

# **HALF-YEAR REPORT OF 2023 ON SCIENTIFIC AND SCIENTIFIC-ORGANIZATIONAL ACTIVITY OF THE DEPARTMENT OF “EQUATIONS OF MATHEMATICAL PHYSICS” OF INSTITUTE OF MATHEMATICS AND MECHANICS OF MINISTRY OF SCIENCE AND EDUCATION OF THE REPUBLIC OF AZERBAIJAN**

In the department of “Equations of Mathematical Physics” 11 workers, 8 of whom are research workers. Of them 3 **doctors of sciences, professors:**

1. Akhundov Adalat Ya. – chief researcher associate, (full time).
2. Mammadov Farman I. – chief researcher associate, (full time).
3. Kerimov Nazim B. – chief researcher associate, (a part time).

## **4 doctors of philosophy in mathematics:**

4. Guliyev Abdurrahim F. – head of department, leading researcher associate, (full time).
5. Bagirov Shirmail H. – leading researcher associate, ass. prof., (a part time).
6. Mammadov Elchin M. – leading researcher associate, ass. prof., (full time).
7. Hasanova Aynur H. – senior researcher associate, ass. prof., (full time).
8. Mammadli Sayali M. – researcher associate, kandidat for a degre, (full time).

## **3 laboratory assistants:**

9. Mustafayeva Lala M. – laboratory assistant, (full time).
10. Abdullayeva Aydan J. – laboratory assistant, doctoral student, (full time).
11. Jabrailova Aynur F. – laboratory assistant, doctoral student, (a part time).

## **I. SCIENTIFIC PART.**

**In 2023, according to the approved plan, the department conducts 7 research works on the topic “Solvability of initial-boundary value problems for various types of differential equations, qualitative properties of solutions and their applications”.**

**Work 1: ”Qualitative properties of the Wiener type of solutions to second-order parabolic equations”.**

**Executor: head of department A.F. Guliyev**

During the reporting period, a new proof of the Wiener criterion for the heat equation was given. In addition, for parabolic equations of second-order, a sufficient condition for the regularity of the boundary point is given in terms of the potential.

**Published papers:**

1. Ə. Quliyev, N. Qəhramanova, M. Kərimov, *Riyaziyyat 11*. Textbook, Radius Publishing, 2023, pp. 320.
2. Abdurrahim F. Quliyev, *A new proof of Wiener's criterion for the heat equation*. Proceedings of the International Conference on the topic "Modern Problems of Mathematics and Mechanics", dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 184-186.  
<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>
3. Abdurrahim F. Guliyev, Aydan J. Abdullayeva, *The sufficient condition for the regularity of the boundary point for the parabolic equation in terms of potential*. Proceedings of the International Conference on the topic "Modern Problems of Mathematics and Mechanics", dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 187-188.  
<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>

**Work 2: "The inverse problem of determining the unknown coefficient depending on a spatial variable in a parabolic equation in domains with a variable boundary".**

**Executor: prof. A.Ya. Akhundov.**

During the reporting period, a theorem on the uniqueness and stability of the solution of the posed inverse problem was proved.

**Published papers:**

1. A.Ya. Akhundov, A.Sh. Habibova, *On the Solvability of an Inverse Problem for a Hyperbolic Heat Equation*. Azerbaijan Journal of Mathematics V. 13, No 1, 2023, pp. 205-215.

<https://azjm.org/volumes/1301/pdf/1301-13.pdf>

2. Adalat Ya. Akhundov, Arasta Sh. Habibova, *On an inverse problem for a parabolic equation in a domain with moving boundaries*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 50-52.  
<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>

**Work 3: ”Qualitative properties of non-uniformly degenerate nonlinear elliptic and parabolic equations”.**

**Executors: prof. F.I. Mammadov, S.M. Mammadli.**

During the reporting period, the Dirichlet problem was considered for degenerate elliptic equations of fractional order in weighted Sobolev spaces of fractional order. In addition, results on the existence of weak solutions for nonuniformly degenerate elliptic and parabolic equations are studied.

