ANNUAL REPORT OF 2022 ON SCIENTIFIC AND SCIENTIFIC-ORGANIZATIONAL ACTIVITY OF THE DEPARTMENT OF "EQUATIONS OF MATHEMATICAL PHYSICS" OF INSTITUTE OF MATHEMATICS AND MECHANICS OF ANAS

In the department of "Equations of Mathematical Physics" 12 workers, 9 of whom are research workers. Of them 3 **doctors of sciences, professors:**

1. Akhundov Adalat Ya. – chief researcher associate, (full time).
2. Mammadov Farman I. – chief researcher associate, (full time).
3. Kerimov Nazim B. – chief researcher associate, (a part time).

5 doctors of philosophy in mathematics:

4. Guliyev Abdurrahim F. – head of department, leading researcher associate, (full time).
5. Bagirov Shirmail H. – leading researcher associate, ass. prof., (a part time).
6. Mammadov Elchin M. – leading researcher associate, ass. prof., (full time).
7. Shukurova Shahla Yu. – senior researcher associate, (a part time).
8. Hasanova Aynur H. – senior researcher associate, ass. prof., (full time).

9. Mammadli Sayali M. – researcher associate, kandidat for a degru, (full time).

3 laboratory assistants:

10. Mustafayeva Lala M. – laboratory assistant, (full time).
11. Abdullayeva Aydan J. – laboratory assistant, doctoral student, (full time).
12. Jabrailova Aynur F. – laboratory assistant, doctoral student, (a part time).

I. SCIENTIFIC PART.

In 2022, according to the approved plan, the department conducts 7 research works on the topic "Solvability of initial-boundary value problems for various types of differential equations, qualitative properties of solutions and their applications".

Work 1: "Qualitative properties of solutions to second-order parabolic equations with Hölder coefficients".

Executor: head of department A.F. Guliyev

During the reporting period, growth type theorems were proved for positive solutions of second-order parabolic equations in terms of parabolic capacity and parabolic potential, reflecting the qualitative properties of positive solutions. The results obtained were used to obtain a Wiener-type boundary point regularity criterion for the case of the Dirichlet problem for the heat equation in symmetric domains.

Published papers:

1. Abdurrahim F. Guliyev, Shirmail G. Bagirov, *Absence of global solutions of semilinear biharmonic equation with a singular potential in exterior domains.* **Trans. Nation. Acad. Sci. Azerb. Ser. Phys-tech. Math. Sci. Mathematics**, 42 (1), 2022, pp.72-85.
http://trans.imm.az/volumes/42-1/4201-07.pdf

2. Abdurrahim Farman Guliyev, Aydan Jafar Abdullayeva, *An increase theorem for the positive solutions of the parabolic equation second order.* Republican scientific conference on the topic "Actual problems of mathematics and mechanics", dedicated to the 99th anniversary of the national leader Heydar Aliyev, Baku State University, May 11-13, 2022, p.193.

3. A.F. Guliyev, *The Wiener criterion for heat equation in terms of the potential.* International Scientific Conference on "Modern Problems of Mathematics and Mechanics" dedicated to the 101st anniversary of the birth of Academician Ibragim Ibragimov, Institute of Mathematics and Mechanics of ANAS, Azerbaijan State Pedagogical University, June 29-July 1, 2022, p. 88-90.
http://mpmm.imm.az/wp-content/uploads/2022/06/Abstract.pdf

4. A.F. Guliyev, A.J. Abdullayeva, *A sufficient condition for the regularity of boundary points for solutions of second-order parabolic equations with respect to the Dirichlet problem.* International Scientific Conference on "Modern Problems of Mathematics and Mechanics" dedicated to the 101st anniversary of the birth of Academician Ibragim Ibragimov, Institute of Mathematics and Mechanics of ANAS, Azerbaijan State Pedagogical University, June 29-July 1, 2022, p. 91-92.
http://mpmm.imm.az/wp-content/uploads/2022/06/Abstract.pdf

5. A.F. Guliyev, A.J. Abdullayeva, A.V. Mammadova, *The bilaterial estimates of parabolic potentials with polar value in special domains.* International Scientific Conference on "Modern Problems of Mathematics and Mechanics" dedicated to the 101st anniversary of the birth of Academician Ibragim

Ibragimov, Institute of Mathematics and Mechanics of ANAS, Azerbaijan State Pedagogical University, June 29-July 1, 2022, p. 93-94. http://mpmm.imm.az/wp-content/uploads/2022/06/Abstract.pdf

Work 2: ”Some inverse problems for the hyperbolic heat equation”.

