

**ANNUAL REPORT OF 2023 ON SCIENTIFIC AND SCIENTIFIC-
ORGANIZATIONAL ACTIVITY OF THE DEPARTMENT OF “EQUATIONS
OF MATHEMATICAL PHYSICS” OF INSTITUTE OF MATHEMATICS
AND MECHANICS OF MINISTRY OF SCIENCE AND EDUCATION OF THE
REPUBLIC OF AZERBAIJAN**

In the department of “Equations of Mathematical Physics” 11 workers, 8 of whom are research workers. Of them 3 **doctors of sciences, professors:**

1. Akhundov Adalat Ya. – chief researcher associate, (full time).
2. Mammadov Farman I. – chief researcher associate, (full time).
3. Kerimov Nazim B. – chief researcher associate, (a part time).

4 doctors of philosophy in mathematics:

4. Guliyev Abdurrahim F. – head of department, leading researcher associate, (full time).
5. Bagirov Shirmail H. – leading researcher associate, ass. prof., (a part time).
6. Mammadov Elchin M. – leading researcher associate, ass. prof., (full time).
7. Hasanova Aynur H. – senior researcher associate, ass. prof., (full time).
8. Mammadli Sayali M. – researcher associate, kandidat for a degru, (full time).

3 laboratory assistants:

9. Mustafayeva Lala M. – senior laboratory assistant, (full time).
10. Abdullayeva Aydan J. – laboratory assistant, (full time).
11. Jabrailova Aynur F. – laboratory assistant, doctoral student, (a part time).

I. SCIENTIFIC PART.

In 2023, according to the approved plan, the department conducts 7 research works on the topic “Solvability of initial-boundary value problems for various types of differential equations, qualitative properties of solutions and their applications”.

Work 1: ”Qualitative properties of the Wiener type of solutions to second-order parabolic equations”.

Executor: head of department A.F. Guliyev

During the reporting period, a new proof of the Wiener criterion for the heat equation was given. In addition, for parabolic equations of second-order, a sufficient condition for the regularity of the boundary point is given in terms of the potential.

Published papers:

1. Ə. Quliyev, N. Qəhramanova, M. Kərimov, *Riyaziyyat 11*. Textbook, Radius Publishing, 2023, 320 pp.
2. Ə. F. Quliyev and others. *Riyaziyyat. SEC-2023*, Collection of tests, Part I (in Azerbaijani and Russian languages), “CN Poliqraf” MMC, 272 pp.
3. Abdurrahim F. Quliyev, *A new proof of Wiener’s criterion for the heat equation*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 184-186.
<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>
4. Abdurrahim F. Guliyev, Aydan J. Abdullayeva, *The sufficient condition for the regularity of the boundary point for the parabolic equation in terms of potential*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 187-188.
<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>

Work 2: ”The inverse problem of determining the unknown coefficient depending on a spatial variable in a parabolic equation in domains with a variable boundary”.

Executor: prof. A.Ya. Akhundov.

During the reporting period, a theorem on the uniqueness and stability of the solution of the posed inverse problem was proved.

Published papers:

1. A.Ya. Akhundov, A.Sh. Habibova, *On the Solvability of an Inverse Problem for a Hyperbolic Heat Equation*. Azerbaijan Journal of Mathematics V. 13, No 1, 2023, pp. 205-215.
<https://azjm.org/volumes/1301/pdf/1301-13.pdf>
2. Adalat Ya. Akhundov, Arasta Sh. Habibova, *On an inverse problem for a parabolic equation in a domain with moving boundaries*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 50-52.
<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>
3. Adalat Ya. Akhundov, *On an inverse problem for an elliptic equation*. Materials of the International scientific conference entitled “Ali Kushchu – the great ambassador of the scientific school of Mirzo Ulugbek”, dedicated to the 620th anniversary of the birth of Ali al-Kushchu and the 80th anniversary of the creation of the Academy of Sciences of Uzbekistan, held at Samarkand State University, September 21-22, 2023, Samarkand, p. 78.
4. Akhundov A.Ya., Habibova A.Sh., *On an inverse problem for a parabolic equation in a domain with moving boundaries*. Materials of the 8th International Scientific Conference “Current Problems of Applied Mathematics and Information Technologies – Al-Khwarizmi 2023”, dedicated to the 105th anniversary of the National University of Uzbekistan, the 1240th anniversary of Musa al-Khwarizmi and the 620th anniversary of Ali al-Kushchu, held at Samarkand State University, September 25-26, 2023, Samarkand, p. 176.
<file:///D:/Abstracts-of-Al-Khwarizmi-2023.pdf>

Work 3: “Qualitative properties of non-uniformly degenerate nonlinear elliptic and parabolic equations”.

