j \right) \\ \dfrac{1}{n} \sum_{j=1}^{n} \left(Q^j(0) - \tilde{Q}_{2N}^j \right) \\ \dots \\ \dfrac{1}{n} \sum_{j=1}^{n} \left(Q^j(0) - \tilde{Q}_{2N}^j \right) \end{bmatrix}. \tag{46}$$

From the second equation of the system (45) for finding the poarameter $a_{2,i}^1$, $i = \overline{1,m}$ we obtain the following system of linear algebraic equations:

$$\begin{bmatrix} 8\rho_2^2 F_2^2 l_1^{\,2} - 2\alpha & 8\rho_2^2 F_2^2 l_2^{\,2} & \dots 8\rho_2^2 F_2^2 l_m^{\,2} \\ 8\rho_2^2 F_2^2 l_1^{\,2} & 8\rho_2^2 F_2^2 l_2^{\,2} - 2\alpha & \dots 8\rho_2^2 F_2^2 l_m^{\,2} \\ 8\rho_2^2 F_2^2 l_1^{\,2} & 8\rho_2^2 F_2^2 l_2^{\,2} & \dots \; 8\rho_2^2 F_2^2 l_m^{\,2} - 2\alpha \end{bmatrix} \times$$

$$\times\begin{bmatrix} a_{2,1}^1 \\ a_{2,2}^1 \\ \cdots\cdot \\ a_{2,n}^1 \end{bmatrix} = \begin{bmatrix} B_1^0(a_{2,1}^0,\ldots,a_{2,m}^0) \\ B_2^0(a_{2,1}^0,\ldots,a_{2,m}^0) \\ \ldots \\ B_m^0(a_{2,1}^0,\ldots,a_{2,m}^0) \end{bmatrix}. \qquad (47)$$

Then using, (43) with respect to the parameter μ in a first approximation we find the parameter $a_{2,i}^1$, $i=\overline{1,m}$. At last using $a_{2,i}$, $i=\overline{1,m}$, we find the hydraulic resistance factor $\lambda_{2,i}$, $i=\overline{1,m}$

$$\lambda_{2,i} = \frac{2a_{2,i}D}{\omega} - \frac{2gD}{\omega^2}.$$

Theorem 5. Assume that the parameters $a_{2,i}^0$, $a_{2,i}^1$, $i=\overline{1,m}$ are the solutions of the system of algebraic equations (46), (47) respectively. Then the hydraulic resistance factors in various parts of the lift found in the form (43) afford a minimum to the functional (40) of the problem (34)-(36).

In subchapter 2 the length of the lifting pipe is divided into different parts an identification problem is considered to determine the hydraulic resistance factor in both parts and using the last square method the following functional is structured:

$$I(\lambda_1,\lambda_2) = \sum_{j=1}^{n}\left[Q^j(2) - \tilde{Q}_2^j\right]^2 + \beta a_{2,1}^2 + \beta a_{2,2}^2 \to \min.$$

Then, if we continue similarly as in subchapter I of chapter II for finding the parameters $a_{2,1} = a_{2,1}^0 + \mu a_{2,1}^1$, $a_{2,2} = a_{2,2}^0 + \mu a_{2,2}^1$ we obtain the following system of linear algebraic equations:

$$\begin{bmatrix} (4l_1\rho_2F_2 + \dfrac{\beta}{nl_1\rho_2F_2}) & 4l_2\rho_2F_2 \\ 4l_1\rho_2F_2 & (4l_2\rho_2F_2 + \dfrac{\beta}{nl_2\rho_2F_2}) \end{bmatrix} \cdot \begin{bmatrix} a_{2,1}^0 \\ a_{2,2}^0 \end{bmatrix} = \begin{bmatrix} \dfrac{2}{n}\sum\limits_{j=1}^{n}\left[Q^j(l_0) - \tilde{Q}_2^j\right] \\ \dfrac{2}{n}\sum\limits_{j=1}^{n}\left[Q^j(l_0) - \tilde{Q}_2^j\right] \end{bmatrix}$$