Executor: prof. A.Ya. Akhundov.

A theorem on the uniqueness and stability of the solution of the considered inverse problem for the hyperbolic heat equation is proved.

Published papers:

1. A.Ya. Akhundov, A.Sh. Habibova, On one inverse problem for a hyperbolic equation. **Ukrainian Mathematical Bulletin,** 19 (2022), No. 3, 305-314.

2. Ədalət Axundov, Həmdulla Aslanov, *Riyaziyyatın bəzi tətbiqləri haqqında.* Republican scientific-practical conference on the topic "Some actual problems of mathematical science and teaching of mathematical subjects", dedicated to the 99th anniversary of the birth of national leader Heydar Aliyev, Baku Business University, May 26, 2022, p. 8.

3. Ədalət Yavuz oğlu Axundov, Sahilə Firudin qızı Gülməmmədova, *Elliptik tənlikdə sağ tərəfin tapılması haqqında tərs məsələ.* Republican scientific conference on the topic "Actual problems of mathematics and mechanics", dedicated to the 99th anniversary of the national leader Heydar Aliyev, Baku State University, May 11-13, 2022, p. 25.

4. Adalat Ya. Akhundov and Arasta Sh. Habibova, *On an inverse problem for a parabolic equation in domain with moving boundaries.* International Scientific Conference on "Modern Problems of Mathematics and Mechanics" dedicated to the 101st anniversary of the birth of Academician Ibragim Ibragimov, Institute of Mathematics and Mechanics of ANAS, Azerbaijan State Pedagogical University, June 29-July 1, 2022, p. 32. http://mpmm.imm.az/wp-content/uploads/2022/06/Abstract.pdf

Work 3: ”Study of some qualitative properties of degenerate and nonlinear elliptic and parabolic equations”.

Executors: prof. F.I. Mammadov, Sh.Yu. Shukurova, S.M. Mammadli.

During the reporting period, sufficiency conditions were found for weights in terms of quasimetric spheres for the fulfillment of new non-uniformly degenerate Sobolev-type inequalities. Such results are used in the study of qualitative properties of nonuniform elliptic equations.

In addition, in the work "Harnack's inequality for solving elliptic equations with double divergence" elliptic equations with double divergence were considered.

$$\sum_{i,j=1}^{n} \frac{\partial^2}{\partial x_i \partial x_k} \big(a_{ik}(x)\, u \big) = 0, \ \ x \in \Omega \subset R_n \tag{1}$$

Here the coefficients $a_{ik}(x)$ are continuous and satisfy the Cordes condition.

Speaking about the solution of equation (1), we mean the function $u(x) \in \Omega$ for which

$$\sum_{i,k=1}^{n} \int_{\Omega} u(x)\, a_{ik}(x) \frac{\partial^2 \varphi}{\partial x_i \partial x_k}\, dx = 0 \tag{2}$$

for any function $\varphi \in C^1(\overline{\Omega}) \cap C^2(\Omega)$ satisfying the condition

$$\varphi\big|_{\partial\Omega} = 0 \ , \ \ \frac{\partial \varphi}{\partial \nu}\bigg|_{\partial\Omega} = 0 \ .$$

In this paper, we study Harnack's inequality for a positive solution of Eq. (1).