Executors: prof. F.I. Mammadov, S.M. Mammadli.

During the reporting period, the Dirichlet problem was considered for degenerate elliptic equations of fractional order in weighted Sobolev spaces of

fractional order. In addition, results on the existence of weak solutions for nonuniformly degenerate elliptic and parabolic equations are studied.

Published papers:

1. Farman Mamedov, *On qualitative properties of some nonuniform elliptic equations*. 6th International HYBRID Conference on Mathematical Advances and Applications, Yıldız Technical University, İstanbul, Turkey, May 10-13, 2023, p. 222.
<https://2023.icomaas.com/wp-content/uploads/2023/05/ICOMAA-2023-ABSTRACT-BOOKLast-version.pdf>
2. [Jasarat Gasimov](#), [Farman Mamedov](#), *On Harnack inequality to the homogeneous nonlinear degenerate parabolic equations*. Cornell University, arXiv.org, 2023, 27 pp.
<https://doi.org/10.48550/arXiv.2310.02026>
<https://arxiv.org/pdf/2310.02026.pdf>
3. [Farman Mamedov](#), [Jasarat Gasimov](#), *On existence of two positive solutions for the nonlinear subelliptic equations involving nonuniformly p -Laplacian*. Cornell University, arXiv.org, 2023, 23 pp.
<https://doi.org/10.48550/arXiv.2303.14774>
<https://arxiv.org/pdf/2303.14774.pdf>
4. F.I. Mamedov, Kh. Gasimova, *On dirichlet problem for a class of non-uniformly parabolic equations with measure data*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, p. 252.
<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>
5. N.M. Mammadzade, S.M. Mammadli, *On the Dirichlet problem for the non-homogeneous Dirichlet problem for a degenerate fractional order Laplace equation*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of

Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, p. 265.

<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>

6. F.I. Mamedov, N.M. Mammadzada, S.M. Mammadli, *To the Weak Solvability of Dirichlet Problem for a Fractional Order Degenerate Elliptic Equation. Azerbaijan Journal of Mathematics*, V. 14, NO. 1 (2024). (in the press)
http://azjm.org/volumes/1401/abstract/V14I1_6.html

Work 4: "Basis properties of high-order ordinary differential operators with periodic boundary conditions in the spaces L_p of a system of spectral and eigenfunctions and associated functions".

Executor: prof. N.B. Kerimov.

The paper investigates the spectral properties of the operator L_0 generated by the differential expression $l_0(y) = (-1)^m y^{(2m)} + q(x)y$, $0 < x < 1$, and the boundary conditions $y^{(s)}(1) - y^{(s)}(0) = 0$ ($s = 0, 1, \dots, 2m - 1$), where $m \in \mathbb{N}$ and $q(x)$ is an arbitrary complex-valued function from the class $L_1^+(0,1) = \left\{ q(x) \in L_1(0,1) : \int_0^1 q(t) e^{-2\pi ikt} dt = 0, k \leq 0 \right\}$.

The boundary conditions of the problem under consideration are regular, but not strongly regular.

The structure of the system of eigenvalues and the system of root functions is studied. It is shown that the system of root functions of the operator L_0 can have an infinite number of associated functions. It is proved that the system of root functions of this operator, chosen according to a certain rule, forms a basis in the space $L_p(0,1)$, $1 < p < \infty$, and this basis is unconditional in the case $p=2$.

In addition, during the reporting period, the eigenvalue problem for fourth-order ordinary differential equations with a spectral parameter in two boundary conditions is considered. This problem describes small bending vibrations of an Euler-Bernoulli beam with a fixed left end fixed and an inertial load at the right end attached by two springs and subjected to a longitudinal force in the cross section.