The article [To the Weak Solvability of Dirichlet Problem for a Fractional Order Degenerate Elliptic Equation](#) is presented in the **Azerbaijan Journal of Mathematics**. (V. 14, NO 1 (2024))

[http://azjm.org/volumes/1401/abstract/V14I1\\_6.html](http://azjm.org/volumes/1401/abstract/V14I1_6.html)

**Published papers:**

1. Farman Mamedov, *On qualitative properties of some nonuniform elliptic equations*. 6th International HYBRID Conference on Mathematical Advances and Applications, Yıldız Technical University, İstanbul, Turkey, May 10-13, 2023, p. 222.  
<https://2023.icomaas.com/wp-content/uploads/2023/05/ICOMAA-2023-ABSTRACT-BOOKLast-version.pdf>
2. F.I. Mamedov, Kh. Gasimova, *On dirichlet problem for a class of non-uniformly parabolic equations with measure data*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, p. 252.

<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>

3. N.M. Mammadzade, S.M. Mammadli, *On the Dirichlet problem for the non-homogeneous Dirichlet problem for a degenerate fractional order Laplace equation*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, p. 265.

<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>

**Work 4: ”Basis properties of high-order ordinary differential operators with periodic boundary conditions in the spaces  $L_p$  of a system of spectral and eigenfunctions and associated functions”.**

**Executor: prof. N.B. Kerimov.**

The paper investigates the spectral properties of the operator  $L_0$  generated by the differential expression  $l_0(y) = (-1)^m y^{(2m)} + q(x)y$ ,  $0 < x < 1$ , and the boundary conditions  $y^{(s)}(1) - y^{(s)}(0) = 0$  ( $s = 0, 1, \dots, 2m - 1$ ), where  $m \in \mathbb{N}$  and  $q(x)$  is an arbitrary complex-valued function from the class  $L_1^+(0,1) = \left\{ q(x) \in L_1(0,1) : \int_0^1 q(t) e^{-2\pi i k t} dt = 0, k \leq 0 \right\}$ .

The boundary conditions of the problem under consideration are regular, but not strongly regular.

The structure of the system of eigenvalues and the system of root functions is studied. It is shown that the system of root functions of the operator  $L_0$  can have an infinite number of associated functions. It is proved that the system of root functions of this operator, chosen according to a certain rule, forms a basis in the space  $L_p(0,1)$ ,  $1 < p < \infty$ , and this basis is unconditional in the case  $p=2$ .

In addition, during the reporting period, the eigenvalue problem for fourth-order ordinary differential equations with a spectral parameter in two boundary conditions is considered. This problem describes small bending vibrations of an Euler-Bernoulli beam with a fixed left end fixed and an inertial load at the right end attached by two springs and subjected to a longitudinal force in the cross section.

The general characteristics of the location of the eigenvalues of this problem on the real axis (in the complex plane) are given, the orders of their repetition are determined, the oscillatory properties of the eigenfunctions are studied, asymptotic formulas for the eigenvalues and eigenfunctions are given, the basis properties in the

space  $L_p, 1 < p < \infty$ , of subsystems of the system of eigenfunctions and associated functions are studied, and the uniform convergence of expansions in Fourier series in these systems is studied.

Article “*On convergence of spectral expansions for the equation of a vibrating beam, at one end of which an elastically fixed inertial load is concentrated*” presented in **Journal of Mathematical Analysis and Applications**, article “*Minimality conditions for Sturm-Liouville problems with a boundary condition depending affinely or quadratically on an eigenparameter*” is presented in the journal **Filomat** (Faculty of Sciences and Mathematics, University of Nis, Serbia).