$$\begin{bmatrix} -(8nl_1^2\rho_2^2F_2^2 + 2\beta) & -8nl_1l_2\rho_2^2F_2^2 \\ -8nl_1l_2\rho_2^2F_2^2 & -(8nl_2^2\rho_2^2F_2^2 + 2\beta) \end{bmatrix} \cdot \begin{bmatrix} a_{2,1}^1 \\ a_{2,2}^1 \end{bmatrix} = \begin{bmatrix} B_1^0(a_{21}^0, a_{22}^0) \\ B_2^0(a_{21}^0, a_{22}^0) \end{bmatrix}$$

Finally using $a_{2,1}$, $a_{2,2}$, we use the following formula to determine the hydraulic resistance factor $\lambda_{2,i}$, $i=\overline{1,2}$.

In **3.1** we consider a problem on definition of fractional order derivative when the oscillatory process is in the inside the Newton fluid. At first the fluid damped oscillatory process is described by a constant coefficient fractional order linear ordinary differential equation:

$$y''(x)+aD^{\alpha}y(x)+by(x)=f(x)\ (x\geq x_0>0) \tag{48}$$

$$y(x_0)=y_0,\ y'(x_0)=y_1,\ \alpha\in(1,2), \tag{49}$$

here the expression $aD^{\alpha}y(x)$ is a mathematical expression reflecting the action of fractional derivative liquid damper, α is a fraction.

Then by means of the definition of Riemann-Liouville fractional order derivative, the given fractional order linear ordinary differential equation is reduced to the second order Volterra integral equation:

$$y(x)+\int_{x_0}^{x}K_{\alpha}(x-t)y(t)dt=F(x), \tag{50}$$

To solve the given problem, we consider a problem of discretization of the equation (43). Assume that the equation (41) was determined on the interval $[t_0,l]$. We divide this interval into n parts by means of the constant step h, i.e. replace $t_k=t_0+kh,\ t_n=l$ and the integral equation (43) by a finite sum y_1 the finite solution y_n is determined by the following expression.

$$y_n=-K_{\alpha}(t_n-t_1)y_1h-K_{\alpha}(t_n-t_2)y_2h-...-K_{\alpha}(t_n-t_{n-1})y_{n-1}h+F_n \tag{51}$$

To find the parameter $\alpha\in(1,2)$ we accept $y^i_{n,st}\ (i=\overline{1,k-1})$ -as a statistical data and build the following quadratic functional:

$$J=\sum_{i=1}^{k-1}(y_n-y_n^i)^2\to\min. \tag{52}$$

Considering the solution y_n in the functional (45), derivative and calculating the $\frac{\partial J}{\partial \alpha}$ of the obtained functional and equate it to zero, we obtain:

$$\frac{\partial J}{\partial \alpha} = \sum_{i=1}^{k-1} (-\sum_{m=0}^{n-1} K_\alpha (t_n - t_m) y_m^i h + F_n - y_n^i) \times$$

$$\times \sum_{m=0}^{n-1} a \frac{-(t_n - t_m)^{1-\alpha} \ln(t_n - t_m)\Gamma(2-\alpha) + (t_n - t_m)^{1-\alpha} \int_0^\infty e^{-t} t^{1-\alpha} \ln t dt}{\Gamma^2 (2-\alpha)} \times$$

$$\times y_m^i h = 0, \tag{53}$$

Solving the algebraic equation (46) in this or other way, we obtain the parameter α and can give calculation algorithm for solving the given problem. On the example from the practice it is shown that depending on the informativity of y_n^i the order α is included either into the interval $(0,1)$ or $(1,2)$, respectively. The algorithm for finding the roots is implemented by dividing a segment into two parts.

