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ABSTRACT

of the dissertation for the degree of Doctor of Science

ON OPTIMAL CONSTANTS IN SOBOLEV INEQUALITIES AND THEIR APPLICATION ABSTRACT

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GENERAL CHARACTERISTICS OF THIS WORK

Rationale of the work.

The mathematical model of the processes of light wave propagation in nonlinear media is a Cauchy problem for the nonlinear Schrodinger equation:

$$i\frac{\partial u}{\partial t} + i\beta|u|^q u = \Delta u + \alpha|u|^p u \quad \text{in} \quad R^n \times R_+, \tag{1}$$

$$u/_{t=0} = u_0(x), \tag{2}$$

where $q > 0, p > 0, \beta \geq 0, \alpha \neq 0, u_0 -$ is a function given in R^n. For equation (1) we set a mixed problem as well.

For $\beta = 0$ the problem (1)-(2) is studied very intensively in the scientific literature. For $\beta = 0$ quite a lot of works were devoted to different properties of problem (1)-(2). The papers of Jiber A.V., Shabat A.G., Kudryashev O.I., Sakbayeva V.J., Jidkova P.E., Gilassey R.T., Merle F., Tsutsumi S., Tsvetkov N., Strauss W., Nawa H., Cazenave T., Weissler F.B. and other were devoted to blow up of solutions of problem (1)-(2) for $\beta = 0, \alpha > 0$. Local solvability of problem (1)-(2) for $\beta = 0$ in various function spaces were studied in the papers of Ginibre J. and Velo G., Baillon J.B., Gazenave T., Figueiza, Shabat A.B. and others.

Solvability of the first mixed problem with a homogeneous boundary condition for the equation for $\beta > 0, \alpha = 0$ was studied by Lions J.L. for $\beta > 0, \alpha \neq 0$ by Vladimirov M.V. and others.

It should be noted that solvability of problem (1)-(2) for $\beta = 0, \alpha > 0$ in the supercritical case are unknown to the author in the scientific literature. The blow up rate of the solutions of problem (1)-(2) for $\beta = 0, \alpha > 0$ in critical and supercritical cases in the scientific literature, the author is unknown.

In the papers of Vladimirov M.V., the first mixed problem with a homogeneous boundary condition for equation (1) was studied without determining the sign of the parameter α and the behavior of solution as $t \to \infty$ was not researched. Then global solvability of the Cauchy problem for the Schrodinger-Hartry equation was not studied in critical and supercritical cases.

In scientific literature there is no studies on self-canalization of the solutions of problem (1)-(2) for $\beta = 0, \alpha > 0$ with a nonlinear term of the form $f\left(|u|^2\right)u$ instead of $|u|^p u$ under appropriate conditions on the function $f\left(|u|^2\right)$.

When solving global solvability of problem (1)-(2) for $\beta = 0, \alpha > 0$ in supercritical and critical cases there arises a problem of determination of the best constant and its estimation in Giliardo-Nirenberg-Sobolev inequality.

Proceeding from these arguments, we can conclude that the topic of the dissertation work was devoted to solvability, blow-up, self-canalization and behavior of solution as $t \to \infty$ of the Schrodinger, Schrodinger-Hartry and Ginzburg-Landau evolution equation and creation of appropriate mathematical tool is urgent and is of great interest both from theoretical and practical point of view.

Object and subject of research.

The main object of the dissertation work is the calculation of optimal constants in Sobolev inequalities and their application to Schrödinger, Schrödinger-Hartry and Ginzburg-Landau nonlinear evolution equations.

The goal and objectives of the study.

The goal of the work is in solving the following main problems.

1. Studying solvability, blow-up, behavior as $t \to \infty$ of the solutions of problem (1)-(2).

2. Studying solvability, blow-up of the solutions of the first boundary value problem with a homogeneous boundary condition for equation (1).

3. Studying solvability and blow-up solutions of the Cauchy problem for the Schrodinger –Hartry equation.

4. Studying blow-up of the solutions of the first mixed problem with homogeneous boundary condition for a nonlinear Schrodinger type evolution equation.

5. Studying blow-up of solution of the first mixed problem with a homogeneous boundary condition for a nonlinear Ginsburg-Landau-Schrodinger type evolution equation.

6. Calculating the exact constant in Giliardo-Nirenberg-Sobolev inequality and obtaining a priori estimation of the exact constant .

7. Creation of a mathematical tool for studying solvability of the Cauchy problem and the first mixed problem with a homogeneous boundary condition for equation (1) and the Schrodinger-Hartry equation.

General technique of studies.

In the work, the methods of mathematical physics, theory of ordinary differential equation, theory of function spaces, imbedding and functional analysis theorems, theory of Fourier transform were used.

Main provisions of dissertation.

1. The best constants in Sobolev's inequalities are calculated, and they are applied to the study of global solvability and the destruction of solutions of the Cauchy problem for the nonlinear evolution Schrodinger and Schrodinger-Hartry equations.

2. The exact constants in some inequalities of modern mathematical physics are calculated and their internal connection is investigated.

3. A method that allows to prove the absence of global solutions of the mixed problem for nonlinear Ginzburg-Landau and Schrödinger type evolution equations is proposed.

4. A method that allows to prove global solvability and destruction of solutions of the Cauchy problem for the nonlinear Schrodinger and Schrodinger–Hartry evolution equations in the supercritical and critical cases is proposed.

Scientific novelty.

1. A new exact integral inequality applied to the proof of entropy inequality, was proved.

2. One generalization of entropy inequality was proved.

3. Internal relation between some fundamental inequalities of mathematical physics, was established.

4. Exact constant in two Sobolev inequalities was calculated.

5. A priori estimations of the exact constant in one Giliardo-Nirenberg-Sobolev inequality were obtained.

6. A new proof of the Gross-Sobolev logarithmic inequality was offered.

7. Upper bound of the blow-up time $t_{\max}$ of problem (1) for $\beta \le 0, \alpha > 0$ was estimated.

8. Sufficient conditions for global solvability of the weak generalized solution of the first mixed problem for equation (1) were obtained.

9. For equation (1) for $p = 2, q = 2$ the first mixed homogeneous problem was stated and smoothness of the generalized weak solution and its behavior as $t \to \infty$ was studied.

10. For the problem (1)-(2) for $\beta = 0, \alpha > 0$ it was proved that it smooth solutions blow-up for $p \ge 4/n$ for some initial data and lower bounds of the blow-up for u_0 for some initial data and lower bounds of the blow-up rate in some norms were obtained.

11. For a system of nonlinear Schrodinger evolution equations the Cauchy problem was stated, its global solvability and blow-up was studied.

12. In the bounded domain, the upper bound of the best constant in the Sobolev and Sobolev inequalities were given. These estimations were applied to the proof of no nontrivial generalized solutions of the first homogeneous boundary value problem for a homogeneous Schrodinger stationary equation and eigen functions of a spectral problem for the Laplace operators.

13. The smoothness of the generalized solution of the first mixed problem of equation (1) for $\beta = 0, \alpha > 0$ in two-dimensional domain and their blow-up for star domains was studied.

14. A generalized equation (1) for $\beta = 0$ was offered and a homogeneous mixed problem of first kind was considered in a bounded domain of many-dimensional Euclidean space and blow-up of solutions of this problem was proved.

15. Blow-up and no global solutions of the mixed problem for a nonlinear Ginsburg-Landau type evolution equation was proved.

16. A new interpolational Sobolev inequality applied to global solvability of the Cauchy problem for the Schrodinger-Hartry equation, was proved. Critical and supercritical cases were studied..

17. Trudinger type inequality in the unbounded domain that was applied to problem (1), (2), for $\beta = 0$, for any $\alpha \neq 0$ was proved.

18. A sufficient condition for self-canalization of the solutions of problem (1), (2) for $\beta = 0, \alpha > 0$ with the nonlinear term $f\left(|u|^2\right)u$ was obtained.

19. Global solvability and blow-up of solution of problem (1), (2) for $\beta = 0, \alpha > 0$ in a supercritical case were studied.

20. Blow-up of the solutions of the first mixed problem with a homogeneous boundary condition was studied for a nonlinear Ginsburg-Landau type evolution equation.

Theoretical and practical value of the study.

The dissertation work is of theoretical and practical character. It develops a theory and methods for solving the Cauchy problem and the first mixed problem for nonlinear evolution equations. The methods of this work may be extended to the problems close by their statement to the problem studied in the present work.

The results of the dissertation may be used in scientific studies and when developing numerical methods for solving mathematical physics problems..

Approbation and application.

The main results of the work were reported at the seminars of "Differential equations" department of IMM of ANAS (doctor of physics-mathematical sciences, prof. A.B.Aliyev), at the seminar of "Mathematical analysis" department (corr. member of ANAS, prof. V.S.Quliyev), at the seminar of "Nonlinear analysis" department (corr. member of ANAS, prof. B.T.Bilalov), at the seminar of "Mathematical physics" chair of BSU (acad. of ANAS, prof. Y.A.Mamedov), at the institute seminar of SRI "Applied mathematics" of BSU (acad. of ANAS, prof. F.A.Aliyev).

Personal contribution of the author.

All the results obtained in the work are the personal contribution of the author.

Publications of the author.

The main results of the work were published in 39 papers.

The name of the institution where the dissertation was completed.