Published papers:

1. F. Mamedov, J. Gasimov, *Positive Solutions of Nonuniformly Elliptic Equations with Weighted Convex-Concave Nonlinearity*. **Mathematical Notes** 112 (1), 251-270, 2022.
https://link.springer.com/article/10.1134/S0001434622070288

2. Farman Mamedov, Nazire Memmedzade, *A Steady Flow of The Viscous Compressible Liquid in Vertical Pipe*. **Journal of Advanced Research in Fluid Mechanics and Thermal Sciences** Volume 89, Issue 2, Pages 150-159, 2022.
https://doi.org/10.37934/arfmts.89.2.150159
https://semarakilmu.com.my/journals/index.php/fluid_mechanics_thermal_sciences/article/view/171/86

3. F. Mamedov, *On Harnack inequality and Hölder continuity to the degenerate parabolic equations*. **Journal of Differential Equations** 340, 521-556, 2022.
https://doi.org/10.1016/j.jde.2022.09.002
https://www.sciencedirect.com/science/article/abs/pii/S0022039622005265

4. F.I. Mamedov, V.A. Mamedova, *On Sobolev-Poincare-Friedrichs Type Weight Inequalities.* **Azerbaijan Journal of Mathematics** 2022, 12 (2), pp. 92-108. https://azjm.org/volumes/1202/pdf/1202-6.pdf

5. F. Mamedov, S. Monsurrò, *Sobolev inequality with non-uniformly degenerating gradient.* **Electronic Journal of Qualitative Theory of Differential Equations**, 2022, No. 24, pp. 1-19. https://doi.org/10.14232/ejqtde.2022.1.24 www.math.u-szeged.hu/ejqtde/ https://www.math.u-szeged.hu/ejqtde/p9821.pdf

6. Narmin R. Amanova, Farman I. Mamedov, *On Harnack's inequality for positive solutions of linear elliptic equations with discontinuous coefficients.* **Transactions of National Academy of Sciences**, Azerbaijan, Ser. Phys.-Tech. Math. Sci. Mathematics, 2022, 42 (2), pp. 1-13. http://trans.imm.az/inpress/4204-03.pdf

7. Farman İ. Mamedov and Narmin R. Amanova, *Phragmen-Lindelof theorem for a class of no-uniformly elliptic equations.* **Proceedings of the Institute of Mathematics and Mechanics, National Academy of Sciences of Azerbaijan**, Volume 48, Number 1, 2022, pp. 40-49. https://doi.org/10.30546/2409-4994.48.1.2022.40 http://proc.imm.az/volumes/48-1/48-01-03.pdf

8. Farman Mamedov, *On Local Properties of weak Solutions of degenerate Parabolic Equations.* (İnvited speaker) 5th International E-Conference on Mathematical Advances and Applications, May 11-14, 2022, Istanbul / TURKEY, p. 21. https://2022.icomaas.com/wp-content/uploads/2022/09/ICOMAA-2022-ABSTRACT-BOOK.pdf

9. Farman Mamedov, *Existence of solutions to the space-fractional, degenerate nonlinear Schrodinger equation with nonlinearity.* (Online) International Symposium on Applied Mathematics and Engineering (ISAME 2022), January 21-23, 2022, Istanbul-Turkey, p. 34. https://ntmsci.com/Areas/Conferences/FilesAndImages/19/ISAME-1.pdf

10. S.M. Memmedli, *On the generalized Hardy inequality and the best constant.* International Scientific Conference on "Modern Problems of Mathematics and Mechanics" dedicated to the 101st anniversary of the birth of Academician Ibragim Ibragimov, Institute of Mathematics and Mechanics of ANAS, Azerbaijan State Pedagogical University, June 29-July 1, 2022, p. 148. http://mpmm.imm.az/wp-content/uploads/2022/06/Abstract.pdf

<u>**Work 4:**</u> **"Spectral properties of a class of higher order ordinary differential operators with periodic (antiperiodic) boundary conditions, including the basis property in L_p spaces of systems of eigenfunctions and associated functions".**

Executor: prof. N.B. Kerimov.

In this work, for ordinary differential equations of the fourth order, we consider an eigenvalue problem, two boundary conditions of which include a spectral parameter. This problem describes small bending vibrations of an Euler-Bernoulli beam with a fixed left end fixed and an inertial load at the right end attached by two springs and subjected to a longitudinal force in the cross section.

General characteristics of the location of the eigenvalues of this problem on the real axis (in the complex plane) are given, their repeating powers are determined, the oscillatory properties of the eigenfunctions are studied, asymptotic formulas for the eigenvalues and eigenfunctions are obtained, the properties of the basis property in the space $L_p, 1 < p < \infty$, of subsystems of the system of eigenfunctions and associated functions are studied and the uniform convergence of expansions in Fourier series in these systems is studied.

