YEAR REPORT OF 2025 ON SCIENTIFIC AND SCIENTIFIC-ORGANIZATIONAL ACTIVITY OF THE DEPARTMENT OF "EQUATIONS OF MATHEMATICAL PHYSICS" OF INSTITUTE OF MATHEMATICS AND MECHANICS OF MINISTRY OF SCIENCE AND EDUCATION OF THE REPUBLIC OF AZERBAIJAN

In the department of ***"Equations of Mathematical Physics"*** 13 workers, 10 of whom are research workers. Of them 4 **doctors of sciences (3 professors):**

1. Akhundov Adalat Ya. – chief researcher associate, (full time).
2. Mammadov Farman I. – chief researcher associate, (full time).
3. Kerimov Nazim B. – chief researcher associate, (a part time).
4. Bagirov Shirmail H. – leading researcher associate, ass. prof., (a part time).

5 doctors of philosophy in mathematics:

5. Guliyev Abdurrahim F. – head of department, leading researcher associate, (full time).
6. Mammadov Elchin M. – leading researcher associate, ass. prof., (full time).
7. Jafarov Nazim J. – senior researcher associate, (full time).
8. Mammadova Vafa A. – leading researcher associate, ass. prof., (full time).
9. Hasanova Aynur H. – senior researcher associate, ass. prof., (full time).
10. Mammadli Sayali M. – researcher associate, kandidat for a degru, (full time).

3 laboratory assistants:

11. Mustafayeva Lala M. – senior laboratory assistant, (full time).
12. Abdullayeva Aydan J. – laboratory assistant, doctoral student, (full time).
13. Jabrailova Aynur F. – laboratory assistant, doctoral student, (a part time).

I. SCIENTIFIC PART.

In 2025, according to the approved plan, the department conducts 8 research works on the topic "Solvability of initial-boundary value problems for various types of differential equations, qualitative properties of solutions and their applications".

Work 1: "Qualitative properties of solutions of degenerate parabolic equations".

Executor: head of department A.F. Guliyev

During the reporting period, sub- and super-solutions were constructed for a certain class of degenerate parabolic equations. Corresponding to these solutions, relations expressing the growth property of solutions were derived in terms of capacity. Using these relations, necessary and sufficient conditions for boundary point regularity for the Dirichlet problem were found. A report abstract based on the results was published.

Published papers:

1. Abdurrahim F. Guliyev, Aishen V. Mammadova, Aydan J. Namatova, ***On the R-Identity Class of Parabolic Operators of the Second Order***. Abstracts of the XII International Scientific Conference "Modern Problems of Mathematics and Mechanics", September 03-06 2025, Baku, Azerbaijan, pp. 84-86. https://mpmm.imm.az/abstract-2025.pdf

2. Abdurrahim F. Guliyev and others, ***"Mathematics"*** – english translation of the SEC "Mathematics" textbook for Landau School, 531 p. ***(in print).***

Work 2: "The inverse problem of determining the unknown right-hand side, depending on the spatial variable, in a system of Burgers-type equations".

Executor: prof. A.Ya. Akhundov.

During the reporting period, a theorem on the uniqueness and stability of the solution of the posed inverse problem with an additional integral condition for finding an unknown right-hand side was proven. Based on the obtained results, theses for reports (abstracts) and an article were prepared and published.

Published papers:

1. Adalat Akhundov, Arasta Habibova and Nahid Pashaev, ***Inverse Problem for a Burgers Type Parabolic Equation***. International Conference on Management and Control in Solving Engineering Problems (MaCoSEP 2025), BMU, March 13-15 2025, Baku, Azerbaijan, vol. 2, pp. 1-3. https://macosep.cyber.az/2025/papers/04.14.html https://doi.org/10.30546/MaCoSEP2025.089

2. Adalat Ya. Akhundov, Arasta Sh. Habibova, ***On an Inverse Problem for a Parabolic Equation of Burgers Type***. Abstracts of the XII International

Scientific Conference "Modern Problems of Mathematics and Mechanics", September 03-06 2025, Baku, Azerbaijan, pp. 272-273. https://mpmm.imm.az/abstract-2025.pdf

3. Adalat Akhundov, Arasta Habibova, Nahid Pashaev, ***Inverse Problem for a Burgers Type Parabolic Equation***. Journal of Baku Engineering University – Mathematics and Computer Science, 2025, vol. 9, iss. 1, pp. 3-8. https://doi.org/10.30546/09090.2025.210.012 https://mcs.beu.edu.az/articles/17

Work 3: "Qualitative properties of non-uniform elliptic equations of the second order and degenerate parabolic equations".