The work was performed at the Scientific Research Institute of Applied Mathematics of Baku State University.

Volume and structure of the dissertation (in signs, indicating the volume of each structural unit separately).

The total volume of dissertation work is 439439 characters (title page - 425 characters, content 3537 characters, introduction - 68000 characters, first chapter - 80000 characters, second chapter - 68000 characters, third chapter - 92000 characters, fourth chapter - 78000 characters, fifth chapter - 48000 characters, conclusions - 1477 characters). The list of used literature consists of 133 items.

THE CONTENT OF THE DISSERTATION

Rationale of the topic is substantiated, review of the papers concerning the dissertation theme is given, brief content of the work is stated in introduction.

Chapter I of the work is devoted to the proof of some integral inequalities and their application. This chapter consists of 6 sections.

In section 1.1 one interpolational inequality is proved and applied to the proof of entropy inequality.

For convenience of further statement we accept the following denotation: $\|u\|_p = \left\{ \int_{R^n} |u(x)|^p dx \right\}^{1/p}$, $p \geq 1$, is the norm in $L_p(R^n)$, in $\|\cdot\|_p$ we will omit p for $p = 2$, i.e. we will write $\|\cdot\|$. For the given ρ from the interval $(0, \rho_0)$, where $\rho_0 = +\infty$ for $n = 1,2$, $\rho_0 = 4/(n-2)$ for $n \geq 3$, we determine $\alpha = 0{,}5\rho n/(\rho + 2)$. For the given $\alpha \in (0,1)$ we determine $\chi = \sqrt{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$. let

$$\forall \theta > 0 \quad \Gamma(\theta) = \int_0^{+\infty} e^{-t} \cdot t^{\theta - 1} dt$$

be the Euler gamma-function;

$$\forall \beta > 0, \gamma > 0 \quad B(\beta, \gamma) = \int_0^1 t^{\beta - 1}(1-t)^{\gamma - 1} dt$$

be the Euler beta-function, $\sigma_n = \dfrac{2\pi^{n/2}}{\Gamma\left(\frac{n}{2}\right)}$,

$$k_g(\alpha) = \chi^{-1}\left[0{,}5\sigma_n B\left(\frac{n}{2}, \frac{n(1-\alpha)}{2\alpha}\right)\right]^{\frac{\alpha}{n}} = \\ = \chi^{-1} \cdot \pi^{\alpha/2}\left[\frac{\Gamma\left[\frac{(n-n\alpha)}{2\alpha}\right]}{\Gamma\left(\frac{n}{2\alpha}\right)}\right]^{\alpha/n}. \qquad (3)$$

Lemma 1. *Let* ρ, α *be above determined numbers,* $\nu(x) \in L_2(R^n)$, $r\nu \in L_2(R^n)$ *then the following interpolational inequality is valid:*

$$\|\nu\|_{(\rho+2)/(\rho+1)} \leq k_g(\alpha) \|r\nu\|^{\alpha} \|\nu\|^{1-\alpha}, \quad (4)$$

where $k_g(\alpha)$ *- is a constant determined by formula (3).*

The constant k_g *is exact: Inequality (4) passes to the equality for where* $\omega_1, \omega_2, \omega_3$ *are arbitrary positive numbers.*

By means of lemma 1 we prove the following theorem

Theorem 1. *Let* $u \in L_2(R^n), ru \in L_2(R^n)$. *Then the following entropy inequality is valid:*

$$-\int_{R^n} \frac{|u(x)|^2}{\|u\|^2} \ln \frac{|u(x)|^2}{\|u\|^2} dx \leq \frac{n}{2} \ln \left[\frac{2\pi \|ru\|^2}{n \|u\|^2} \right]. \quad (5)$$

Inequality (5) is exact: it passes to the equality for

$$u(x) = a \exp\left(-b \left| x - \overset{\circ}{x} \right|^2 \right),$$

where $a, b -$ *are arbitrary positive constant,* $\overset{\circ}{x} \in R^n$ *is arbitrary.*

In section 1.2 we consider one generalized entropy inequality.

Let k be any given positive number.

Let ρ be a given positive number such that for $n - k \leq 0$ ρ is any for $n - k > 0$, $\rho < 2k/(n-k)$. Assume $\alpha = n\rho/[k(\rho+2)]$, for the given α we will determine $\chi = \sqrt{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$.

Let $\forall \theta > 0$, $\Gamma(\theta) = \int_0^{+\infty} e^{-t} t^{\theta-1} dt$, be the Euler gamma-function;

$$\forall \beta > 0, \ \forall \gamma > 0, \ B(\beta, \gamma) = \int_0^1 t^{\beta-1}(1-t)^{\gamma-1} dt,$$

the Euler beta-function; $\sigma_n = 2\pi^{n/2} / \Gamma(n/2)$,

$$k_g(\alpha) = \frac{1}{\chi}\left[\frac{\sigma_n}{k} B\left(\frac{n}{k}, \frac{n(1-\alpha)}{k\alpha}\right)\right]^{\alpha k/2n} =$$

$$= \frac{1}{\chi}\left[\frac{\frac{\sigma_n}{k}\Gamma\left(\frac{n}{k}\right)\Gamma\left(\frac{n(1-\alpha)}{k\alpha}\right)}{\Gamma\left(\frac{n}{k\alpha}\right)}\right]^{\alpha k/2n} . \qquad (6)$$

Lemma 2. *Let* k, ρ, α *be above determined numbers,* $V(x) \in L_2(R^n)$, $r^{k/2}V(x) \in L_2(R^n)$, $r = |x|$.

Then the following integral inequality is valid:

$$\|V\|_{(\rho+2)/(\rho+1)} \le k_g(\alpha)\left\|r^{k/2}V\right\|^{\alpha}\|V\|^{1-\alpha}, \qquad (7)$$

where $k_g(\alpha)$ *is a constant defined by formula (6). The constant* $k_g(\alpha)$ *is exact; inequality (7) passes to the equality for* $V(x) = V_0(r) = \omega_1/(\omega_2 + \omega_3 r^k)^{1+1/\rho}$, *where* $\omega_1, \omega_2, \omega_3$ *are arbitrary positive numbers*

By means of lemma 2 we prove the following theorem

Theorem 2. *Let* $u \in L_2(R^n)$, $r^{k/2}u \in L_2(R^n)$, $\forall k > 0$.

Then the following entropy inequality is valid:

$$-\int_{R^n} \frac{|u(x)|^2}{\|u\|^2}\ln\left(\frac{|u(x)|^2}{\|u\|^2}\right)dx \le \frac{n}{k}\ln\left[\frac{ek\left(\frac{\sigma_n}{k}\Gamma(n/k)\right)^{k/n}\left\|r^{k/2}u\right\|^2}{n\|u\|^2}\right]. \qquad (8)$$

Inequality (8) is exact, it passes to the equality for

$$u(x) = u_0(r) = a\exp\left(-b\left|x - \overset{0}{x}\right|^k\right),$$

where a, b *are arbitrary positive constant,* $\overset{0}{x} \in R^n$ *is arbitrary.*

For $k = 2$ inequality (8) passes to the known entropy inequality.

Some remarks on fundamental Gross-Sobolev, Hirshman, Pauli-Heizenberg-Weil inequalities and entropy inequality are stated in section 1.3.

In this section the n dimensional analog of the Hirshman inequality with an exact constant in it is proved. Exact constant in the Pauli-Heisenberg-Weil inequality is calculated. Equivalence of two forms of logarithmic Gross-Sobolev inequalities is proved and a new proof of this inequality is found based on entropy and Hirshman inequality. The Panli-Geizenberg-Weil inequality with an exact constant in is proved by another two methods as well. These methods reveal its internal relation with entropy Gross-Sobolev and Hirshman inequalities. A new method for proving entropy inequality is offered.

Now we state the obtained results. The following propositions are known:

Proposition 1[1]. *Let* $\forall f(x) \in H^1(\mathrm{R}^n)$.
The following Gross-Sobolev logarithmic inequality is valid

$$I(f) + n\left[1 + \ln\left(\sqrt{\pi\lambda}\right)\right] \le \lambda^2 \frac{\|\nabla f\|^2}{\|f\|^2}, \tag{9}$$

where $\lambda > 0$,

Here and in the sequel $H^1(\mathrm{R}^n) \equiv W_2^1(\mathrm{R}^n)$ is a Sobolev space, $\|\cdot\| -$ is a norm in $L_2(\mathbb{R}^n)$.

Proposition 2[2]. *Let* $\forall f(x) \in H^1(\mathrm{R}^n)$. *Then the following logarithmic Gross-Sobolev inequality is valid:*

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[1] F.B. Weissler, Logarithmic Sobolev inequalities for the heat-diffusion semigroup, Trans. Amer. Math. Soc., 237(1978), 255-269

[2] *L. Gross, Logarithmic Sobolev inequalities, Amer. J. Math., 97(1975), 1661-1683*

$$I(f) \leq \frac{n}{2} \ln\left(\frac{2}{\pi e n} \frac{\|\nabla f\|^2}{\|f\|^2} \right). \quad (10)$$

Statement 1. *Inequalities (9) and (10) are equivalent: (10) follows from (9) while (9) follows from (10).*

Statement 2. *Let* $\forall f(x) \in L_1(\mathrm{R}^n) \cap L_2(\mathrm{R}^n)$ $\hat{f}(\xi)$ *be the Fourier image of the function* $f(x)$ *:*

$$\hat{f}(\xi) = \frac{1}{(2\pi)^{n/2}} \int e^{-ix\xi} f(x) dx.$$

Then the following Hirshman inequality is valid:

$$I(f) + I(\hat{f}) \leq -n \ln(\pi e) \ . \quad (11)$$

Inequality (11) is exact: the equality in it is attained iff $f = ae^{-b|x-\bar{x}|^2}, \forall a > 0, \ \forall b > 0, \forall \bar{x} \in \mathrm{R}^n$.