In another paper, are studied the spectral properties of the differential expression $l_0(y) = (-1)^m y^{(2m)} + q(x)y, \ 0 < x < 1$ and the operator L_0 generated by the boundary conditions $y^{(s)}(1) - y^{(s)}(0) = 0 \ (s = 0,1,...,2m-1)$. Here $m \epsilon N$ and $q(x)$ is an arbitrary complex-valued function from the class

$$L_1^+(0,1) = \left\{ q(x) \epsilon L_1(0,1): \int_0^1 q(t) e^{-2\pi i k t} dt = 0, \ k \leq 0 \right\}.$$

The boundary conditions of the problem under consideration are regular, but not strongly regular. The structure of the system of eigenvalues and the system of root functions is studied. It is shown that the system of root functions of the operator L_0 can have an infinite number of associated functions. It is proved that the system of root functions of this operator, chosen according to a certain rule, forms a basis in the space $L_p(0,1), \ 1 < p < \infty$, and this basis is unconditional in the case $p= 2$.

Published papers:

Опубликованные статьи:

1. Ziyatkhan S. Aliyev, Nazim B. Kerimov, Vuqar A. Mehrabov, *On convergence of spectral expansions for the equation of a vibrating beam, at one end of which*

an elastically fixed inertial load is concentrated. Journal of **Mathematical Analysis and Applications**. (in printing)

2. Н.Б. Керимов, *О спектральных свойствах дифференциальных операторов высокого порядка с периодическими краевыми условиями.* **Differential Equations**. (in printing)

<u>Work 5:</u> "Investigation of the existence of a global solution of equations of elliptic, parabolic types of higher orders with a singular potential".

Executor: ass. prof. Sh.H. Bagirov.

During the reporting period, the problem of the existence in the outer infinite region of the ball of a global solution of a semilinear elliptic equation of the fourth order with a singular potential and a biharmonic Baouendi-Grushin type operator in the main part was studied, a result of the Fujita type was obtained, and a sufficient condition was found to ensure the absence of a global solution. The example shows that the found sufficient estimate is exact. At the same time, the existence of a weak solution to a boundary value problem for a high-order semilinear equation in a bounded domain was proved.

Published papers:

1. Shirmail G. Bagirov, Abdurrahim F. Guliyev, *Absence of global solutions of semilinear biharmonic equation with a singular potential in exterior domains.* **Trans. Nation. Acad. Sci. Azerb. Ser. Phys-tech. Math. Sci. Mathematics**, 42 (1), 2022, pp.72-85.
   http://trans.imm.az/volumes/42-1/4201-07.pdf

2. Bağırov Ş.H., Haşımova. A.R., *Baş hissə biharmonik operator olan yarımxətti elliptik tənlik üçün sərhəd məsələsinin müsbət həllinin varlığı.* Republican scientific conference on the topic "Actual problems of mathematics and mechanics", dedicated to the 99th anniversary of the national leader Heydar Aliyev, Baku State University, May 11-13, 2022, p. 25.

3. Bağırov Ş.H., Cəbrayılova A.F., *Qeyri məhdud oblastda yarım xətti elliptik tənliyin mənfi olmayan həllinin yoxluğu.* Republican scientific conference on the topic "Actual problems of mathematics and mechanics", dedicated to the 99th anniversary of the national leader Heydar Aliyev, Baku State University, May 11-13, 2022, p. 26.

4. Bağırov Ş.H., Quliyeva K.Ə., *Yüksək tərtib yarım xətti elliptik tənlik üçün sərhəd məsələsinin həllinin varlığı.* Republican scientific conference on the topic "Actual problems of mathematics and mechanics", dedicated to the 99th anniversary of the national leader Heydar Aliyev, Baku State University, May 11-13, 2022, p. 27.

5. Bağırov Ş.H., Abbasov V.M., *Yarım xətti parabolik tənlik üçün birinci qarəşıq məsələnin həllinin yoxluğu.* Republican scientific conference on the topic "Actual problems of mathematics and mechanics", dedicated to the 99th anniversary of the national leader Heydar Aliyev, Baku State University, May 11-13, 2022, p. 5.