In **3.2** it is assumed that the value of the mass m is rather large. In this case, in a first approximation a small parameter for the inverse value of mass is accepted $\left(\varepsilon = \frac{1}{m}\right)$ and in a first approximation for $f \equiv 0$ we decribe the solution of the equation (41) in the asymptotic form as follows:

$$\ddot{y}(t) + \varepsilon a \dot{y}(t) + by(t) = 0. \tag{54}$$

Then to determine the $y(t)$ we obtain the following integral equation:

$$y(t) + \int_{t_0}^{t} K_\alpha (t - z, \varepsilon) y(z) dz = y_1(t - t_0), \tag{55}$$

Thus, the equation (48) within the initial conditions $y(t_0)=0$, $y'(t_0)=y_1$ in a first approximation is of the following form:

$$y(t,\varepsilon)=y_1(t-t_0)+\varepsilon\, y_1\left[-\frac{a(t-t_0)^{3-\alpha}}{(3-\alpha)!}+\frac{5b(t-t_0)^3}{6}\right]. \tag{56}$$

Assume that different initial conditions $y_i'(t_0)=y_{1i}$, $(i=1,2,\ldots,k)$ have finite values y_i and at the end of the interval $y(T)=y_T$. Denote the solution of the equation within the initial conditions $y(t_0)=0$, $y'(t_0)=y_1$ by $y(t,\alpha,y_{1i})$. y and y_{1i} are accepted as statistical data. Then we can write the expression (56) as follows:

$$y(T,\alpha,y,\varepsilon)=y_{1i}(t-t_0)+\varepsilon\, y_{1i}\left[\frac{a(T-t_0)^{3-\alpha}}{(3-\alpha)!}-\frac{5b(T-t_0)^3}{6}\right]. \tag{57}$$

Thus, we construct the following quadratic functional:

$$I=\sum_{i=1}^{k}\left(y(t,\alpha,y_{1i})-y_{Ti}\right)^2. \tag{58}$$

It is required to find such $\alpha=\alpha^*$ that the functional (52) takes minimum value.

For that we calculate the derivative of the functional (52) with respect to α equate to zero and for finding the parameter α we obtain:

$$\frac{a(T-t_0)^{3-\alpha}}{\Gamma(2-\alpha)}-5b\frac{(T-t_0)^3}{6}+\frac{1}{\varepsilon}\left[-(T-t_0)+\frac{y_{T1}y_{11}+\ldots+y_{Tk}y_{1k}}{y_{11}^2+\ldots+y_{1k}^2}\right]=0. \tag{59}$$

The **3.3** we consider the difference equation included in the discrete system and determined from the appropriate system of linear algebraic equations and characterizing the Rosser model that shows perturbation:

$$\begin{bmatrix}P_i^{j+1}\\ Q_{i+1}^{j}\end{bmatrix}=\begin{bmatrix}A_{11}(\lambda) & A_{12}(\lambda)\\ A_{21}(\lambda) & A_{22}(\lambda)\end{bmatrix}\begin{bmatrix}P_i^{j}\\ Q_i^{j}\end{bmatrix}+\begin{bmatrix}B_{11} & B_{12}\\ B_{21} & B_{22}\end{bmatrix}\begin{bmatrix}W_i^{j}\\ X_i^{j}\end{bmatrix}. \tag{60}$$

Then using equation (60) we get the following expressions:

$$\begin{cases} P_i^{j+1} = A_{11}(\lambda)P_i^j + A_{12}(\lambda)Q_i^j + B_{11}W_i^j + B_{12}\mathrm{X}_i^j \\ Q_{i+1}^j = A_{21}(\lambda)P_i^j + A_{22}(\lambda)Q_i^j + B_{21}W_i^j + B_{22}\mathrm{X}_i^j . \end{cases} \tag{61}$$

We consider the following quadratic functional:

$$J(\lambda) = \sum_{i=1}^{m}\left(P(i,n) - \overline{P}(i,n)\right)^2, \quad I(\lambda) = \sum_{j=1}^{m}\left(Q(m,j) - \overline{Q}(m,j)\right)^2.$$