Executors: prof. F.I. Mammadov, ass. prof. V.A. Mammadova, S.M. Mammadli.

During the reporting period, the qualitative properties of non-uniform second-order elliptic and parabolic equations were studied.

Based on the results obtained, an article was prepared and submitted for publication.

Published papers:

1. Farman Mamedov, ***On the first boundary value problem for the nonlinear elliptic and parabolic equations of second order***. 8th International HYBRID Conference on Mathematical Advances and Applications (ICOMAA 2025), May 07-09 2025, Yildiz Technical University, Istanbul, Turkey, p. 15. https://2025.icomaas.com/wp-content/uploads/2025/06/2025-Icomaa-AbstractBook.pdf

2. Farman Mamedov, Khayala Akhundova, ***On Approximate Solvability of The Nonlinear Non-Uniformly Elliptic Equations with $L^1(D)$ Data***. 8th International HYBRID Conference on Mathematical Advances and Applications (ICOMAA 2025), May 07-09 2025, Yildiz Technical University, Istanbul, Turkey, p. 151. https://2025.icomaas.com/wp-content/uploads/2025/06/2025-Icomaa-AbstractBook.pdf

3. Ferman Mamedov, ***On the L_1 data Dirichlet problem for the nonuniform parabolic equations of second order***. TURAN25 – TURAN-Fundamental Sciences Symposium, June 23-25 2025, Yildiz Technical University, Istanbul, Turkey, p. 31. ***(Invited plenary speakers)***

https://www.researchgate.net/publication/393790108_Structural_Properties_of_the_h_X_Class_of_X-Valued_Harmonic_Functions

4. Farman Mamedov and Nazire Memmedzade, ***To the Flow of Immiscible Viscous Fluids in the Vertical Pipe***. International Congress on New Trends in Mechanics, September 02-05 2025, Azerbaijan State Pedagogical University Baku, Azerbaijan, pp. A4-1-A4-6 (pp. 114-119). ***(Invited plenary speakers)*** https://adpu.edu.az/index.php/az/science/conferences

Work 4: "Some spectral properties of high-order ordinary differential operators with periodic and antiperiodic boundary conditions".

Executor: prof. N.B. Kerimov.

The paper considers an eigenvalue problem for fourth-order ordinary differential equations, in which the spectral parameter appears in two of the boundary conditions. This problem describes the small flexural vibrations of an Euler-Bernoulli beam, the left end of which is fixed, while the right end has an inertial load attached by two springs and is subjected to a longitudinal force.

A general characterization of the location of eigenvalues on the real axis (in the complex plane) is provided, their multiplicity orders are determined, the oscillation properties of eigenfunctions are studied, and asymptotic formulas for the eigenvalues and eigenfunctions are derived.

The basis properties of subsystems of the system of eigenfunctions and associated functions in the space L_p, $1 < p < \infty$, and the uniform convergence of Fourier series with respect to these systems are investigated.

Based on the results obtained, one article is in the process of submission to a journal.

In addition, a 176-page book entitled ***"American Mathematical Competitions (2000–2024)"*** has been prepared for publication (scheduled for publication in the first quarter of next year).

This book is a collection of problems posed in the 2000–2024 AMC 10 competition, which is held as part of the American Mathematical Competition (AMC) for US high school students and is considered one of the most prestigious mathematics olympiads. Results from these olympiads are highly valued internationally. Leading universities such as MIT, Harvard, Stanford, and others give special preference to the winners of these olympiads in their admissions. Since the organization of the Republican Subject Olympiad in Mathematics rounds (up to the final stage) and the problems posed therein comply with AMC standards, this book can be useful for

students in grades 7–11 preparing for mathematics olympiads, as well as for teachers conducting olympiad preparation classes.

Published papers:

1. Ziyatkhan S. Aliyev, Nazim B. Kerimov, Vuqar A. Mehrabov, ***On convergence of spectral expansions for the equation of a vibrating beam, at one end of which an elastically fixed inertial load is concentrated***. Journal of Mathematical Analysis and Applications, **546**:2 (2025), 129349. **(WOS, Scopus), (IF 1.2, SJR 0.86, Q1, CiteScore 2.5, SNIP 1.142)**
   https://doi.org/10.1016/j.jmaa.2025.129349

Work 5: "Existence and asymptotics of global solutions of nonlinear elliptic and parabolic equations and systems of equations, comparison theorems. Finding sufficient conditions to ensure the absence of global solutions of nonlinear elliptic and parabolic equations and systems of equations".