6. Sh.G. Bagirov, M.J. Aliyev, *On the absence of global solutions of inhomogeneous evolution semilinear inequalities with a biharmonic operator in the main part.* International Scientific Conference on "Modern Problems of Mathematics and Mechanics" dedicated to the 101st anniversary of the birth of Academician Ibragim Ibragimov, Institute of Mathematics and Mechanics of ANAS, Azerbaijan State Pedagogical University, June 29-July 1, 2022, p. 65.
   http://mpmm.imm.az/wp-content/uploads/2022/06/Abstract.pdf

Work 6: **"Investigation of qualitative properties of solutions of one class of nonlinear equations given by linear and nonlinear boundary conditions".**

Executor: ass. prof. E.M. Mammadov.

During the reporting period, for a nonlinear problem posed for equations of the third and fourth orders, results were obtained the stabilization of the solution with respect to *t* under certain smoothness conditions imposed on the nonlinear functions specified in the equation and the boundary condition.

Published papers:

1. Эльчин Муса оглы Мамедов, *Об одной смешанной задаче для одного нелинейного уравнения четвертого порядка.* Republican scientific conference on the topic "Actual problems of mathematics and mechanics", dedicated to the 99th anniversary of the national leader Heydar Aliyev, Baku State University, May 11-13, 2022, p. 298-299.

2. Mamedov E.M., *Behaviour of solutions of nonlinear third order equations with nonlinear boundary conditions.* International Scientific Conference on "Modern

Problems of Mathematics and Mechanics" dedicated to the 101st anniversary of the birth of Academician Ibragim Ibragimov, Institute of Mathematics and Mechanics of ANAS, Azerbaijan State Pedagogical University, June 29-July 1, 2022, p. 131.

http://mpmm.imm.az/wp-content/uploads/2022/06/Abstract.pdf

Work 7: **"Investigation of the existence of a solution to one inverse problem for a second-order parabolic equation".**

Executor: ass. prof. A.H. Hasanova.

The work is devoted to the study of an approximate solution of an inverse problem with a nonlinear Neumann boundary condition for a second-order semilinear equation of parabolic type by the method of successive approximations. The substantiation of the proposed algorithm for the approximate solution of the considered inverse problem and the proof of the theorem on the convergence of the approximate solution to the exact solution are provided.

II. ORGANIZATIONAL ACTIVITY.

Head of the department, Ph.D. Abdurrahim Guliyev is the scientific secretary of the Dissertation Council of the Institute of Mathematics and Mechanics of ANAS, lectures to the master's students of the Institute of Mathematics and Mechanics on the subject "Modern problems of mathematics". Head of the department Abdurrahim Guliyev works as a teacher at Baku State University and in Azerbaijan State Oil and Industry University (ASOIU), is an expert in the SEC on the subject of "mathematics".

Chief researcher associate of the department prof. Adalat Akhundov is the deputy director on Scientific affairs, the head of the Education Department, a member of the Scientific Council, the deputy chairman of the Dissertation Council, a member of the editorial board of the journals Proceedings of Mathematics and Mechanics Institute, "Scientific works" of Baku University for girls. Professor Adalat Akhundov lectures to the master's students of the Institute of Mathematics and Mechanics in the specialties "Differential Equations" and "Equations of Mathematical Physics", works as a professor at Lankaran State University.

Chief researcher associate of the department professor Farman Mammadov is a member of the Dissertation Council, a member of the Expert Council of the HAC (Higher Attestation Commission), a member of the editorial board of the journals Azerbaijan Journal of Mathematics, Proceedings of Mathematics and Mechanics

Institute, Journal of Contemporary Applied Mathematics, Universal Journal of Applied Mathematics, a reviewer of the journal of Mathematical Reviews of American Mathematical Society.

Chief researcher associate of the department professor Nazim Kerimov is a professor at the Khazar University, a member of the editorial board of the journals Proceedings of Mathematics and Mechanics Institute, Transactions issue mathematics of Mathematics and Mechanics Institute, Azerbaijan Journal of Mathematics.