Then determining the gradient $\frac{dI(\lambda)}{d\lambda}$ equating in to zero, we get the required hydraulic resistance factor:

$$\frac{dI(\lambda)}{d\lambda} = 2\sum_{j=1}^{n}\Big[(1 + A_0(\lambda))^{j-1}\Big(1 + A_0(\lambda)P_m^0 - A_0(\lambda)Q_m^0\Big) +$$

$$+ \sum_{k=0}^{j-1} f_{mk} - A_0(\lambda)g_{m-1k} - A_0(\lambda)(1 - A_0(\lambda))Q_{m-1}^k(\lambda) -$$

$$- A_0^2(\lambda)P_{m-1}^k(\lambda)(1 + A_0(\lambda))^{j-1-k} +$$

$$+ (1 - A_0(\lambda))Q_{m-1}^j(\lambda) + g_{m-1j} - \overline{Q}(m,j)\Big] \cdot \frac{h\omega}{2FD}P_{m-1}^j(\lambda) +$$

$$+ A_0(\lambda)\frac{dP_{m-1}^j(\lambda)}{d\lambda} - \frac{h\omega}{2FD}Q_{m-1}^j(\lambda) + (1 - A_0(\lambda))\frac{dQ_{m-1}^j(\lambda)}{d\lambda}.$$

Conclusions

Summarizing the researches carried out in the work we note the following:

1. A new effective method for finding the hydraulic resistance factor in the lifting pipe, is offered.
2. When the lifting pipe is long enough, a new algorithm for determining the hydraulic resistance factor is offered.
3. When the load is large enough, its inverse is accepted as a small parameter and an asymptotic method for determining the fractional order is given.
4. Accepting the liquid as a damper in oil recovery process in rod pumping units, the motion of the object is written with fractional order ordinary differential equations, and an inverse problem for determining the fractional order is solved.
5. The discrete Rosser model for the gas-lift process is structured and the hydraulic resistance factor is found by means of the discrete Rosser model.

The basic results of the dissertation work are in the following works:

1. Алиев, Ф.А. Об одной задаче идентификации по определению параметров динамических систем / Ф.А.Алиев, Н.А.Исмаилов, А.А.Намазов [и др.] // Proceeding of IAM, - Баку: -2014. V.3, №2, - с.139-151.
2. Aliev, F.A., Ismailov, N.A., Gasimov, Y.S., Namazov, A.A., Rajabov, M.F. A method to solving some identification problem// The 5th International Conference on Control and Optimization with Industrial Applications, - Baku: -27 august -29 august, -2015, - p.237-239.
3. Aliev, F.A., Ismailov, N.A., Namazov, A.A. Asymptotic method for finding the coefficient of hydraulic resistance in lifting of fluid on tubing // - Novosibirsk: Journal of Inverse and Ill-posed Problems, - 2015. Volume 23, Issue 5, - p. 511–518.
4. Baigereyev, D. On an Identification Problem on the Determination of the Parameters of the Dynamic System / D.Baigereyev, N.A.Ismailov, A.A.Namazov [et.al.] // Mathematical Problems in Engineering, -2015. V.2015, -8p.
5. Aliev, F.A., Ismailov, N.A., Namazov, A.A. Algorithm for calculating the parameters of formation of gas-liquid mixture in the shoe of gas lift well // -Baku: Applied and Computational Mathematics, - 2016, V.15, № 3, - p.370-376.
6. Алиев, Ф.А. Алгоритм вычисления параметров образования газожидкостной смеси на башмаке газлифтной скважины / Ф.А.Алиев, Н.А.Исмаилов, А.А.Намазов // Proceeding of IAM, - Баку: -2016. V.5, №1, - с.123-132.
7. Алиев, Ф.А. Асимптотический метод решения задачи идентификации для нелинейных динамических систем / Ф.А.Алиев, Н.А.Исмаилов, А.А.Намазов [и др.] // Proceeding of IAM, - Баку: -2016. V.5, №1, - с.84-97.
8. Гаджиева, Н.С. Алгоритм решения задачи идентифика-ции для определения параметров дискретных динамических