Executor: ass. prof. Sh.H. Bagirov.

During the reporting period, the problem of the existence of a global solution to a system of high-order semilinear parabolic equations with respect to time and a singular potential, including a biharmonic operator in spatial arguments, was studied in an infinite cylinder whose base is the outer part of a compact set. Sufficient conditions were found to ensure the nonexistence of such solutions. Simultaneously, a system of n semilinear parabolic equations with a singular potential was considered. A similar problem was studied for this system, and a critical value for the nonexistence of a global solution was found.

A plenary talk on the obtained results was given at the conference *"Actual Problems of Algebra, Analysis, Topology, and Computational Mathematics"*, held May 30–31 in Tashkent, Uzbekistan. The results were published as an abstract and an article.

Published papers:

1. Bagirov Sh.G., Guluyeva K.A., ***Absence of global solutions of a system of n semilinear parabolic equations with a singular potential in exterior domain***. Abstracts of the International Scientific Conference "Actual Problems of Algebra, Analysis, Topology and Computational Mathematics", Tashkent, May 30-31 2025, pp. 104-105.

2. Shirmayil G. Bagirov, ***Absence of global solutions of a system of higher order semilinear evolution equations with a singular potential in exterior domain***.

Proc. of the Inst. of Math. and Mech., Nat. Acad. of Sci. of Azerb., 2025, vol. 51, iss. 2, pp. 245-258. **(Q2, SJR 0.38, IF 1.045)**
https://proc.imm.az/volumes/51-2/pimm0302.pdf
https://doi.org/10.30546/2409-4994.2025.51.2.4101

Work 6: "Study of global properties of the solution of a mixed problem for one class of nonlinear hyperbolic equations".

Executor: ass. prof. E.M. Mammadov.

During the reporting period, results were obtained on the qualitative properties of a solution for a third-order equation with a nonlinear boundary condition. Under certain conditions imposed on the initial data, nonlinearity in the equation, and boundary condition, the solution stabilized about time and destruction over a finite time interval. At the same time, a result was obtained on the existence of a local solution for a third-order nonlinear equation with strong dissipation. Based on these results, a paper was prepared and submitted for publication.

Published papers:

1. Мамедов Э.М., ***О разрешимости задачи Коши для нелинейного уравнения третьего порядка с сильной диссипацией***. Abstracts of the International Scientific Conference "Actual Problems of Algebra, Analysis, Topology and Computational Mathematics", Tashkent, May 30-31 2025, pp. 257-258.

2. Elchin M. Mamedov, ***On blow up and stabilization of solution for the equation of fourth order with nonlinear boundary conditions.*** Abstracts of the XII International Scientific Conference "Modern Problems of Mathematics and Mechanics", September 03-06 2025, Baku, Azerbaijan, pp. 284-287.
https://mpmm.imm.az/abstract-2025.pdf

Work 7: "Unique weak solvability of the first boundary value problem in paraboloid-type domains for second-order parabolic equations of non-divergent structure of the Gilbarg-Serrin type with the right-hand side belonging to a certain class of generalized functions".

Executor: N.J. Jafarov.

During the reporting period, an estimate for the Green's function of the first boundary-value problem for second-order parabolic equations with divergent structure

and discontinuous coefficients in paraboloid-type domains was proved. At the same time, a criterion for the membership of the solution to the Dirichlet problem in a Hölder space for second-order parabolic equations with divergent structure and discontinuous coefficients was found in paraboloid-type domains.

Based on these results, the article ***"The Green's Function for the Class of Second-Order Parabolic Equations in Paraboloid-Type Domains"*** was submitted for publication.

Work 8: "Existence and uniqueness of a solution to one inverse problem for a second-order parabolic equation with a boundary condition".

Executor: ass. prof. A.H. Hasanova.

During the reporting period, an inverse problem for a second-order parabolic equation was considered. A theorem on the existence and uniqueness of a solution to the problem was proven. A paper based on the results obtained is being prepared for publication.

II. ORGANIZATIONAL ACTIVITY.

Head of the department, Ph.D. Abdurrahim Guliyev is the scientific secretary of the Dissertation Council of the Institute of Mathematics and Mechanics of Ministry of Science and Education of the Republic of Azerbaijan, works at Azerbaijan State Oil and Industry University (ASOIU) and is an expert in the SEC on the subject of "mathematics".