Leading researcher of the department associate professor Shirmail Bagirov lectures to the master's students of the Institute of Mathematics and Mechanics of ANAS on the subject "Nonlinear differential equations", is the deputy dean for scientific affairs of the Faculty of Mechanics and Mathematics of the Baku State University, works as an associate professor at Baku State University and at the National Aviation Academy.

Leading researcher of the department associate professor Elchin Mammadov is a member of the commission for control of the IMM trade union organization, works as an associate professor at Baku State University.

Senior researcher of the department Shahla Shukurova works as a teacher in Azerbaijan State Oil and Industry University (ASOIU).

Senior researcher of the department associate professor Aynur Hasanova is a member of the working group created to use the platform Web of Science of the Clarivate Analytics and collect information.

Classes of master's students of the department are held in accordance with the programs and schedules. Doctoral students and dissertators under the guidance of scientific leaders (Abdurrahim Guliyev, Adalat Akhundov, Farman Mammadov, Shirmail Bagirov) continue their research on the approved topics.

This year, an employee of the department, Shirmail Bagirov, was an opponent on the defense of one thesis.

On February 16, 2022, the leading researcher of the department, associate professor Shirmail Bagirov, at the institute-wide seminar of the Institute of Mathematics and Mechanics of ANAS, made a presentation on the topic "The absence of a global solution to some semilinear elliptic and parabolic equations in an infinite region".

 Head of the department, Abdurrahim Guliyev and professor Farman Mammadov continuing their cooperation with Turkish scientists, expanded their

scientific ties. Also, professor Farman Mammadov continues to cooperate with Italian scientists. In addition, employees of our department cooperate with scientists from Russia, Ukraine, Turkey, America, Sweden and other countries.

Every week, on Wednesdays, under the leadership of the head of the department Abdurrahim Guliyev, and on Mondays, under the leadership of professor Farman Mammadov, the department's seminar on the topic "Modern problems of mathematical physics" is traditionally held.

KONFERENCES

1. Chief researcher associate of the department Professor Farman Mammadov, as a plenary speaker on the topic "Existence of solutions to the space-fractional degenerate nonlinear Schodinger equation with nonlinearity", took part online in the International Symposium "International Symposium on Applied Mathematics and Engineering" (ISAME 2022), jointly organized by Biruni University and BISKA BİLİSİM Company, was held on February 21-23, 2022 in Istanbul, Turkey.
   https://ntmsci.com/Conferences/ISAME2022/PlenarySpeakers

2. The chief researcher of the department, Professor Farman Mammadov spoke online at the 5th International E-Conference on Mathematical Advances and Applications (ICOMAA-2022) with a report on the topic "On Local Properties of weak Solutions of degenerate Parabolic Equations" organized by Yildiz Technical University on May 11-14, 2022 in Istanbul, Turkey.
   https://2022.icomaas.com/wp-content/uploads/2022/06/ICOMAA-2022-ABSTRACT-BOOK.pdf

3. Employees of the department Abdurrahim Guliyev, Adalat Akhundov, Shirmail Bagirov, Elchin Mammadov, Aydan Abdullayeva took part in the Republican scientific conference on the topic "Actual problems of mathematics and mechanics", dedicated to the 99th anniversary of the national leader Heydar Aliyev, held on May 11-13 2022 at Baku State University.

4. On June 29-July 1, 2022, employees of the department Abdurrahim Guliyev, Adalat Akhundov, Shirmail Bagirov, Elchin Mammadov, Sayali Mammadli, Aydan Abdullayeva took part in the International Scientific Conference on the topic "Modern Problems of Mathematics and Mechanics", dedicated to the 101st anniversary of birthday of Academician Ibragim Ibragimov, organized jointly by

the Institute of Mathematics and Mechanics of ANAS and the Azerbaijan State Pedagogical University.

5. Professor Adalat Akhundov, an employee of the department, took part in the Republican scientific-practical conference on the topic "Some actual problems of mathematical science and teaching of mathematical subjects", dedicated to the 99th anniversary of the national leader Heydar Aliyev, which was held on May 26, 2022 at Baku University business.

Thus, in 2022, employees of the department published 26 papers, of which 9 scientific papers (5 abroad), included in journals from the Web of Science and Scopus list, 17 abstracts (2 abroad).

Head of department: **Ph.D. Abdurrahim Guliyev**