Statement 3. *Let* $\forall f(x) \in H^1(\mathbb{R}^n),\ rf \in L_2(\mathbb{R}^n),\ r = |x|$.

Then the following Pauli-Heizenberg-Weil inequality is valid:

$$\|f\| \leq \sqrt{\frac{2}{n}} \|\nabla f\|^{1/2} \|rf\|^{1/2} = \sqrt{\frac{2}{n}} \left\| |\xi| \hat{f} \right\|^{1/2} \|rf\|^{1/2}. \quad (12)$$

Inequality (12) is exact; the equality in it is attained iff f is a Gauss function: $f = ae^{-b|x-\bar{x}|^2}, \ \forall a > 0, \ \forall b > 0, \forall \bar{x} \in \mathrm{R}^n$.

Statement 4. Let $\forall f \in H^1(\mathrm{R}^n), rf \in L_2(\mathrm{R}^n)$.

Then the following entropy inequality is valid

$$I(f) \geq -\frac{n}{2} \ln\left(\frac{2\pi e}{n} \frac{\|rf\|^2}{\|f\|^2} \right). \quad (13)$$

Inequality (13) is exact; the equality in it is attained in it iff f is a Gauss function $f = ae^{-b|x-\bar{x}|^2}, \ \forall a > 0, \forall b > 0, \forall \bar{x} \in \mathrm{R}^n$.

Under the weak global solution of problem (14)–(16) we understand the following: the function $u \in C([0,T]; H^{-1}) \cap$

$L^{\infty}([0,T];\overset{\circ}{H}{}^{1}(\Omega))$ satisfies (14)–(16) in the sense of distribution for $\forall T>0$.

The following theorem is valid

Theorem 4. (on global existence).

A) Let $g<0, \rho\in(0,2), u_0\in\overset{\circ}{H}{}^{1}(\Omega)$. *Then problem (14)–(16) has a weak global solution.*

Б) Let $g<0, \rho=2.$ *Then assume that the initial function* $u_0\in\overset{\circ}{H}{}^{1}(\Omega)$ *is such that for it the following condition is fulfilled*

$$|g|\cdot\|u_0\|^2<\|\psi_0\|^2,$$

where ψ_0 *is a radially-symmetric function,* $\psi_0=\psi_0(r), r=|x,|$ *that is the positive solution of the following boundary value problem:*

$$\begin{cases}\dfrac{d^2}{dr^2}\psi_0(r)+\dfrac{1}{r}\dfrac{d\psi_0}{dr}-\psi_0+\psi_0^3=0, \quad 0<r<\infty,\\ \psi_0'(0)=0, \psi_0(\infty)=0, \psi_0(r)\in H^1(R^2)\cap C^2([0,\infty)).\end{cases}$$

Then problem (14)–(16) has a weak global solution.

Remark 1. For $g>0$ problem (14)–(16) has a weak global solution $\forall u_0\in\overset{\circ}{H}{}^{1}(\Omega)$.

Theorem 5. *Let* $\Omega\subset R^2$ *be a bounded or unbounded domain. Let* $u(x)$ *be any function from the space* $\overset{\circ}{H}{}^{1}(\Omega)$.

Then the following inequality is valid:

$$\int\limits_{\Omega}\left[\exp\left(\theta\frac{|u|^2}{\|\nabla u\|^2}\right)-1\right]dx\le C_T\,\frac{\|u\|^2}{\|\nabla u\|^2},$$

where

$$0<\theta<\frac{4\pi}{e}, \quad C_T=\sum_{m=1}^{\infty}\frac{\theta^m}{m!}\frac{1}{\pi^{m-1}}\frac{m^{3m-2}}{(2m-1)^{2m-1}}.$$

Theorem 6. *The weak solution of problem (14)–(16) is unique.*

In section 1.5 one interpolational Sobolev inequality is proved.

Then a new proof of the logarithmic Gross-Sobolev inequality is proved based on interpolational Sobolev inequality. The following theorem is valid

Theorem 7. *Let* $u(x) \in H^1(R^n)$, *where* $H^1(R^n) \equiv W_2^1(R^n)$ *is a Sobolev space. Let* ρ, α *be certain numbers from section 1.1,* $K_g(\alpha)$ *be determined by formula (3). Then the following Galiardo-Nirenberg-Sobolev multiplicative inequality is valid*

$$\|u\|_{\rho+2} \le \overline{K_0} \|\nabla u\|^{\alpha} \|u\|^{1-\alpha}.$$

Here $\overline{K_0} = K_g(\alpha) K_B\left(\frac{\rho+2}{\rho+1}\right)$, *where* K_B *is determined in the following way*

$$K_B(p) = \left[\left(\frac{p}{2\pi}\right)^{\frac{1}{p}} \left(\frac{p'}{2\pi}\right)^{-\frac{1}{p'}}\right]^{n/2},$$

$$1 \le p \le 2,\ \frac{1}{p} + \frac{1}{p'} = 1. \tag{17}$$

The following theorem is proved by means of theorem 6

Theorem 8. *Let* $u(x) \in H^1(R^n)$. *Then the following logarithmic Gross-Sobolev inequality is valid:*

$$\int_{R^n} \frac{|u|^2}{\|u\|^2} \ln \frac{|u(x)|^2}{\|u\|^2} dx \le \frac{n}{2} \ln\left(\frac{2\|\nabla u\|^2}{\pi e n \|u\|^2}\right). \tag{18}$$

Inequality (18) is exact: it passes to equality for

$$u(x) = a \exp(-b|x - \dot{x}|^2),$$

where a, b *are arbitrary positive constants,* $\dot{x} \in R^n$ *is arbitrary.*

In section 1.6 we prove one interpolational Sobolev inequality. Then we offer one generalized logarithmic Sobolev inequality based on interpolational Sobolev inequality

We have

Theorem 9. *Let* $\hat{U}(\xi)$ *be a Fourier transform of the function* $U(x)$

$$\hat{U}(\xi) = \frac{1}{(2\pi)^{n/2}} \int_{R^n} e^{-i(x,\xi)} U(x)dx, \quad \xi \in R^n .$$

Let k, ρ, α *be certain numbers from section 1.2,* $U(x) \in L_2(R^n),\ |\xi|^{k/2}\hat{U}(\xi) \in L_2(R^n).$

Then the following multiplicative Sobolev inequality is valid

$$\|U\|_{\rho+2} \le \overline{K}_0 \left\|\xi^{k/2}\hat{U}(\xi)\right\|^{\alpha} \|U\|^{1-\alpha} .$$

Here $\overline{K}_0 = K_g(\alpha) K_B\left(\dfrac{\rho+2}{\rho+1}\right)$, *where* $K_g(\alpha)$ *was determined in section 1.2 by formula (6), while* K_B *was determined in section 1.5 by formula (17).*

For $k = 2$ from theorem 9 it follows theorem 7 from section 1.5, for $k = 4$ by the relation $\left\| |\xi|^2 \hat{U} \right\| = \|\Delta U\|$ from theorem 9 we have the following

Theorem 10. *Let* $\rho \in (0, \infty)$ *for* $n \le 4$, *while for* $n > 4$ $\rho \in \left(0, \dfrac{8}{n-4}\right)$ $\alpha = n\rho/[4(\rho+2)]$.

Then, let $U(x) \in L_2(R^n),\ \Delta U \in L_2(R^n)$. *Then the following interpolational Sobolev inequality is valid*

$$\|U\|_{\rho+2} \le \overline{K}_0 \|\Delta U\|^{\alpha} \|U\|^{1-\alpha} .$$

Here
$$\overline{K}_0 = K_g(\alpha) K_B\left(\frac{\rho+2}{\rho+1}\right),$$

where

$$K_g(\alpha) = \frac{1}{\chi}\left[\frac{\sigma_n}{4} B\left(\frac{n}{4}, \frac{n(1-\alpha)}{4\alpha}\right)\right]^{2\alpha/n},$$

K_B *was determined by formula (17).*

Theorem 11. *Let* k *be an arbitrary positive number. Let* $U(x) \in L_2(R^n)$, $|\xi|^{k/2}\hat{U}(\xi) \in L_2(R^n)$.

Then the following logarithmic Gross-Sobolev inequality is valid:

$$\int_{R^n} \frac{|U|^2}{\|U\|^2} \ln\left(\frac{|U|^2}{\|U\|^2}\right) dx \le \frac{n}{k} \ln\left[\frac{k\left(\frac{\sigma_n}{k}\Gamma\left(\frac{n}{k}\right)\right)^{k/n} \left\||\xi|^{k/2}\hat{U}\right\|}{n\pi^k e^{k-1}\|U\|^2}\right].$$

For $k = 2$ from theorem 11 it follows logarithmic Gross-Sobolev inequality, while for $k = 4$ it follows the following.