The general characteristics of the location of the eigenvalues of this problem on the real axis (in the complex plane) are given, the orders of their repetition are determined, the oscillatory properties of the eigenfunctions are studied, asymptotic formulas for the eigenvalues and eigenfunctions are given, the basis properties in the space $L_p, 1 < p < \infty$, of subsystems of the system of eigenfunctions and associated

functions are studied, and the uniform convergence of expansions in Fourier series in these systems is studied.

Article “*On convergence of spectral expansions for the equation of a vibrating beam, at one end of which an elastically fixed inertial load is concentrated*” presented in **Journal of Mathematical Analysis and Applications**, article “*Minimality conditions for Sturm-Liouville problems with a boundary condition depending affinely or quadratically on an eigenparameter*” has been accepted for publication in the journal **Contemporary Mathematics**.

Published papers:

1. Н.Б. Керимов, *О спектральных свойствах дифференциальных операторов высокого порядка с периодическими краевыми условиями*. Differential Equations, 2023, Vol. 59, № 3, pp. 314–332. (Russian version of the article)
https://sciencejournals.ru/view-article/?j=degrus&y=2023&v=59&n=3&a=DeqRus_2303314Kerimov

N.B. Kerimov, *On the spectral properties of high-order differential operators with periodic boundary conditions*. ISSN 0012-2661, Differential Equations, 2023, Vol. 59, No. 3, pp. 312–331. (English version of the article)
<https://link.springer.com/article/10.1134/S0012266123030035>

2. Y.N. Aliyev, N.B. Kerimov, *Minimality conditions for Sturm-Liouville problems with a boundary condition depending affinely or quadratically on an eigenparameter*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 86-88.
<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>

Work 5: “**Existence of a global solution to a system of semilinear parabolic equations with a singular potential in an infinite domain**”.

Executor: ass. prof. Sh.H. Bagirov.

During the reporting period, the existence of positive global solutions of second-order semilinear parabolic equations with periodic coefficients in time, which have a general non-linearity, was studied. Sufficient conditions are found to ensure the

absence of such solutions. The result obtained in the course of the study was published as a conference material and submitted to print as an article.

Published papers:

1. Sh.G. Bagirov, Aynur F. Jabrailova, *The absence of positive global periodic solution of a second-order semi-linear parabolic equation with time-periodic coefficients*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 139-141.

<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>

Work 6: ”Investigation of the existence and qualitative properties of solutions of a mixed problem for a class of nonlinear hyperbolic equations”.

Executor: ass. prof. E.M. Mammadov.

During the reporting period, results were obtained on the behavior of the global solution of a 4th order equation with nonlinear boundary conditions. Under certain conditions imposed on the initial data, on the nonlinearity in the equation and the boundary condition, stabilization of the solution with respect to t is obtained, and the destruction of the solution under certain conditions is shown.

In addition, during the reporting period, for a third-order equation with nonlinearity in the main part, the problem of the destruction of the solution in a finite period of time was considered and a sufficient condition for the destruction of the solution was obtained under certain conditions imposed on the nonlinear functions in the equation and the boundary condition.

Since July of this year, been conducting remote research work with a teacher at the Kokand State Pedagogical Institute of the Republic of Uzbekistan U. Abdurrahmonov via the Internet and as a result of joint work, one scientific article was prepared. In addition, the article “Behavior of solutions for one nonlinear problem” was published in the journal “Scientific Bulletin” of Namangan State University.

Published papers:

1. Мамедов Э.М. *О поведении решений для одной нелинейной задачи*, Scientific Bulletin of NamSU, Namangan, Uzbekistan, ISSN: 2181-0427, 2023, issue 11, pp. 74-78.
https://api.scienceweb.uz/storage/publication_files/1912/14465/65547baf1aa0c_Axborotnoma%202023_11-son.pdf
2. E.M. Mamedov, *Some results on the global behavior for the solution of a nonlinear fourth order equation with nonlinear boundary conditions*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 248-249.
<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>

Work 7: ”Investigation of the existence of a solution to one inverse problem for a semilinear second-order parabolic equation with a Neumann-type boundary condition”.