Chief researcher associate of the department prof. Adalat Akhundov is the deputy director on Scientific affairs, a member of the Scientific Council, the deputy chairman of the Dissertation Council, a member of the editorial board of the journals Proceedings of Mathematics and Mechanics Institute, "Scientific works" of Baku University for girls. Prof. Adalat Akhundov works as a professor at Lankaran State University.

Chief researcher associate of the department prof. Farman Mammadov is a member of the Dissertation Council, a member of the Expert Council of the HAC (Higher Attestation Commission), a member of the editorial board of the journals Azerbaijan Journal of Mathematics, Proceedings of Mathematics and Mechanics Institute, Journal of Contemporary Applied Mathematics, Universal Journal of Applied Mathematics, a reviewer of the journal of Mathematical Reviews of American Mathematical Society. Prof. Farman Mammadov works as a professor at Azerbaijan State Oil and Industry University (ASOIU).

Chief researcher associate of the department prof. Nazim Kerimov is a professor at the Khazar University, a member of the editorial board of the journals Proceedings of Mathematics and Mechanics Institute, Transactions issue mathematics of Mathematics and Mechanics Institute, Azerbaijan Journal of Mathematics.

Leading researcher of the department associate professor Shirmail Bagirov is a member of the Expert Council of the HAC (Higher Attestation Commission), the Deputy Dean for Research of the Faculty of Mechanics and Mathematics of the Baku State University, works as an Associate Professor of the Department of "Differential and Integral Equations" at Baku State University.

Leading researcher of the department associate professor Elchin Mammadov is a member of the commission for control of the IMM trade union organization, works as an associate professor at Baku State University and Baku University for Girls.

Senior researcher of the department associate professor Aynur Hasanova is a member of the working group created to use the platform Web of Science of the Clarivate Analytics and collect information.

Doctoral students and dissertators of the department under the guidance of scientific leaders (Abdurrahim Guliyev, Adalat Akhundov, Farman Mammadov, Shirmail Bagirov) continue their research on the approved topics. Under the supervision of Professor Farman Mamedov, graduate student Nazira Mamedzade completed her dissertation for a PhD in mathematics, and the work is currently being submitted for defense.

On April 16, 2025, Leading Researcher of the Department, Doctor of Mathematical Sciences, Associate Professor Shirmail Bagirov, delivered a talk titled ***"Absence of a Global Solution for a System of Semilinear Parabolic Equations in an Infinite Domain"*** at the Institute-wide seminar of the Institute of Mathematics and Mechanics of the Ministry of Science and Education of the Republic of Azerbaijan.

On October 29, 2025, Head of Department, Ph.D. in Physical and Mathematical Sciences Abdurrahim Guliyev, delivered a talk titled ***"On the Life and Scientific Activity of Prof. Ilham Mammadov"*** at a scientific session dedicated to the 70th anniversary of the Corresponding Member of ANAS, Doctor of Physical and Mathematical Sciences, Prof. Ilham Mammadov, at the Institute of Mathematics and Mechanics of the Ministry of Science and Education of the Republic of Azerbaijan.

On November 5, 2025, Chief Researcher of the Department, Prof. Farman Mammadov, delivered a talk titled ***"On the Harnack Inequality for Elliptic and***

Parabolic Equations" at the Institute-wide seminar of the Institute of Mathematics and Mechanics of the Ministry of Science and Education of the Republic of Azerbaijan.

This year, the department's chief researcher, Professor Farman Mamedov, participated as an opponent in two PhD dissertation defenses in mathematics at Baku State University. During the reporting period, Professor Farman Mamedov also reviewed articles submitted to prestigious scientific journals.

Head of the department, Abdurrahim Guliyev and prof. Farman Mammadov continuing their cooperation with Turkish scientists, expanded their scientific ties. Also, Professor Farman Mamedov continues his scientific collaboration with Italian scientists and, throughout the current year, actively participated in scientific seminars at UFAZ, held with the participation of Turkish and Azerbaijani scientists. In addition, employees of our department cooperate with scientists from Russia, Ukraine, Turkey, Uzbekistan, America, Sweden and other countries.

Every week, on Wednesdays, under the leadership of the head of the department Abdurrahim Guliyev and of prof. Farman Mammadov, the department's seminar on the topic ***"Modern problems of mathematical physics"*** is traditionally held.