Theorem 12. *Let* $U(x) \in H^2(R^n)$.

Then the following logarithmic Gross-Sobolev inequality is valid:

$$\int_{R^n} \frac{|U|^2}{\|U\|^2} \ln\left(\frac{|U|^2}{\|U\|^2}\right) dx \le \frac{n}{4} \ln\left[\frac{4\left(\frac{\sigma_n}{4}\Gamma\left(\frac{n}{4}\right)^{4/n}\right)\|\Delta U\|^2}{n\pi^4 e^3\|U\|^2}\right].$$

Chapter II was devoted to calculation of optimal constants in two Sobolev inequalities and their application. This chapter consists of four sections.

In sections 2.1 we get two upper bounds of the best constant in one interpolational Sobolev inequality and study their exactness. For the given ρ from the interval $(0, \rho_0)$, where $\rho_0 = \infty$ for $n = 1,2$, $\rho_0 = 4/(n-2)$ we determine $\alpha = 0{,}5\rho n/(\rho + 2)$. For the best constant in the Sobolev inequality

$$\|U\|_{\rho+2} \le \overline{K}_0 \|\nabla U\|^{\alpha} \|U\|^{1-\alpha} , \qquad (19)$$

where K_0 is the best constant, the following estimation is valid

$$K_0 < \overline{K}_0 = K_g(\alpha) K_B\left(\frac{\rho+2}{\rho+1}\right).$$

Here $K_g(\alpha)$ was determined by formula (3), K_B was determined by formula (17).

Then, one more upper bound for K_0 is given

$$K_0 < \overline{\overline{K}}_0 = \frac{1}{\chi}\sqrt{K_B\left(\frac{\rho+2}{2}\right) K_B^2\left(\frac{\rho+2}{\rho+1}\right) \|G\|_{\frac{\rho+2}{2}}} ,$$

where $\chi = \sqrt{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$, $G(|x|) = K_{\frac{n-2}{2}}(|x|)/|x|^{\frac{n-2}{2}}$, $K_{\frac{n-2}{2}}(|x|)$ is the McDonald function, $n \ge 2$.

Theorem 13. *For* $\rho \ge 2$ *the estimation* $\overline{\overline{K}}_0 \le \overline{K}_0$, *for* $0 < \rho \le 2$ *the estimation* $\overline{K}_0 \le \overline{\overline{K}}_0$ *is valid.*

In section 2.2 we study interrelation between optimal constants in two Sobolev inequalities.

Theorem 14. *Between optimal constant* K_0 *in Sobolev inequality (19) and optimal constant* K_c *in the following Sobolev inequality*

$$\|u\|_{\rho+2} \le K_c \|u\|_{H^1(R^n)}$$

there exists the following relation

$$K_c = \chi K_0 .$$

In section 2.3 the optimal constant K_0 in inequality (19) is calculated.

Theorem 15. *For the optimal constant* K_0 *in Sobolev inequality (19) the following formula is valid:*

$$K_0 = \frac{1}{\chi}\left(\frac{1-\alpha}{\|\psi_0\|^2}\right)^{\frac{\alpha}{n}},$$

where $\psi_0(r)$ is the main state (positive solution from the class $C^2([0,\infty))\cap H^1(R^n)$), with minimal norm $\|\psi_0\|$) of the following boundary value problem:

$$\left.\begin{aligned} &\frac{d^2\psi_0}{dr^2}+\frac{n-1}{r}\frac{d\psi_0}{dr}-\psi_0+\psi_0^{\rho+1}=0,\\ &\frac{d\psi_0}{dr}\Big/_{r=0}=0,\ \psi_0(\infty)=0. \end{aligned}\right\} \tag{20}$$

Then inequality (19) is applied to the study of solvability of the Cauchy problem for the nonlinear Schrodinger equation

$$\begin{aligned} &iu_t+\Delta u=\omega|u|^{\rho}u \quad in \quad R^n\times R_+,\\ &u(0,x)=u_0(x). \end{aligned} \tag{21}$$

Here $\omega\in R_1, \rho\in R_+, u_0(x)$ is a function given in R^n. We accept the following denotation:

$$\|Du(t,\cdot)\|^2 = A(u(t)), B(u(t)) = 2(\rho+2)^{-1}\|u(t,\cdot)\|_{\rho+2}^{\rho+2},$$

$$E(u(t)) = A(u(t)) + \omega B(u(t)).$$

Let the condition on ρ, indicated in section 2.1. be fulfilled. By the imbedding $H^1(R^n)\subset L_{\rho+2}(R^n)$ the mapping $u\to E(u)$ is a continuous functional. Let

$$\omega<0, \eta=\rho n/4>1, \lambda>0, d=\inf\sup_{\mu\geq 0} E(\mu^{n/2}u(\mu x)),\ u\in H^1(R^n),$$

$\|u\|=\lambda;$ we define the following sets

$$M_\lambda=\{u\,|\,u\in H^1(R^n), \|u\|=\lambda,\ B(u)<\theta A^{\eta}(u), E(u)<d\},$$

$$M_\lambda^+=\{u\,|\,u\in H^1(R^n), \|u\|=\lambda,\ E(u)<d,\ A(u)>\eta B(u)\},$$

$$M_\lambda^-=\{u\,|\,u\in H^1(R^n), \|u\|=\lambda,\ E(u)<d,\ A(u)<\eta B(u)\},$$

$$V_\lambda^+ = \{u \mid u \in H^1(R^n), \|u\| = \lambda,\ E(\mu^{n/2}u(\mu x)) < d, \forall \mu \in [0,1]\},$$
$$V_\lambda^- = \{u \mid u \in H^1(R^n), \|u\| = \lambda,\ E(\mu^{n/2}u(\mu x)) < d, \forall \mu \in [1,\infty)\},$$

where $\theta = \lambda^{4\eta/n+2(1-\eta)} / \|\psi_0\|^{4\eta/n}, \psi_0$ is the main state of equation (20).

Statement 5.

1) The formula $d = (1-\eta^{-1})(1/\eta|\omega|\theta)^{1/(\eta-1)}$ is valid;

2) We have the equalities $M_\lambda = M_\lambda^+ \cup M_\lambda^-, V_\lambda^+ = M_\lambda^+,\quad V_\lambda^- = M_\lambda^-$.

Put $V^+ = \bigcup\limits_{\lambda>0} V_\lambda^+,\quad V^- = \bigcup\limits_{\lambda>0} V_\lambda^-$.

By c we will denote various constants independent of t and $u(x,t)$.

Theorem 16. *Let* $4/n < \rho < \rho_0$, *where* $\rho_0 = 4/(n-2)$ *for* $n \geq 3$ *(* $\rho_0 = \infty$ *for* $n = 1,2$*). Let be above determined stability sets. Then:*

1) For $u_0 \in V^+$ *problem (21) has a unique global solution,* $u(t) \in C([0,\infty), H^1(R^n))$, *and for* $\forall t \in [0,\infty)$ *the inclusion* $u \in V^+$ *is valid. For the entropy* $\varepsilon(t) = -\int |u(x,t)|^2 \ln|u(x,t)| dx$ *of nonlinear Schrodinger dynamics (21) the estimation* $\varepsilon(t) \leq c + c\ln(1+t),$ $\forall t \in [0,\infty)$ is valid.

2) Let $u \in V^-, ru_0 \in L_2(R^n)$*. Then problem (21) has no solution* $u(t) \in C([0,T), H^1(R^n))$, *on the whole, i.e. for* $\forall T > 0$.

Theorem 17. *Let* $\rho = 4/n, \omega < 0$.

Let $u_0 \in H^1(R^n)$, $ru_0 \in L_2(R^n)$ *and for the initial function* $u_0(x)$ *the condition* $|\omega|^{n/2}\|u_0\| < \|\psi_0\|$, *be fulfilled, where* ψ_0 *is the main state of equation (20). Then problem (21) has a unique solution* $u(t) \in C([0,\infty), H^1(R^n))$ *on the whole and for this solution for* $\forall t \in [0,\infty)$ *the following estimation is valid.*

$$c(1+t)^{-n/n+2} \leq \|u(t)\|_{(2n+4)/n} \leq c(1+t)^{-n/(n+2)},$$

$$c + c\ln(1+t) \le \varepsilon(t) \le c + c\ln(1+t).$$

Remark 2. For $n=1$ equation (20) is solved exactly, namely: $\psi_0 = \left[(\theta+1)/\theta\right]^{\theta/2} / ch^{\theta}(x/\theta), \theta = 2/\rho,$ by the some token $\|\psi_0\|^2 = 2^{2\theta-1}\left[(\theta+1)/\theta\right]^{\theta}\theta B(\theta,\theta),$ where $B(\theta,\theta)$ -is Eulers beta-function.

In section 2.4 we obtained some sufficient conditions for no nontrivial generalized solutions of Dirichlet's internal problem with a homogeneous Schrodinger type equation. To this end, we established some upper bounds of the exact constant in the known Sobolev space in an unbounded domain, and also in the known Steklov inequality.

Let $\Omega \subset R^n$ be an arbitrary bounded open domain with the boundary $\partial\Omega \subset C^{1,\mu}$, $0 < \mu \le 1$.