### Published papers:

1. Н.Б. Керимов, *О спектральных свойствах дифференциальных операторов высокого порядка с периодическими краевыми условиями*. Differential Equations, 2023, Vol. 59, № 3, pp. 314–332. (Russian version of the article)  
[https://sciencejournals.ru/view-article/?j=deqrus&y=2023&v=59&n=3&a=DeqRus\\_2303314Kerimov](https://sciencejournals.ru/view-article/?j=deqrus&y=2023&v=59&n=3&a=DeqRus_2303314Kerimov)

N.B. Kerimov, *On the spectral properties of high-order differential operators with periodic boundary conditions*. ISSN 0012-2661, Differential Equations, 2023, Vol. 59, No. 3, pp. 312–331. (English version of the article)  
<https://link.springer.com/article/10.1134/S0012266123030035>

2. Y.N. Aliyev, N.B. Kerimov, *Minimality conditions for Sturm-Liouville problems with a boundary condition depending affinely or quadratically on an eigenparameter*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 86-88.  
<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>

**Work 5:** “**Existence of a global solution to a system of semilinear parabolic equations with a singular potential in an infinite domain**”.

**Executor: ass. prof. Sh.H. Bagirov.**

During the reporting period, the existence of positive global solutions of second-order semilinear parabolic equations with periodic coefficients in time, which have a general non-linearity, was studied. Sufficient conditions are found to ensure the

absence of such solutions. The result obtained in the course of the study was published as a conference material and submitted to print as an article.

#### **Published papers:**

1. Sh.G. Bagirov, Aynur F. Jabrailova, *The absence of positive global periodic solution of a second-order semi-linear parabolic equation with time-periodic coefficients*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 139-141.

<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>

**Work 6: ”Investigation of the existence and qualitative properties of solutions of a mixed problem for a class of nonlinear hyperbolic equations”.**

**Executor: ass. prof. E.M. Mammadov.**

During the reporting period, results were obtained on the behavior of the global solution of a 4th order equation with nonlinear boundary conditions. Under certain conditions imposed on the initial data, on the nonlinearity in the equation and the boundary condition, stabilization of the solution with respect to  $t$  is obtained, and the destruction of the solution under certain conditions is shown.

#### **Published papers:**

1. E.M. Mamedov, *Some results on the global behavior for the solution of a nonlinear fourth order equation with nonlinear boundary conditions*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 248-249.

<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>

**Work 7: ”Investigation of the existence of a solution to one inverse problem for a semilinear second-order parabolic equation with a Neumann-type boundary condition”.**

**Executor: ass. prof. A.H. Hasanova.**

The work is devoted to the study of the existence of a solution to one inverse problem with a nonlinear Neumann boundary condition for a second-order semilinear equation of parabolic type. The substantiation of the proposed algorithm for the approximate solution of the considered inverse problem and the proof of the theorem on the convergence of the approximate solution to the exact solution are provided.

#### **Published papers:**

1. Aynur H. Hasanova, *Uniqueness and stability of the solution of the inverse problem for equation of parabolic type*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 196-197.  
<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>

#### **Doctoral students:**

1. **Abdullayeva Aydan (scientific leaders Abdurrahim Guliyev)**
2. **Jabrailova Aynur (scientific leaders Shirmail Bagirov)**

#### **Published abstracts:**

1. Abdurrahim F. Guliyev, Aydan J. Abdullayeva, *The sufficient condition for the regularity of the boundary point for the parabolic equation in terms of potential*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 187-188.  
<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>
2. Sh.G. Bagirov, Aynur F. Jabrailova, *The absence of positive global periodic solution of a second-order semi-linear parabolic equation with time-periodic coefficients*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 139-141.  
<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>



## II. ORGANIZATIONAL ACTIVITY.

Head of the department, Ph.D. Abdurrahim Guliyev is the scientific secretary of the Dissertation Council of the Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, lectures to the master's students of the Institute of Mathematics and Mechanics on the subject "Modern problems of mathematics". Head of the department Abdurrahim Guliyev works as a teacher at Baku State University and in Azerbaijan State Oil and Industry University (ASOIU), is an expert in the SEC on the subject of "mathematics".

Chief researcher associate of the department prof. Adalat Akhundov is the deputy director on Scientific affairs, the head of the Education Department, a member of the Scientific Council, the deputy chairman of the Dissertation Council, a member of the editorial board of the journals Proceedings of Mathematics and Mechanics Institute, "Scientific works" of Baku University for girls. Professor Adalat Akhundov lectures to the master's students of the Institute of Mathematics and Mechanics in the specialties "Differential Equations" and "Equations of Mathematical Physics", works as a professor at Lankaran State University.