Executor: ass. prof. A.H. Hasanova.

The work is devoted to the study of the existence of a solution to one inverse problem with a nonlinear Neumann boundary condition for a second-order semilinear equation of parabolic type. The algorithm proposed for the approximate solution of the inverse problem under consideration was justified, and a theorem on the convergence of the approximate solution to the exact solution was proved.

Published papers:

1. Aynur H. Hasanova, *Uniqueness and stability of the solution of the inverse problem for equation of parabolic type*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 196-197.
<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>

Doctoral students:

Jabrailova Aynur (scientific leaders Shirmail Bagirov)

Published abstract:

1. Sh.G. Bagirov, Aynur F. Jabrailova, *The absence of positive global periodic solution of a second-order semi-linear parabolic equation with time-periodic coefficients*. Proceedings of the International Conference on the topic “Modern Problems of Mathematics and Mechanics”, dedicated to the 100th anniversary of the national leader Heydar Aliyev, Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, April 26-28, 2023, Baku, pp. 139-141.
<https://mpmm.imm.az/wp-content/uploads/2023/05/Abstract-Aliyev-100-2023.pdf>

II. ORGANIZATIONAL ACTIVITY.

Head of the department, Ph.D. Abdurrahim Guliyev is the scientific secretary of the Dissertation Council of the Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan. Head of the department Abdurrahim Guliyev works as a teacher at Baku State University and at Azerbaijan State Oil and Industry University (ASOIU), is an expert in the SEC on the subject of “mathematics”.

Chief researcher associate of the department prof. Adalat Akhundov is the deputy director on Scientific affairs, a member of the Scientific Council, the deputy chairman of the Dissertation Council, a member of the editorial board of the journals Proceedings of Mathematics and Mechanics Institute, “Scientific works” of Baku University for girls. Prof. Adalat Akhundov works as a professor at Lankaran State University.

Chief researcher associate of the department prof. Farman Mammadov is a member of the Dissertation Council, a member of the Expert Council of the HAC (Higher Attestation Commission), a member of the editorial board of the journals Azerbaijan Journal of Mathematics, Proceedings of Mathematics and Mechanics Institute, Journal of Contemporary Applied Mathematics, Universal Journal of Applied Mathematics, a reviewer of the journal of Mathematical Reviews of American Mathematical Society. Prof. Farman Mammadov works as a professor at Azerbaijan State Oil and Industry University (ASOIU).

Chief researcher associate of the department prof. Nazim Kerimov is a professor at the Khazar University, a member of the editorial board of the journals Proceedings of Mathematics and Mechanics Institute, Transactions issue mathematics of Mathematics and Mechanics Institute, Azerbaijan Journal of Mathematics.

Leading researcher of the department associate professor Shirmail Bagirov is the deputy dean for scientific affairs of the Faculty of Mechanics and Mathematics of the Baku State University, works as an associate professor at Baku State University and at the National Aviation Academy.

Leading researcher of the department associate professor Elchin Mammadov is a member of the commission for control of the IMM trade union organization, works as an associate professor at Baku State University.

Senior researcher of the department associate professor Aynur Hasanova is a member of the working group created to use the platform Web of Science of the Clarivate Analytics and collect information.

Doctoral students and dissertators of the department under the guidance of scientific leaders (Abdurrahim Guliyev, Adalat Akhundov, Farman Mammadov, Shirmail Bagirov) continue their research on the approved topics.

On February 8, 2023, the head of the department, leading researcher Abdurrahim Guliyev in the all-institute seminar of IMM gave a report on the topic “Growth-type theorems for non-negative solutions of the heat equation and their applications”.

On October 4, 2023, leading researcher of the department associate professor Shirmail Bagirov in the all-institute seminar of IMM gave a report on the topic “Absence of positive global solutions to semilinear elliptic, parabolic equations and systems of equations in external domains”.

On December 6, 2023, chief researcher associate of the department prof. Farman Mammadov in the all-institute seminar of IMM gave a report on the topic “Existence and uniqueness of some problems for elliptic equations, the lowest term of which depends on the gradient”.