We consider the Dirichlet problem for the Schrodinger operator $\Delta + q(x)$

$$\begin{aligned} &\Delta u + q(x)u = 0 \ \ в \ \ \Omega, \\ &u = 0 \quad на \quad \partial\Omega , \end{aligned} \tag{22}$$

where $\Delta = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$, $q(x)$ is a function given in Ω. We will consider problem (22) in the following generalized sense.

Definition 1. The function $u(x) \in \overset{0}{W}{}_2^1(\Omega)$ is a generalized solution of problem (22), if it satisfies the integral idently

$$\int_{\Omega} \left[-\sum_{i=1}^{n} \frac{\partial u(x)}{\partial x_i} \frac{\partial \psi(x)}{\partial x_i} + q(x)u(x)\psi(x) \right] dx = 0$$

for any function $\psi(x) \in \overset{0}{W}{}_2^1(\Omega)$.

We accept the following denotation: For the given ρ from the interval $(0,\rho_0)$, where $\rho_0 = \infty$ for $n = 1{,}2$, $\rho_0 = \frac{4}{n-2}$ for $n \ge 3$, we determine $\alpha = 0{,}5\frac{\rho n}{\rho + 2}$.

Assume $\overline{K}_0 = K_g K_B\left(\dfrac{\rho+2}{\rho+1}\right)$, where $K_g(\alpha)$ was determined by formula (3), K_B was determined by formula (17). Then, $\overline{K}_c(\Omega,\alpha) = \overline{K}_0^{1/\alpha}(\alpha)|\Omega|^{1-\alpha/n}$, $|\Omega| = mes\,\Omega$.

The following theorems are valid.

Theorem 18. Let ρ be above determined number, $q(x) \neq const$ - be a function given in Ω for the class $L_{(\rho+2)/\rho}(\Omega)$. Then let

$$\Lambda = \inf J(\nu),\quad J(\nu) = \frac{\|\nabla\nu\|}{\|\nu\|_{\rho+2}},\quad \nu \in \overset{0}{W}{}_2^1(\Omega)/\{0\}.$$

Let $q(x)$ be such that

$$\|q\|_{(\rho+2)/\rho} < \Lambda^2 . \tag{23}$$

Then problem *(22)* has no nontrivial generalized solution in the class $\overset{0}{W}{}_2^1(\Omega)$.

Theorem 19. Let ρ and α be above determined numbers, $q(x) \neq const$ be a function in Ω from the class $L_{(\rho+2)/\rho}(\Omega)$. Let $q(x)$ be such that for the norm $\|q\|_{(\rho+2)/\rho}$ the following condition be fulfilled

$$\|q\|_{(\rho+2)/\rho} < \Lambda_*^2 ; \tag{24}$$

here $\Lambda_* = \dfrac{1}{\overline{k}_c}$.

Then problem (22) has no nontrivial generalized solution in the class $\overset{0}{W}{}_2^1(\Omega)$.

Remark 3. Compared to condition (24) condition (23) is more exact as $\Lambda_* < \Lambda$. It is clear that condition (24) is more practical from the point of view of calculation than condition (23). In theorems 18 and 19 if we substitute $\|q\|_{(\rho+2)/\rho}$ by $\|q^*\|_{(\rho+2)/\rho}$, where $q^* = \max(0, q(x))$, we get the best estimation. It is clear that if $q(x) \le 0$ a.e. in Ω, then problem (22) has no solution in the class $\overset{0}{W}{}_2^1(\Omega)$.

When proving theorems 18 and 19 we use the following theorem.

Theorem 20. Let ρ be the above determined number, $\nu(x)$ be any function from the class $\overset{0}{W}{}_2^1(\Omega)$.

Then the following Sobolev inequality is valid:

$$\|\nu\|_{\rho+2} \le \overline{K}_C \|\nabla \nu\|.$$

Chapter III is devoted to solvability of problem for nonlinear evolutional Schrodinger equations and blow-up, self canalization of their solutions. This chapter consists of six sections.

In 3.1 a mixed problem is considered for Schrodingers evolutional cubic equation with a cubic dissipative term.

Let $\Omega \subset R^2$ be a bounded or unbounded domain with smooth boundary $\partial\Omega$.

We consider the following mixed problem:

$$i\frac{\partial u}{\partial t} + i\beta|u|^2 u = \Delta u + \alpha |u|^2 u, \quad x \in \Omega, t > 0, \tag{25}$$

$$u\big|_{t=0} = u_0(x), \quad x \in \Omega, \tag{26}$$

$$u = 0, \quad \text{на} \quad \partial\Omega, \quad t \ge 0. \tag{27}$$

Here $\{\alpha, \beta\} \in R^1$ are the parameters of equation (25), u_0 is a function given in Ω.

Assume $H := L_2(\Omega; C)$; $H_0^1 := \overset{0}{W}{}_2^1(\Omega; C)$, $H^2 := W_2^2(\Omega; C)$ -are Sobolev’s Hilbert spaces; $B = H^2 \cap H_0^1$.

Let $\Psi_0(r),\ r=|x|$ be a positive monotonically decreasing function from the class $C^2[0,\infty)$ with the finite norm $\|\Psi_0\|_{H^1}$, that is a unique solution (main state) of the following nonlinear boundary value problem

$$\left.\begin{array}{l}\Psi_0''(r)+\dfrac{1}{r}\Psi_0'(r)-\Psi_0(r)+\Psi_0^3(r)=0,\ 0<r<\infty,\\ \Psi_0'(0)=0,\ \ \Psi_0(\infty)=0.\end{array}\right\} \quad (28)$$

We have the following theorem on global solvability of problem (25)-(27).

Theorem 21. *Let* $\beta>0,\ \alpha,u_0\in B$ *be such that one of the following conditions be fulfilled* $\left(\alpha-\sqrt{3}\beta\right)\|u_0\|^2<P_k$, $k=1,2$, *where*

$$P_1=\|\psi_0\|^2\ ,\ \ P_2=\frac{27\pi}{8},$$

ψ_0 *is the main state of problem (28). For* $\alpha<0$ u_0 *any of* B. *Then problem (25)-(27) has a global strong solutions and this solution is unique,* $u(x,t)$ *is from the class* $C^0([0,\infty);B)\cap C^1([0,\infty);H)$.

In section 3.2 we consider global solvability and smoothness, asymptotics as $t\to\infty$ of the mixed problem

$$iu_t+i\beta|u|^q u=\Delta u+\alpha|u|^p u,\ \ (x,t)\in Q=G\times[0,T]; \quad (29)$$

$$u\big|_{s=0}=0,\quad S=\partial G\times[0,T]; \quad (30)$$

$$u(x,0)=u_0(x),\quad x\in G. \quad (31)$$

Here G is an arbitrary bounded domain of n dimensional domain R^n with a smooth boundary, $\beta\in R_+,\alpha\in R,\ \ q,p\in R_+,u_0$ is a function given in R^n.

We have the following theorem

Theorem 22. *Let* $\beta>0,p>0,q>0,\alpha<0$ *(for* $\alpha>0$ *we additionally assume that* $q>p$*). Let* $u_0\in\overset{\circ}{H}{}^1(G)\cap L_{\nu+2}(G)$ *where*

$\nu = \max(p, q)$. *Then there exists a unique solution of problem (29)-(31) such that*

$$u \in L_\infty\left(0, T; \overset{\circ}{H}{}^1(G) \cap L_{\nu+2}(G)\right) \cap L_{p+q+2}(Q),$$

$$u_t \in L_\infty\left(0, T; H^{-1}(G) + L_{\frac{\nu+2}{\nu+1}}(G)\right).$$

In section 3.3 we consider self-canalization of the solutions of a nonlinear evolution Schrodinger equation.

Let for the nonlinear Schrodinger evolution equation the Cauchy problem be posed:

$$i\frac{\partial u}{\partial t} = \Delta u + f\left(|u|^2\right)u \quad in \quad R^n \times R_+, \tag{32}$$

$$u\big|_{t=0} = u_0(x) \quad in \quad R^n. \tag{33}$$

Here $f(s)$ is a function given on $[0, \infty)$ u_0 is a function given in R^n.

We well accept standard denotation :

$H(R^n) = L^2(R^n)$, $H^1(R^n) = W_2^1(R^n)$ are complex Hilbert spaces of Sobolev; $\|\cdot\|$ is a norm in $L_2(R^n)$, $\|\cdot\|_p$ is a norm in $L_p(R^n)$, $p \geq 1$.

Put

$$A(t) = A(u(t)) = \|\nabla u(\cdot, t)\|^2,$$

$$E(t) = E(u(t)) = A(t) - B(t), \text{ where } B(t) = \int_{R^n} F(|u(x,t)|^2)dx,$$

where $F(s) = \int_0^S f(\tau)d\tau$; $a(t) = a(u(t)) = \|u(\cdot, t)\|^2$,

$$P_k(t) = P_k(u(t)) = \frac{i}{2}\int_{R^n}\left(u\frac{\partial \bar{u}}{\partial x_k} - \bar{u}\frac{\partial u}{\partial x_k}\right)dx, \quad k = 1, 2, \ldots, n.$$

Definition 2. *We say that the global solution* $u(x,t)$ *of problem (32), (33) from the class*

$$C^0([0,+\infty); H^1(R^n)) \cap C^1([0,+\infty); H^{-1}(R^n))$$

is self-canalized if for $\forall t \in [0,+\infty)$ *the following bilateral estimation is fulfilled for it:* $const \le \|u(\cdot,t)\|_4 \le const$, *where const denotes various positive constants independent of* t *and* $u(x,t)$.