KONFERENCES

1. Head of the Department Abdurrahim Guliyev presented a report on the topic ***"On the R-Identity Class of Parabolic Operators of the Second Order"*** at the International Scientific Conference *"Modern Problems of Mathematics and Mechanics"*, held at the Institute of Mathematics and Mechanics on September 3–6, 2025.

2. The department's chief researcher, Professor Adalat Akhundov, presented a paper on ***"Inverse Problem for a Burgers Type Parabolic Equation"*** at the International Scientific Conference *"Management and Control in Solving Engineering Problems"* (MaCoSEP 2025), held at Baku Engineering University on March 13–15, 2025. The conference proceedings will be indexed in the Institute of Electrical and Electronics Engineers (IEEE) database, which is part of Scopus and Web of Science.
   https://macosep.cyber.az/2025/Program_MaCoSEP2025_%2812.03.2025%29.pdf?utm_source (s. 31)

3. The department's chief researcher, Professor Adalat Akhundov, presented a paper on ***"On an Inverse Problem for a Parabolic Equation of Burgers Type"*** at the International Scientific Conference *"Modern Problems of Mathematics and Mechanics"*, held at the Institute of Mathematics and Mechanics on September 3–6, 2025.

4. Chief Researcher of the Department, Prof. Farman Mamedov, presented a paper on ***"A first boundary value problem for the non-uniformly degenerate parabolic equations"*** online on May 7–9, 2025, as an invited speaker at the *"8th International HYBRID Conference on Mathematical Advances and Applications"*, organized by Yildiz Technical University in Istanbul, Turkey. ***(Invited plenary speakers)***
https://2025.icomaas.com/invited-speakers/

5. Chief Researcher of the Department, Prof. Farman Mamedov, presented a paper entitled ***"On the L_1 data Dirichlet problem for the nonuniform parabolic equations of second order"*** as an invited speaker at the TURAN25 Fundamental Science Symposium, held at Yildiz Technical University in Istanbul, Turkey, on June 23–25, 2025. ***(Invited plenary speakers)***
https://www.researchgate.net/publication/393790108_Structural_Properties_of_the_h_X_Class_of_X-Valued_Harmonic_Functions

6. The department's chief researcher, Professor Farman Mamedov, presented a paper entitled ***"To the Flow of Immiscible Viscous Fluids in the Vertical Pipe"*** as an invited speaker at the 1st International Scientific Conference *"New Trends in Mechanics"*, held at ASPU, on September 2–5, 2025. ***(Invited plenary speakers)***
https://adpu.edu.az/images/2025/08/29/Programme_ICNTM25_24UMK.pdf (s. 10)
https://adpu.edu.az/index.php/az/science/conferences (s. 26)

7. Leading researcher of the department, Associate Professor Shirmail Bagirov, presented a report online on May 30–31, 2025, entitled ***"Absence of global solutions of a system of n semilinear parabolic equations with a singular potential in exterior domain"*** at the International Scientific Conference *"Actual Problems of Algebra, Analysis, Topology and Computational Mathematics"*, held in Tashkent, Uzbekistan.

8. Leading researcher of the department, Associate Professor Elchin Mamedov, presented a report online on May 30–31, 2025, entitled ***"On the Solvability of the Cauchy Problem for a Third-Order Nonlinear Equation with Strong Dissipation"*** at the International Scientific Conference *"Actual Problems of Algebra, Analysis, Topology and Computational Mathematics"*, held in Tashkent, Uzbekistan.

9. Leading researcher of the department, Associate Professor Elchin Mamedov, presented a paper entitled ***"On blow up and stabilization of solution for the equation of fourth order with nonlinear boundary conditions"*** at the International Scientific Conference *"Modern Problems of Mathematics and Mechanics"*, held at the Institute of Mathematics and Mechanics from September 3–6, 2025.

Thus, in 2025, employees of the department published 13 articles, of which:

3 were scientific articles (1 abroad), 2 of which were published in journals included in the Web of Science and Scopus databases;

10 were abstracts (5 abroad).

4 articles have been submitted for publication.

The book "American Mathematical Competitions (2000–2024)" has been prepared for publication, 176 p. (N. Kerimov) (in print).

"Mathematics" –English translation of the SEC mathematics textbook for Landau School, 531 p. (A. Guliyev) (in print).

The total number of citations to staff articles in 2025 is 108.

Head of department: **Ph.D. Abdurrahim Guliyev**