The head of the department, Abdurrahim Guliyev, was the scientific editor of the textbook “Mathematics 11”.

Chief researcher of the department prof. Adalat Akhundov was on a scientific trip to the Republic of Uzbekistan on September 17-27, 2023. September 18, 19, 20, 2023 in Tashkent, at the Institute of Mathematics named after. V.T. Romanovsky

Academy of Sciences of the Republic of Uzbekistan, he held meetings with the director, deputy director for scientific work, scientific secretary, got acquainted with the activities of scientific departments, signed an agreement on scientific activity with the director, academician Sh.A. Ayupov. Signed an agreement on joint scientific activities between Samarkand State University (rector prof. R.Yu. Khalmuradov) and the Samarkand branch of the Tashkent University of Information Technologies (director associate prof. Z. Karshiev) and IMM.

Leading researcher of the department, ass. prof. Elchin Mamedov was on a scientific trip to the city of Kokand in the Republic of Uzbekistan on November 4-15, 2023 and lectured at the optional sections of mathematics at the Kokand State Pedagogical Institute.

In the current year, the employees of the department prof. Nazim Kerimov, prof. Farman Mammadov, ass. prof. Shirmail Baghirov were official opponents at the defense of one dissertation.

<https://www.imm.az/exp/wp-content/uploads/2023/05/Gunel-referat-az.pdf>

Head of the department, Abdurrahim Guliyev and prof. Farman Mammadov continuing their cooperation with Turkish scientists, expanded their scientific ties. Also, prof. Farman Mammadov continues to cooperate with Italian scientists. In addition, employees of our department cooperate with scientists from Russia, Ukraine, Turkey, Uzbekistan, America, Sweden and other countries.

Every week, on Wednesdays, under the leadership of the head of the department Abdurrahim Guliyev, and on Mondays, under the leadership of prof. Farman Mammadov, the department's seminar on the topic “Modern problems of mathematical physics” is traditionally held.

KONFERENCES

1. Employees of the department Abdurrahim Guliyev, Adalat Akhundov, Farman Mammadov, Nazim Kerimov, Shirmail Bagirov, Elchin Mammadov, Aynur Hasanova, Sayali Mammadli, Aydan Abdullayeva, Aynur Jabrailova took part in the International Conference on the topic “Modern problems of mathematics and mechanics”, dedicated to the 100th anniversary of the birth of national leader Heydar Aliyev, held on April 26-28, 2023 at the Institute of Mathematics and Mechanics of the Ministry of Science and Education of the Republic of Azerbaijan.

2. Chief researcher of the department prof. Farman Mammadov made an online report on the topic “On qualitative properties of some nonuniform elliptic equations” at the International Conference “6th International HYBRID Conference on Mathematical Advances and Applications”, organized by the Yildiz Technical University in Istanbul, Turkey, May 10-13, 2023.
3. Chief researcher of the department prof. Adalat Akhundov made a scientific report on the topic “On an inverse problem for an elliptic equation” at the plenary session of the International scientific conference “Ali Kushchu – Great Ambassador of the scientific school of Mirzo Ulugbek”, dedicated to the 620th anniversary of the birth of Ali al-Kushchu and the 80th anniversary of the founding of the Academy of Sciences of Uzbekistan, organized at Samarkand State University on September 21-22, 2023.
4. Chief researcher of the department prof. Adalat Akhundov made a scientific report on the topic “On an inverse problem for a parabolic equation in a domain with moving boundaries” at the 8th International Scientific Conference “Current Problems of Applied Mathematics and Information Technologies – Musa al-Khwarizmi 2023”, dedicated to the 105th anniversary of the National University of Uzbekistan, the 1240th anniversary of Musa al-Khwarizmi and the 620th anniversary of Ali al-Kushchu, held on September 25-26, 2023 at Samarkand State University.

Thus, in 2023, employees of the department published 19 articles, of which 1 was a textbook, 1 was a collection of tests, 5 were scientific articles (4 abroad), (2 were published in journals included in the list of the Web of Science database and Scopus), 12 abstracts (3 abroad).

Head of department:

Ph.D. Abdurrahim Guliyev