The following theorems are valid.

Theorem 23. *(on global existence).*

Let $u_0 \in H^1(R^n)$, $f(0)=0$, $f(s) \in C^2([0;\infty))$ *on the interval* $[0;+\infty)$ *be a positive increasing, downwards convex function, i.e.* $f(s) \ge 0$, $f'(s) > 0$, $f''(s) < 0$ $\forall s \in [0,+\infty)$.

Then for $n=1$ *problem (32), (33) has a global solution from the class*

$$C^0([0,+\infty); H^1(R^n)) \cap C^1([0,+\infty); H^{-1}(R^n)).$$

For $n=2$ *the said one remains valid subject to the condition*

$$\|u_0\|^2 < \frac{27\pi}{16 f'(0)}.$$

Theorem 24. *(on self-canalization).*

Let all the conditions of theorem 23 be fulfilled. Let the initial function u_0 *be such that the following inequality be fulfilled:* $E(u_0) < \dfrac{P^2(u_0)}{a(u_0)}$, where $P^2(u_0) = \sum_{k=1}^{n} P_k^2(u_0)$. Then the solution of problem (32), (33) is self-canalized.

In section 3.4 we consider the Cauchy problem for a cubic evolution Schrodinger equation:

$$i\frac{\partial u}{\partial t} = \Delta u + |u|^2 u \quad in \quad R^2 \times R_+, \tag{34}$$

$$u|_{t=0} = u_0 \quad in \quad R^2. \tag{35}$$

It is proved that under some initial data the solution of problem (34), (35) blows-up after some finite time whose exact value is estimated from above.

In section 3.5 we consider the Cauchy problem for a nonlinear evolution Schrodinger equation:

$$i\frac{\partial u}{\partial t} = \Delta u + |u|^{\rho} u, \quad x \in R^n, \ t > 0, \tag{36}$$

$$u(x,0) = u_0(x) \quad \text{in} \quad R^n \tag{37}$$

Here $\rho > 0$, $n \le 3$, $u_0(x)$ is a function given in R^n.

It is proved that for $\rho \ge 4/n$ and some initial data the solutions of problems (36), (37) blow-up after finite time whose value is estimated from above. Furthermore, the lower bounds of blow-up rate of solutions were obtained in some norms.

In section 3.6 we consider a global existence, asymptotics as $t \to \infty$ and blow-up of the solution of the problem

$$iu_t + \Delta u = kf\left(|u|^2\right)u \quad in \quad R^n \times R_+,$$
$$u(0,x) = u_0(x);$$

here $k \in R^1$, u_0 is a function given in R^n.

Chapter IV was devoted to no global solutions of the first mixed problem for a nonlinear Ginsburg-Landau-Schrodinger type evolution equation, solvability and blow-up solutions of the Cauchy problem for a system of nonlinear Schrodinger evolution equations. This chapter consists of 5 sections.

Section 4.1 studied no global solutions of a mixed problem for a nonlinear Schrodinger type evolution equation. Let $\Omega \subset R^n$ – be an arbitrary bounded domain with a smooth boundary. Consider that following mixed problem:

$$u_t = i\beta\Delta u + f(u, \nabla u), \ x \in \Omega, \ t > 0, \tag{38}$$

$$u(x,0) = u_0(x), \ x \in \overline{\Omega}, \ u(x,t)\big|_{\partial\Omega} = 0, \quad t \ge 0, \tag{39}$$

in which

$$f(u, \nabla u) \ge \omega_1 |u|^{1+\gamma} + \omega_2 |\nabla u|^{1+\mu}, \tag{40}$$

where $\omega_1 \ge 0, \omega_2 \ge 0, \ \omega_1^2 + \omega_2^2 \ne 0, \gamma > 0, \mu > 0, \beta \ne 0.$

It was proved that for "rather large" values of initial data problem (38)-(40) has no global solutions.

In section 4.2 blow-up of solutions of the first mixed problem for a class of nonlinear Ginsburg-Landau-Schrodinger evolution equation is studied..

Let $\Omega \subset R^n$ be a bounded domain with smooth boundary $\partial\Omega$. Consider the following mixed problem:

$$u_t = (\alpha + i\beta)\Delta u + f(u) + (\eta + i\mu)u, \quad x \in \Omega, \quad t > 0, \tag{41}$$

$$u(x,0) = u_0(x), \quad x \in \overline{\Omega}, \tag{42}$$

$$u(x,t)\big|_{\partial\Omega} = 0, \quad t \geq 0. \tag{43}$$

Here $f(u) = (\omega + i\gamma)|u|^{1+p}, \{\alpha, \beta, \omega, \gamma, \eta, \mu\} \in R,\ p \in R_+,$ $\alpha^2 + \beta^2 \neq 0,\ \omega^2 + \gamma^2 \neq 0.$

It was proved that the solutions of problem (41)-(43) under "large values" of initial data blow-up after finite time estimated from above.

In section 4.3 no global solutions of the first mixed problen is studied for a nonlinear Ginsburg-Landau type evolution equation.

Let $\Omega \subset R^n$ be an arbitrary bounded domain with a smooth boundary. Consider the following mixed problem:

$$u_t = (\alpha + i\beta)\Delta u + f(u, \nabla u),\ x \in \Omega, t > 0,$$
$$u(x,0) = u_0(x), x \in \overline{\Omega},\ u(x,t)/_{\partial\Omega} = 0, t \geq 0,$$

here

$$f(u, \nabla u) \geq \omega_1 |u|^{1+\gamma} + \omega_2 |\nabla u|^{1+\mu} \ ,$$

where $\omega_1 \geq 0, \omega_2 \geq 0,\ \omega_1^2 + \omega_2^2 \neq 0, \gamma > 0, \mu > 0, \beta \neq 0, \alpha \in R.$

It was proved that for "rather large initial data" the problem under consideration has no global solutions.

In section 4.4 we consider a mixed problem for a nonlinear Schrodinger evolution equation in two-dimensional domain. Let

$\Omega \subset R^2$ be a bounded domain with a rather smooth boundary Γ. Consider the following mixed problem:

$$i\frac{\partial u}{\partial t} = \Delta u + |u|^{\rho} u,\ x \in \Omega,\ t > 0, \tag{44}$$

$$u(x,t) = 0,\ x \in \Gamma,\ t \geq 0, \tag{45}$$

$$u(x,0) = u_0(x),\ x \in \Omega. \tag{46}$$

Here $\rho > 0$, $u_0(x)$ is a function given on Ω. For problem (44)-(46) we study smoothness of solutions and their blow-up in the starry domain Ω.

Problem (44)-(46) has a global solution in the space of functions

$$u(x,t) \in C\left([0,\infty); \overset{0}{W}{}_2^1(\Omega) \cap W_2^2(\Omega)\right) \cap C^1([0,\infty); L_2(\Omega))$$

for $\rho < 2$ for any

$$u_0(x) \in \overset{0}{W}{}_2^1(\Omega) \cap W_2^2(\Omega).$$

In the case of $\rho = 2$ what has been said remains valid for

$\|u_0\|^2 < \frac{27\pi}{8}$.

Then, for $\rho \geq 2$ a set of initial data u_0 is selected from the space $\overset{0}{W}{}_2^1(\Omega) \cap W_2^2(\Omega)$ whose solution of the problem under consideration blows up for finite time $t_{\max}$, estimated from above. More exactly, let the initial function $u_0(x)$ be such that $E_0 = \|\nabla u_0\|^2 - \frac{2}{\rho+2}\|u_0\|_{\rho+2}^{\rho+2} < 0$, then the solution of the problem (44)-(46)

$$u(x,t) \in (([0,t_{\max}); W_2^1(\Omega) \cap W_2^2(\Omega)) \cap C^1([0,t_{\max}); L_2(\Omega))$$

blows up after finite time $t_{\max}$, estimated from above by a certain number dependent on $u_0(x)$ for a star domain.

The domain Ω is called a star domain if the ray outgoing from any internal point of the domain Ω intersects its boundary $\partial\Omega$ at one point.

In section 4.5 we consider global existence, behavior as $t \to +\infty$ and blow-up of the solution of the Cauchy problem for a system of nonlinear Schrodinger evolution equations

$$i\frac{\partial u_1}{\partial t} + \theta_1 \Delta u_1 = \gamma_1 \bar{u}_2 u_3,$$

$$i\frac{\partial u_2}{\partial t} + \theta_2 \Delta u_2 = \gamma_2 \bar{u}_1 u_3, \text{ in } \mathrm{R}^n \times \mathrm{R}_+, \tag{47}$$

$$i\frac{\partial u_3}{\partial t} + \theta_3 \Delta u_3 = \gamma_3 u_1 u_2,$$

$$u_m(0,t) = u_{0m}(x), \quad x \in \mathrm{R}^n, \; m = 1,2,3. \tag{48}$$

Here $u(x,t) = (u_1(x,t), u_2(x,t), u_3(x,t))$ is a known complex valued vector function; $u_{0m}(x), m = 1,2,3$ are the functions given in $\mathbb{R}^n$; $\theta_m, \gamma_m, m = 1,2,3$ are the given nonzero real constants (parameters of equation (47)), i.e. $\theta_m, \gamma_m \in \mathrm{R}^1 \setminus \{0\}, m = 1,2,3;$ the dash over $\bar{u}_m(x,t)$ means complex conjugation of $u_m(x,t)$.