систем / Н.С.Гаджиева, А.А.Намазов, И.М.Аскеров, [и др.] // Proceeding of IAM, - Баку: -2016. V.5, №2, - с.235-244.
9. Namazov, A.A. Qazlift prosesində diskret Rosser modeli vasitəsilə hidravlik müqavimət əmsalının təyin olunması // Azərbaycan və Türkiyə Universitetləri: təhsil, elm, texnologiya, I Beynəlxalq elmi-praktiki konfransın materialları, - Bakı, - 18-20 dekabr, 2019, III hissə, s. 39-41.
10. Алиев, Ф.А. Асимптотический метод определения коэффициента гидравлического сопротивления на разных участках трубопровода при добыче нефти / Ф.А.Алиев, Н.А.Исмаилов, А.А.Намазов [и др.] // Proceedings of IAM, - Баку: - 2017. V.6, №1, - с. 3-15.
11. Aliev, F.A. Asymptotic Method for Solution of Identification Problem of the Nonlinear Dynamic Systems / F.A.Aliev, N.A.Ismailov, A.A.Namazov [et.al.] //-Serbia: Filomat, -2018. Vol.32, N.3, - p.1025-1033
12. Aliev, F.A., Hajiyeva, N.S., Namazov A.A., Huseynova, NS. Asymptotical method to solution of identification problem for defining the parameters of discrete dynamical system in gas-lift process // 6th International Conference on Control and Optimization with Industrial Applications (COIA), - Baku: -11 july – 13july, - 2018, VOL I , - p.55-57.
13. Aliev, F.A. Identification Problem for Determining the Parameters of a Discrete Dynamic System/ F.A.Aliev, N.S.Hajieva, A.A.Namazov [et.al.] // International Applied Mechanics, -Kiev: - 2019. V.55, №1, - p.110-116
14. Алиев, Ф.А., Муталлимов, М.М., Намазов, А.А. Метод идентификации для определения порядка дробной производной колебательной системы // - Баку: Proceedings of IAM, -2019. V.8, №1, - с.3-13.
15. Намазов, А.А. Вычислительный алгоритм определения порядка дробных производных колебательных систем // - Баку: Proceedings of IAM, -2019. V.8, №2, - с.202-210.
16. Намазов, А.А. Определение коэффициента гидравлического сопротивления с помощью модели россера при

газлифтном процессе // - Баку: Proceedings of IAM, -2019. V.8, №1, - с.99-105.
17. Aliev, F.A. Algoritm for Solving the İdentification Problem for Determining the Fractional-Order Derevative of an Osccilatory System / F.A.Aliev, N.A.Aliev, Mutallimov M.M. [et.al.] // Applied and Computational Mathematics, -Baku: -2020. v.19, №3, -p.415-422.
18. Aliev, F.A., Hajiyeva, N.S., Namazov, A.A., Safarova, N.A. Asymptotical method to solution the identification problem for determining the parameters of discrete dynamical systems // The 7th International Conference on Control and Optimization with Industrial Applications, - Baku: -26 august -28 august, -2020, - p.92-94.
19. Namazov, A.A., Hajiyeva, N.S., Ismailov, N.A. Determination of the coefficient of hydraulic resistance by roesser model in discrete dynamical systems // 7th International Conference on Control and Optimization with Industrial Applications (COIA), -Baku -26 august-28 august - 2020, v. II , - p.287-289.

The defense will be held on **30 June 2022** at **14^{00}** at the meeting of the Dissertation council ED 1.04 of Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at the Institute of Mathematics and Mechanics of National Academy of Sciences of Azerbaijan.

Address: AZ 1141, Baku, B.Vahabzadeh, 9.

Dissertation is accessible at the Institute of Mathematics and Mechanics of National Academy of Sciences of Azerbaijan Library

Electronic versions of dissertation and its abstract are available on the official website of the Institute of Mathematics and Mechanics of National Academy of Sciences of Azerbaijan.

Abstract was sent to the required addresses on **27 May 2022.**

Signed for print: 13.05.2022
Paper format: 60x84 1/16
Volume: 38493
Number of hard copies: 20