Chief researcher associate of the department professor Farman Mammadov is a member of the Dissertation Council, a member of the Expert Council of the HAC (Higher Attestation Commission), a member of the editorial board of the journals Azerbaijan Journal of Mathematics, Proceedings of Mathematics and Mechanics Institute, Journal of Contemporary Applied Mathematics, Universal Journal of Applied Mathematics, a reviewer of the journal of Mathematical Reviews of American Mathematical Society.

Chief researcher associate of the department professor Nazim Kerimov is a professor at the Khazar University, a member of the editorial board of the journals Proceedings of Mathematics and Mechanics Institute, Transactions issue mathematics of Mathematics and Mechanics Institute, Azerbaijan Journal of Mathematics.

Leading researcher of the department associate professor Shirmail Bagirov lectures to the master's students of the Institute of Mathematics and Mechanics of ANAS on the subject "Nonlinear differential equations", is the deputy dean for scientific affairs of the Faculty of Mechanics and Mathematics of the Baku State University, works as an associate professor at Baku State University and at the National Aviation Academy.



Leading researcher of the department associate professor Elchin Mammadov is a member of the commission for control of the IMM trade union organization, works as an associate professor at Baku State University.

Senior researcher of the department associate professor Aynur Hasanova is a member of the working group created to use the platform Web of Science of the Clarivate Analytics and collect information.

Classes of master's students of the department are held in accordance with the programs and schedules. Doctoral students and dissertators under the guidance of scientific leaders (Abdurrahim Guliyev, Adalat Akhundov, Farman Mammadov, Shirmail Bagirov) continue their research on the approved topics.

On February 8, 2023, the head of the department, leading researcher Abdurrahim Guliyev made a report on the topic "Growth-type theorems for non-negative solutions of the heat equation and their applications" at the institute-wide seminar of the Institute of Mathematics and Mechanics.

The head of the department, Abdurrahim Guliyev, was the scientific editor of the textbook "Mathematics 11".

In the current year, the employees of the department prof. Nazim Kerimov, prof. Farman Mammadov, assoc. Shirmail Bagirov were official opponents at the defense of one dissertation.

<https://www.imm.az/exp/wp-content/uploads/2023/05/Gunel-referat-az.pdf>

Head of the department, Abdurrahim Guliyev and professor Farman Mammadov continuing their cooperation with Turkish scientists, expanded their scientific ties. Also, professor Farman Mammadov continues to cooperate with Italian scientists. In addition, employees of our department cooperate with scientists from Russia, Ukraine, Turkey, America, Sweden and other countries.

Every week, on Wednesdays, under the leadership of the head of the department Abdurrahim Guliyev, and on Mondays, under the leadership of professor Farman Mammadov, the department's seminar on the topic "Modern problems of mathematical physics" is traditionally held.

## **KONFERENCES**

1. Employees of the department Abdurrahim Guliyev, Adalat Akhundov, Farman Mammadov, Nazim Kerimov, Shirmail Bagirov, Elchin Mammadov, Aynur Hasanova, Sayali Mammadli, Aydan Abdullayeva, Aynur Jabrailova took part in the International Conference on the topic "Modern problems of mathematics and mechanics", dedicated to the 100th anniversary of the birth of national

leader Heydar Aliyev, held on April 26-28, 2023 at the Institute of Mathematics and Mechanics of the Ministry of Science and Education of the Republic of Azerbaijan.

2. Chief researcher of the department, professor Farman Mammadov made an online report on the topic “On qualitative properties of some nonuniform elliptic equations” at the International Conference “6th International HYBRID Conference on Mathematical Advances and Applications”, organized by the Yildiz Technical University in Istanbul, Turkey, May 10-13, 2023.

*Thus, in the first half of 2023, employees of the department published 13 papers, of which 1 textbook, 2 scientific papers (1 abroad), included in journals from the Web of Science and Scopus list, 10 abstracts (1 abroad).*

**Head of department:**

**Ph.D. Abdurrahim Guliyev**