Further we will suppose that the parameters $\theta_m, \gamma_m, m = 1,2,3,$ of equation (47) satisfy the conditions

$$\gamma_1 + \gamma_2 = \gamma_3, \tag{49}$$

$$\frac{\omega_1}{|\omega_1|} = \frac{\omega_2}{|\omega_2|} = \frac{\omega_3}{|\omega_3|}, \tag{50}$$

where $\omega_m = \dfrac{\theta_m}{\gamma_m}$, i.e. $sign(\theta_m \gamma_m) = const, m = 1,2,3.$

The following theorem is valid

Theorem 25. *(on global solvability). Let the parameters of equation (47) satisfy conditions (49), (50) and* $n < 4$.

Let $u_{0m} \in H^1$, $m = 1,2,3.$

Then problem (47), (48) has a unique global solution such that

$$u_m(x,t) \in C^0\left([0,+\infty); H^1\right) \cap C^1\left([0,+\infty); H^{-1}\right),\ m = 1,2,3.$$

Chapter V was devoted to solvability and blow-up of the solution of the Cauchy problem for a nonlinear Schrodinger-Hartry evolution equation. This chapter consists of three sections.

In section 5.1 one interpolation inequality containing a convolution was proved. The following theorem is valid.

Theorem 26. *Let* $0 < \lambda < \min(4,n), \forall V(x) \in H^1(R^n)$, *then the following interpolation inequality is valid:*

$$\sqrt[4]{Q(V)} \le \overline{K}_0 \|\nabla V\|^{\theta} \cdot \|V\|^{1-\theta}; \tag{51}$$

here

$$Q(V) = \int_{R^n} \int_{r^n} \frac{|V|^2(x)|V|^2(y)}{|x-y|^{\lambda}} dxdy, \quad \theta = \frac{\lambda}{4};$$

$$\overline{K}_0 = \sqrt[4]{K_L} K_W K_Б,$$

$$K_L = \pi^{\frac{\lambda}{2}} \frac{\Gamma(n/2-\lambda/2)\Gamma(n/2)^{-1+\frac{\lambda}{n}}}{\Gamma(n-\lambda/2)\Gamma(n)};$$

$$K_W = \chi_\theta^{-1}(\sigma_n B/2)^{\frac{\theta}{n}}, \sigma_n = \frac{2\pi^{\frac{n}{2}}}{\Gamma(n/2)},$$

$B = B\left(\frac{n}{2}, \frac{n(1-\theta)}{2\theta}\right)$ *– is a beta-function,* $\Gamma(\cdot)$ *-is Euler gamma-function* $\chi_\theta = \sqrt{\theta^\theta (1-\theta)^{1-\theta}}$;

$$K_B(p,p') = \left[\left(\frac{p}{2\pi}\right)^{\frac{1}{p}} \left(\frac{p'}{2\pi}\right)^{-\frac{1}{p'}}\right]^{\frac{n}{2}}, \quad p = \frac{4n}{2n-\lambda}, \quad p' = \frac{4n}{2n+\lambda}.$$

We study calculation of the exact constant in inequality (51).

Let $0<\lambda<\min(4,n), \forall\theta=\frac{\lambda}{4}, \quad V(x)\in H^1(R^n)$. Consider the functional

$$R_0(V)=\frac{\|\nabla V\|^{\theta}\cdot\|V\|^{1-\theta}}{\sqrt[4]{Q(V)}} \tag{52}$$

and its minimization along the space $H^1(R^n), V\neq 0$. Since the mapping $R_0(V)\to R$, is continuous and the functional $R_0(V)$ is lower bounded by inequality (51), consequently there exists infimum. Assume

$$\frac{1}{K_0}=\Lambda=\inf\{R_0(V)\,|\,V\in H^1, V\neq 0\}, \tag{53}$$

Let W_r be a subspace of $H^1(R^n)$, consisting of the functions $V(x)\in H^1(R^n)$, dependent only on r, positive and decreasing monotonically tend to zero $r\to+\infty$. The following theorem is valid.

Theorem 27. *Let*

$$0<\lambda<\min(4,n),\ \forall\theta=\frac{\lambda}{4},\ \ \forall V(x)\in H^1(R^n).$$

Then let $R_0(V)$ *be determined by relation (52),* K_0 *by relation (53). Then:*

1) The functional $R_0(V)$ *achieves its infimum on the function* $\psi_0(r)\in W_r\cap C^2[0,\infty]$, *that is the positive solution of the following nonlinear boundary value problem:*

$$\psi_0''+\frac{n-1}{r}\psi_0'-\psi_0=\psi_0\int\frac{\psi_0^2(\xi)d\xi}{|x-\xi|^{\lambda}}, \tag{54}$$

$$\psi_0'(0)=0, \quad \psi_0(+\infty)=0;$$

2) For the solution $\psi_0(r)\in W_r\cap C^2[0,\infty)$ *of problem (54) the following relation is valid*

$$\|\nabla\psi_0\|^2 = \theta Q(\psi_0) = \frac{\theta\|\psi_0\|^2}{1-\theta} \geq \frac{\theta}{C_B^2} K_L \|G\|^2_{\frac{2n}{2n-\lambda}},$$

where

$$C_B = K_B\left(\frac{2n}{2n-\lambda}, \frac{2n}{2n+\lambda}\right) K_B^2\left(\frac{4n}{2n+\lambda}, \frac{4n}{2n-\lambda}\right),$$

$$G(r) = \frac{K_{(n-2)/2}(r)}{r^{(n-2)/2}}, \quad n \geq 2,$$

$K_{(n-2)/2}$*-is McDonald's function,* $G(r)$*-is a kernel of the integral operator* $(I-\Delta)^{-1}$*, for which for* $n \geq 2$ *there exist various integral representations, including the above used. As is known, for* $n = 1,3$ $G(r)$ *is expressed by the elementary functions;*

3) The optimal constant K_0 *is determined by the formula*

$$K_0 = \chi_\theta^{-1}\left[\frac{1-\theta}{\|\psi_0\|^2}\right]^{\frac{1}{4}};$$

4) The equality in interpolational inequality (51) holds iff

$$V = \gamma\psi_0\left(\beta|x-\mu|\right), \gamma > 0, \quad \beta \in R^1 - \{0\}, \mu \in R^n.$$

In section 5.2 we consider a Cauchy problem for a nonlinear Schrodinger-Hartry evolution equation:

$$iu_t + \Delta u = \omega f(|u|)u \quad в \quad R^n \times R_+, \tag{55}$$

$$u\big|_{t=0} = u_0(x) \qquad в \quad R^n, \tag{56}$$

here

$$f(|u|) = \int |x-y|^{-\lambda} |u(y,t)|^2 dy, \tag{57}$$

where ω, λ are real positive numbers (parameters of equation (55)), u_0 is a functions given in R^n.

Denotation. $H := L_2(R^n),\ H^1 : W_2^1(R^n)$ is Sobolev's Hilbert space $H^{-1} = (H^1)^*$ is a conjugated space of $H^1; H_\Sigma = \{v \mid v \in H;$ $rv \in H,$ where $r = |x|; \nabla v \in H\}; \|\cdot\|$ is a norm in H, $\|\cdot\|_p$ is a norm in $L_p(R^n),\ p \geq 1; \|\|\cdot\|\|$ is a norm in H^1, $\rho \in (0, \rho_0), \rho_0 = \infty$ for $n = 1,2,\ \rho_0 = \dfrac{4}{n-2}, n \geq 3\ \alpha = 0{,}5\rho n / (2+\rho),\ \theta = \lambda / 4;\ \eta = \lambda / 2$, $\lambda_0 = \min(4, n);\ Q(t) = \dfrac{1}{2}\iint |x-y|^{-\lambda} |u(x,t)|^2 |u(y,t)|^2 dxdy.$

Under repulsive interaction, i.e. for $\omega > 0$ the following theorem is proved.

Theorem 28. *(on global solvability and damping).*

Let $\omega > 0,\ 0 < \lambda < \lambda_0,\ u_0 \in H_\Sigma$ *.Then problem (55)-(57) has a unique global solution* $C^0([0,\infty); H_\Sigma)$ *and for* $\forall t \in [0,\infty)$ *the following estimations are valid:*

1) *for* $\forall n : c(1+t)^{-\lambda} \leq Q(t) \leq c(1+t)^{-\lambda};$

2) *for* $n \geq 3, \quad 2 \leq \lambda < \lambda_0 : c(1+t)^{-\alpha} \leq \|u(t)\|_{\rho+2} \leq c(1+t)^{-\alpha},$
$\|u(\cdot,t)\|_\Omega \leq c(1+t)^{-\max(\alpha,\theta)}, \quad \forall \Omega \subset R^n, mes\Omega < +\infty;$

3) *for* $0 < \lambda < \min(2,n) : c(1+t)^{-\alpha} \leq \|u(\cdot,t)\|_{\rho+2} \leq c(1+t)^{-\alpha\eta},$
$\|u(\cdot,t)\|_\Omega \leq c(1+t)^{\max(\theta,\alpha\eta)}, \forall \Omega \subset R^n,\ mes\Omega < +\infty.$

4) $Q_R(t) \geq c(1+t)^{-\lambda}, \|u(\cdot,t)\|_{L^{\rho+2}(|x| \leq R)} \geq c(1+t)^{-\alpha};$

5) $\varliminf\limits_{t\to+\infty} \|u(\cdot,t)\|_{L^2(|x|\leq ct)} \geq c, \quad \varliminf\limits_{t\to+\infty} \|u(\cdot,t)\|_{L^2(ct \leq |x| \leq ct)} \geq c;$

6) $\varliminf\limits_{t\to+\infty} \left\lfloor t^\alpha \|u(\cdot,t)\|_{L^{\rho+2}(|x|\leq ct)} \right\rfloor \geq c, \quad \varliminf\limits_{t\to+\infty} \left\lfloor t^\alpha \|u(\cdot,t)\|_{L^{\rho+2}(ct\leq|x|\leq ct)} \right\rfloor \geq c;$

7) $\varliminf\limits_{t\to+\infty} \left[t^\lambda Q_{R_1}(t)\right] \geq c, \quad \varliminf\limits_{t\to+\infty} \left[t^\lambda Q_{R_1,R_2}(t)\right] \geq c;$

Here and in the sequel, by c we will denote various positive constants independent of $t, u(x,t)$, *but dependent only in* $u_0, |\omega|, \lambda, n;$ *in* *definition of* $Q_R(t), Q_{R_1}(t), Q_{R_1,R_2}(t)$ – *integration with respect to* x *and* y *in the integral* $\iint |x-y|^{-\lambda}|u(x,t)|^2|u(y,t)|^2 dxdy$ *is conducted in domains:*

$\Omega_R = \{\{x,y\} \in R^n, |x| \le R, |y| \le R\}, \Omega_{R_1} \equiv \Omega_R$ c $R = R_1 = ct,$

$\Omega_{R_1R_2} = \{\{x,y\} \in R^n, R_1 \le |x| \le R_2, R_1 \le |y| \le R_2, R_1 = ct, R_2 = ct\}.$

Consider the case $\omega < 0$, i.e. the attractive interaction. The following theorem is valid.

Theorem 29. *(on global solvability).*

Let $\omega < 0,\ 0 < \lambda < \min(2,n), u_0 \in H^1$. *Then problem (55)-(57) has a unique global solution* $u(x,t)$ *in the class of function* $C^0([0,\infty); H^1)$.

In section 5.3 we study the Cauchy problem for a nonlinear Schrodinger Hartry evolution equation in the critical case

$$iu_t + \Delta u = \omega f(|u|)u \quad in \quad R^n \times R_+, \tag{58}$$

$$u|_{t=0} = u_0(x) \quad in \quad R^n, \tag{59}$$

here

$$f(|u|) = \int |x-y|^{-\lambda}|u(y,t)|^2 dy, \tag{60}$$

where $\omega \in R^1 \setminus \{0\}, \lambda$ is a real positive number u_0 is a function given in R^n.

Problem (58)-(60) is studied in critical value of the parameter $\lambda = 2$ in the case $\omega < 0, n \ge 3$.

Denotation. $H^1 := W_2^1(R^n)$ is Sobolev's Hilbert space, $H^{-1} = (H^1)^*$ is a space conjugated to H^1; $H_\Sigma = \{v \mid v \in L_2(R^n);$ $rv \in L_2(R^n),$ where $r = |x|; \nabla v \in L_2(R^n)\}$; $\|\cdot\|$ is a norm in $L_2(R^n), \|\cdot\|_p$ is a norm in $L_p(R^n), p \ge 1$; $\||\cdot\||$ is a norm in H^1.

Definition 3. *We will call the stationary solutions of equation (58) for* $\omega < 0$ *with a nonlinear term (60) the solution of the form*

$u(x,t) = e^{ikt}\psi(x)$, *where* $k \in R_+, \psi(x)$ *(a real function) is the solution from the class* H^1 *of the following problem:*

$$\Delta\psi - k\psi = \omega\psi\int|x-y|^{-\lambda}|\psi(y)|^2\,dy, \quad x \in R^n,$$

$$\psi(x) \to 0 \quad for \quad |x| \to \infty. \tag{61}$$

Definition 4. *We will denote positive radially-symmetric* $(n \geq 2)$ *solution of problem (61)* $\psi(r)$ *from the class* $C^2([0,\infty))$ *with finite norm* $\|\psi(r)\|$ *by* $\psi_0(r)$ *and call the main state.*

For the state ψ_0 of equation (61) for $k = 1, \omega = -1, \lambda = 2, n \geq 3$ the following relation is valid:

$$\int|\nabla\psi_0|^2\,dx = \frac{1}{2}\iint\frac{|\psi_0(x)|^2|\psi_0(y)|^2}{|x-y|^2}dxdy = \int|\psi_0|^2\,dx.$$

Determine for $\forall v(x) \in H^1(R^n)$ the functional

$$R(v) = \int|\nabla v|^2\,dx - \frac{1}{2}\iint\frac{|v(x)|^2|v(y)|^2}{|x-y|^2}dxdy$$

and the set

$$M = \{v \mid v \in H^1(R^n)/\{0\}, R(v) = 0\}.$$

The following lemma is valid

Lemma 3. *Let* $\psi_0(x)$ *be the main state of equation (61) for* $k = 1, \omega = -1, \lambda = 2, n \geq 3.$ *Then ,*

$$S(\psi_0) = \min_{v \in M} S(v), \quad where \quad S(v) = \int|\nabla v(x)|^2\,dx.$$

In the critical case the following theorems were proved. Without loss of generality in the sequel we will assume $\omega = -1.$

Theorem 30. *Let*

$\lambda = 2, n \geq 3, \omega = -1, \lambda = 2, n \geq 3, \quad u_0 \in H^1(R^n) \setminus \{0\}, ru_0 \in L_2(R^n)$ *and satisfy the condition* $E(u_0) < 0.$

Then the solution $u(t)$ *of problem (58)-(60) blow-up after finite time* $t_{\max}$ *upper bounded by a certain number dependent on the initial function* u_0.

Theorem 31. *Let* $\lambda = 2, \quad n \geq 3, \omega = -1, \ \forall u_0 \in H^1(R^n)$ *satisfy the condition*

$$J(u_0) = \int |\nabla u_0|^2 dx + \int |u_0|^2 dx - \frac{1}{2} \iint \frac{|u_0(x)|^2 |u_0(y)|^2}{|x-y|^2} dxdy < S(\psi_0).$$

Then :

a) if u_0 *satisfies the condition* $R(u_0) < 0$ *and* $ru_0 \in L^2(R^n)$, *then the solution* $u(t)$ *of problem (58)-(60) blows-up after finite time* $t_{\max}$ *, upper bounded by some number dependent on* u_0*;*

b) if

$$R(u_0) > 0,$$

then the solution of problem (58)-(60) globally exists:

$$u(x,t) \in C([0,\infty); H^1(R^n)) \cap C^1([0,\infty); H^{-1}(R^n)).$$

Theorem 32. *Let* $\lambda = 2, \quad n \geq 3, \omega = -1, \ \forall u_0 \in H^1(R^n)$ *satisfy the conditions*

$$E_0 = \int |\nabla u_0|^2 dx - \frac{1}{2} \iint \frac{|u_0(x)|^2 |u_0(y)|^2}{|x-y|^2} dxdy \geq 0.$$

$$J(u_0) < S(\psi_0).$$

Then the solution $u(t)$ *of problem (58)-(60) globally exists,*

$$u(t) \in C([0,\infty); H^1(R^n)) \cap C^1([0,\infty); H^{-1}(R^n))$$

$$\|\nabla u(t)\| < \|\nabla \psi_0\|, \quad \forall t \in [0,\infty).$$

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CONCLUSIONS

The dissertation work is devoted to studying global solvability and blow up of solutions of the Cauchy problem and also a mixed problem for some Schrodinger Ginsburg-Landau and Schrodinger Hartry nonlinear evolution equations. To this end, some exact integral inequalities are proved, the best constant is calculated and its estimation in one interpolational Sobolev inequality is given.

In the work the following results are obtained:

1. New exact integral inequalities applied to the proof of the entropy inequality, are proved. The entropy inequality is generalized. The Trudinger type inequality that is applicable to Schrodinger nonlinear evolution equation is proved for unbounded domains. Exact constants in some inequalities of mathematical physics are calculated.
2. The exact constants are calculated and their estimates are given in some Sobolev inequalities that are aplicable to the proof of solvability of Schrodinger equations.
3. Sufficient conditions for solvability, blow-up and self-canalization of the solution of the Cauchy problem, and also a mixed problem for some nonlinear Schrodinger evolution equations are obtained.
4. Sufficient conditions of no global solutions are obtained a mixed problem for Schrodinger and also Ginsburg-Landau type nonlinear evolution equation.
5. The global solvability and blow-up solutions of the Cauchy problem is studied for a system of Schrodinger evolution equations with quadratic nonlinearity.

6. Global solvability and blow-up, damping of the solutions of the Cauchy problem is studied for Schrodinger-Hartry evolution equation in critical and supercritical cases.

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1. Насибов Ш.М. О численном выделении ограниченных решений систем линейных дифференциальных уравнений в частных производных эволюционного типа, Ж. вычисл. матем. и матем. физ., 17:1 (1977), 119–135
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